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Triplane Framework: Redefining Space-time Visualization in Special Relativity

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Abstract

This research delves into the spatial distribution of speed limits and space-time intervals across various axes within three-dimensional and four-dimensional space-time. A novel theoretical model is proposed based on this distribution, aimed at enhancing the visualization and comprehension of four-dimensional space-time. The primary objective of this study is to offer an intuitive approach to visualizing the spacetime continuum and to present an innovative framework for interpreting four-dimensional space-time. Our investigation revealed that speed limits along different spatial axes may not universally adhere to the speed limit constant, c. Furthermore, we uncovered that the invariant space-time interval can be dissected along these axes, leading to the development of the Triplane Framework. This framework interprets spacetime as three intersecting orthogonal planes rather than four dimensions. Additionally, the study explores the mathematical formalism governing component transformations within this Triplane framework. This research contributes fundamental insights into special relativity by elucidating the distribution of speed limits and space-time intervals within three and four-dimensional space-time. Such insights enrich the interpretation of experimental data in fields like astrophysics and cosmology. Moreover, the proposed Triplane Space-time Framework offers a fresh perspective for visualizing relativistic phenomena, potentially influencing theoretical frameworks in the field.

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INTRODUCTION

At the heart of special relativity [1-3] lies a revolutionary concept that fundamentally alters our understanding of motion and the structure of the universe: the cosmic speed limit imposed by the speed of light. Albert Einstein's seminal theory, introduced in 1905, challenged centuries-old notions of absolute space and time, proposing instead a dynamic framework where space and time intertwine to form a unified entity known as space-time. Central to this framework is the assertion that the speed of light in a vacuum, denoted as c, serves as an immutable barrier that no object or information can surpass. Unlike classical mechanics [4, 5], where velocities can theoretically exceed any limit, special relativity posits c as an unassailable constraint, transcending all frames of reference. This universal speed limit engenders a multitude of profound implications. Time dilation, perhaps the most famous consequence of special relativity, dictates that as an object approaches the speed of light, time slows relative to an observer at rest. Mathematically, time dilation is expressed by the Lorentz factor [6] γ, given by:

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \tag{1}$$

where v is the velocity of the object. As v approaches c, γ approaches infinity, indicating that time dilation becomes increasingly significant at relativistic speeds.

Furthermore, length contraction underscores the relativistic effects, compressing the spatial dimensions of moving objects along their direction of motion. Mathematically, length contraction is described by the Lorentz contraction formula:

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2} \tag{2}$$

where L_0 is the proper length of the object (i.e., the length measured in its rest frame) and L is the contracted length observed in a frame moving relative to the object with velocity v. As v approaches c, the contracted length L approaches zero, indicating that objects moving at relativistic speeds appear highly contracted in the direction of motion.

Experimental verification of these predictions has solidified the status of the speed of light as an absolute cosmic speed limit [7, 8]. Observations of highenergy particles, such as cosmic rays, corroborate the relativistic effects predicted by Einstein's theory, affirming the invariance of c across diverse physical phenomena. Moreover, the cosmic speed limit permeates our understanding of the cosmos, influencing the behaviour of stars, galaxies, and the very fabric of spacetime itself. As humanity ventures into the depths of the universe, grappling with the mysteries of black holes [9], gravitational waves [10], and the nature of dark matter [11], the elucidation of this fundamental constraint remains paramount, guiding our exploration of the cosmic frontier and informing the boundaries of scientific inquiry.

The space-time interval is a foundational concept in the theory of special relativity, offering a unified perspective on the geometry of space and time [12]. Introduced by Hermann Minkowski in 1908 [13], it represents the invariant measure of separation between two events in the four-dimensional continuum of space-time. In the classical framework of Newtonian physics, space and time are treated as separate and absolute entities. However, the advent of special relativity shattered this Newtonian paradigm, revealing that space and time are intimately interconnected. According to Einstein's theory, observers in relative motion will perceive both spatial and temporal dimensions differently, leading to phenomena such as time dilation and length contraction.

The space-time interval provides a rigorous mathematical framework to account for these relativistic effects. It is defined as:

$$(\Delta S)^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \tag{3}$$

Where Δt represents the temporal separation between events, Δx , Δy , and Δz denote the spatial separations along the respective axes, and c is the speed of light in a vacuum.

The classification of space-time intervals into three types – spacelike, timelike, and lightlike – provides further insights into the causal structure of relativistic events. Spacelike intervals occur between events that are separated by a distance greater than the time interval between them. In such cases, no causal influence can propagate between the events at speeds less than that of light. Timelike intervals, on the other hand, signify events with a temporal separation greater than their spatial separation, allowing for causal connections within the light cone of a given event. Lightlike intervals represent events connected by a ray of light, where the interval is zero. Understanding the space-time interval is crucial for elucidating various phenomena in special relativity, including the twin paradox [14], relativistic velocities, and the geometry of spacetime around massive objects [15]. By incorporating both temporal and spatial dimensions into a single metric, the space-time interval offers a comprehensive framework for modeling the fabric of the universe and unraveling its most profound mysteries.

The paper presents a detailed investigation into the distribution of speed limits and space-time intervals in both three-dimensional and four-dimensional spacetime. It begins by focusing on motion in three dimensions (t, x, and y), considering a particle moving with a constant velocity in a specific direction. The speed limits along the x and y axes are derived, taking into account the particle's angle of motion relative to each axis. Space-time intervals along these axes are then analyzed, highlighting the interplay between spatial and temporal dimensions in relativistic physics. The paper then extends its analysis to the four-dimensional spacetime, introducing the z axis alongside t, x, and y. Speed limits and space-time intervals along each axis are discussed, emphasizing the role of angles of motion in determining these parameters. Furthermore, the impact of acceleration on speed limit distribution is explored. Finally, the paper proposes a comprehensive theoretical framework called the Triplane Space-time Framework, which offers a novel visualization of space-time as three intersecting planes representing different spatial dimensions and time. This framework is shown to maintain consistency with established coordinate systems and mathematical processes in relativistic physics. The regions of 3-dimensional space-time enclosed by consecutive planes are named, providing a refined theoretical model for understanding space-time in special relativity. Finally, the mathematical formalism for the Triplane Space-time Framework is established

METHOD

Distribution of Speed Limit and Space-time Interval in Three-dimensional Spacetime

To simplify our analysis, we initially restrict our investigation to three dimensions of spacetime: t, x, and y . Let us consider a particle moving with a constant velocity, denoted by v, along a specific direction, with no velocity along the z direction. Consequently, we confine our examination to the spatial dimensions x and y. According to the principles of special relativity [16], the particle is subject to a spatial speed constraint equivalent to the speed of light, denoted by c, in its direction of motion. However, if the particle possesses velocities in both the x and y directions, the spatial speed limit in these axial directions cannot be identical. If the particle were to maintain a speed limit of c in both the x and y axial directions, its speed limit in the direction of motion would exceed c, violating the fundamental tenets of relativity. Consequently, the speed limits along the axial directions must be lower than the speed of light c.

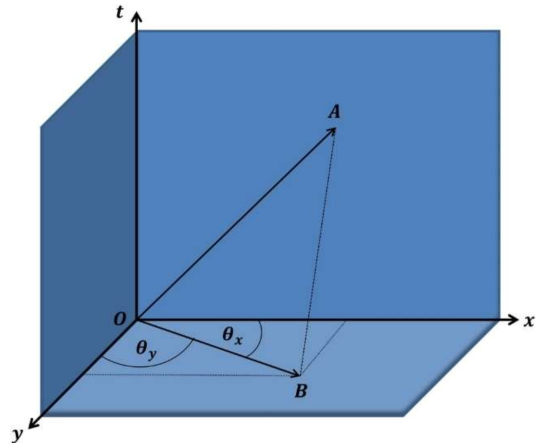

Figure 1. The particle's trajectory in 3-dimensional space-time, represented by line OA, is shown (Z-axis excluded). Its projection onto the X-Y plane, shown as line OB, forms an angle θ_X with the X-axis and an angle θ_Y with the Y-axis. These are, respectively, the angles the particle's trajectory makes with the X- and Y-axes.

Along the X-axis

Consider the trajectory of the particle in relation to the x-axis, forming an angle θ_x. In this scenario, the particle's speed is constrained by the component of the speed along the x-axis, denoted as v_x. According to the principles of special relativity, the maximum attainable speed along the x-axis is determined by the velocity of light c, modulated by the cosine of the angle θ_x. Mathematically, this is expressed as $c\cos\theta_x$. The velocity v_x of the particle along the x-axis falls within the range $-c\cos\theta_x < v_x < c\cos\theta_x$. This range ensures that the particle's velocity remains below the speed of light c along the x-axis, as exceeding this limit would violate the constraints imposed by relativity.

Space-time Interval Along the X-axis

The space-time interval along the x-axis, denoted as $(\Delta S_x)^2$, can be delineated by considering the temporal and spatial components of the particle's motion. Employing the principles of special relativity, the interval is expressed as:

$$(\Delta x)^2 - (c\, cos\, \theta_x\, \Delta t)^2 = (\Delta S_x)^2 \quad (4)$$

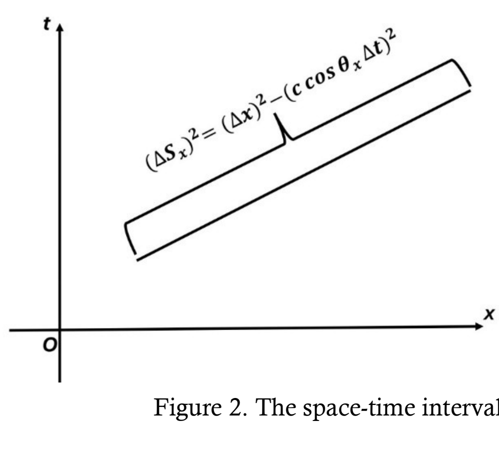

Figure 2. The space-time interval for the X-axis.

Here, the term $(c\cos\theta_x\,\Delta t)^2$ accounts for the temporal dilation effect induced by the particle's velocity along the x-axis. It represents the distortion in the temporal dimension caused by the relativistic motion of the particle. This formulation underscores the fundamental interplay between spatial and temporal dimensions in relativistic physics, wherein the velocity of light acts as a universal speed limit.

Along the Y-axis

When the particle's velocity forms an angle θ_x with the X-axis, it inherently creates an angle $\theta_y = 90° - \theta_x$ with the Y-axis. This geometric relationship arises from the perpendicularity of the X and Y axes in Euclidean space. Consequently, the speed limit along the Y-axis is determined by the sine of the angle θ_x and is given by $c\sin\,\theta_x$. The velocity component v_y of the particle along the Y-axis lies within the range $-c\sin\,\theta_x < v_y < c\sin\,\theta_x$. This range ensures that the particle's velocity remains within the bounds dictated by special relativity, preventing it from exceeding the speed of light c along the Y-axis.

Space-time Interval Along the Y-axis

Analogous to the treatment along the X-axis, the space-time interval along the Y-axis, denoted as $(\Delta S_y)^2$, can be expressed in terms of the spatial and temporal components of the particle's motion. By applying the principles of special relativity, the interval takes the form:

$$(\Delta y)^2 - (c\,sin\,\,\theta_x\,\Delta t)^2 = (\Delta S_y)^2 \tag{5}$$

Here, the term $(c\sin\,\theta_x\,\Delta t)^2$ accounts for the temporal dilation effect resulting from the particle's velocity along the Y-axis. This formulation illustrates the intricate relationship between spatial and temporal dimensions in relativistic physics, wherein the velocity of light serves as a fundamental constraint on the particle's motion along different axes.

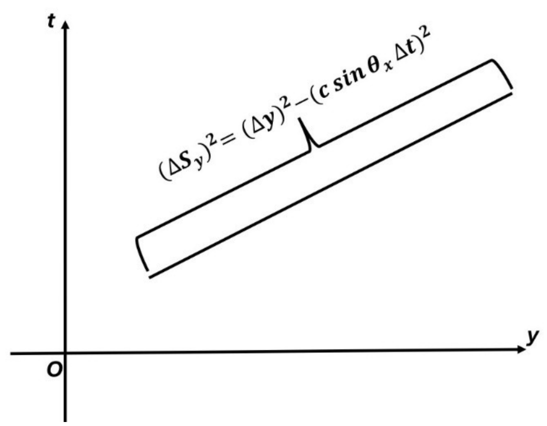

Figure 3. The space-time interval for the Y-axis.

Combined Space-time Interval

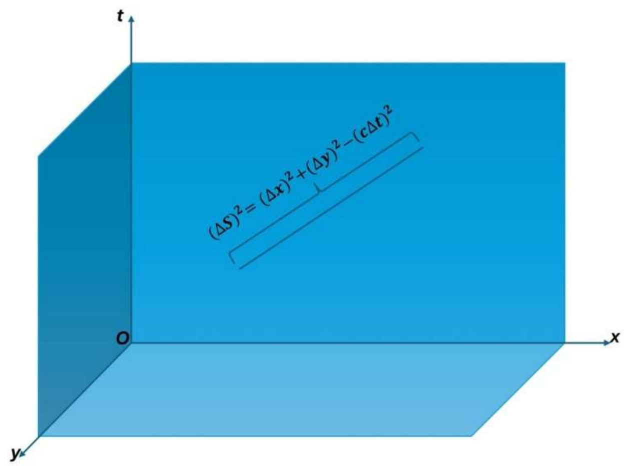

Figure 4. The total space-time interval for 3-dimensional space-time (Z-axis excluded).

Building upon our previous discussions, we have established the space-time intervals along the different axial directions as follows:

$$(\Delta x)^2 - (c\cos\theta_x\,\Delta t)^2 = (\Delta S_x)^2 \tag{6}$$

$$and\ \ (\Delta y)^2 - (c\sin\,\theta_x\,\Delta t)^2 = \left(\Delta S_y\right)^2 \tag{7}$$

Combining the intervals as expressed in equations (6) and (7) yields the total space-time interval, denoted as $(\Delta S)^2$, which is given by:

$$(\Delta S)^2 = (\Delta S_x)^2 + \left(\Delta S_y\right)^2 \tag{8}$$

Alternatively, we can express $(\Delta S)^2$ as:

$$(\Delta S)^2 = (\Delta x)^2 - (c\cos\theta_x\,\Delta t)^2 + (\Delta y)^2 - (c\sin\,\theta_x\,\Delta t)^2 \tag{9}$$

Simplifying further, we obtain:

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 - (c\Delta t)^2 \tag{10}$$

This comprehensive formula encapsulates the space-time interval in three dimensional space-time, excluding the Z-axis component. It demonstrates that the space-time interval comprises two distinct components: one corresponding to motion along the X-axis and another associated with motion along the Y-axis.

Distribution of Speed Limit and Space-time Interval in Four-dimensional Spacetime

Expanding our analysis to encompass the intricacies of four-dimensional spacetime, we introduce the spatial coordinates x, y, and z, alongside the temporal coordinate t. This extension allows for a comprehensive examination of the particle's motion in both space and time.

Speed Limits

In four-dimensional spacetime, the particle's motion along each axis is constrained by the speed of light c, modified by the cosine of the angles it forms with the X, Y, and Z axes. These angles, denoted as θ_x, θ_y, and θ_z respectively, are determined by the particle's spatial displacement components. To derive the cosine of each angle, we employ trigonometry:

$$\cos\theta_x = \frac{\Delta x}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}, \tag{11}$$

$$\cos\theta_y = \frac{\Delta y}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}, \tag{12}$$

$$and\ \cos\theta_z = \frac{\Delta z}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}} \tag{13}$$

So, the speed limits along the X, Y, and Z-axis are $c\cos\theta_x$, $c\cos\theta_y$, and $c\cos\theta_z$, respectively. These expressions provide a precise mathematical representation of the particle's orientation with respect to each spatial axis, enabling the derivation of its speed limits along these directions within the framework of four-dimensional spacetime.

Space-time Intervals

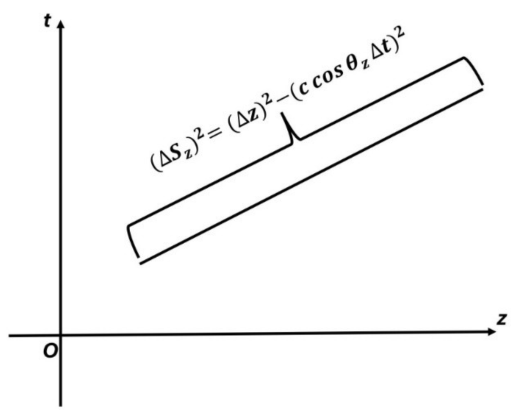

Figure 5. The space-time interval for the Z-axis.

In the realm of four-dimensional space-time, the trajectory of a particle can be comprehensively understood by considering its orientation with respect to the spatial axes (*x*, *y*, and *z*) and the temporal axis (*t*). The angles θ_x, θ_y, and θ_z define the particle's orientation relative to the X, Y, and Z axes, respectively. These angles are crucial in determining the particle's speed limits and the resulting space-time intervals along each axis.

The space-time intervals along the X, Y, and Z axes, denoted as $(\Delta S_x)^2$, $(\Delta S_y)^2$, and $(\Delta S_z)^2$, respectively, can be expressed as follows:

$$(\Delta x)^2 - (c\cos\theta_x \Delta t)^2 = (\Delta S_x)^2, \tag{14}$$

$$(\Delta y)^2 - \left(c \cos\theta_y \, \Delta t\right)^2 = \left(\Delta S_y\right)^2, \tag{15}$$

$$and \ (\Delta z)^2 - (c \cos\theta_z \, \Delta t)^2 = (\Delta S_z)^2 \tag{16}$$

Here, (Δx), (Δy), and (Δz) represent the spatial displacements along the X, Y, and Z axes, respectively, and Δt denotes the temporal displacement. The term $c \cos\theta_i \, \Delta t$ accounts for the temporal dilation effect caused by the particle's velocity along each axis, where i represents X, Y, or Z. Combining these space-time intervals yields the total space-time interval, $(\Delta S)^2$, given by:

$$(\Delta S)^2 = (\Delta S_x)^2 + \left(\Delta S_y\right)^2 + (\Delta S_z)^2 \tag{17}$$

Alternatively, $(\Delta S)^2$ can be expressed as:

$$(\Delta S)^2 = (\Delta x)^2 - (c \cos\theta_x \, \Delta t)^2 + (\Delta y)^2 - (c \sin\, \theta_x \, \Delta t)^2 + (\Delta z)^2 - (c \cos\theta_z \, \Delta t)^2 \tag{18}$$

Upon substitution of the angles θ_x, θ_y, and θ_z into the equations, we arrive at:

$$(\Delta S)^2 = (\Delta x)^2 - \frac{(\Delta x)^2}{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}(c\Delta t)^2 + (\Delta y)^2 - \frac{(\Delta y)^2}{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}(c\Delta t)^2 + (\Delta z)^2 - \frac{(\Delta z)^2}{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}(c\Delta t)^2 \tag{19}$$

Further simplification leads to:

$$(\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - \frac{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}(c\Delta t)^2 \tag{20}$$

$$Or, \qquad (\Delta S)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2 \tag{21}$$

This elaborate equation encapsulates the intricate relationship between spatial displacements and temporal durations in four-dimensional space-time, providing a comprehensive characterization of the space-time interval experienced by the particle.

Constraints on the Speed Limit Distribution under Acceleration

In our previous discussions, we examined the distribution of speed limits for a particle in constant motion. Now, let's delve into how acceleration along a specific axis affects these speed limits. Consider a scenario where we apply acceleration to the particle along the X-axis. We'll explore both positive and negative accelerations and their implications for the distribution of speed limits.

Positive Acceleration Scenario

When we introduce positive acceleration (a_x) along the X-axis, the particle's speed limit along this axis, governed by $c \cos\theta_x$, remains unchanged. This constancy arises because positive acceleration along the X-axis does not directly influence the particle's velocity in the Y and Z axial directions.

However, it's important to note that the particle gaining a velocity higher than $c \cos\theta_x$ along the X-axis would still violate the universal speed limit (c) in its direction of motion. This principle holds for the speed limits along the Y and Z axes as well.

Negative Acceleration Scenario Conversely, when the particle experiences negative acceleration ($-\boldsymbol{a}_x$) along the $\boldsymbol{X}$-axis, the situation changes. Despite negative acceleration not directly affecting the particle's velocity along the $\boldsymbol{Y}$ and $\boldsymbol{Z}$ axes, it can induce modifications in the speed limits along those axes, thereby influencing the overall speed limit distribution. In summary, while positive acceleration along a specific axis preserves the speed limit distribution, negative acceleration introduces complexities. Negative acceleration can indirectly impact the speed limits along other axes, depending on the specific characteristics of the acceleration and the angles involved. These considerations highlight the nuanced relationship between acceleration and the distribution of speed limits in relativistic dynamics.

RESULTS AND DISCUSSION

A Comprehensive Theoretical Framework for Visualizing Space-time in Special Relativity

In the domain of special relativity, particles traverse through the fabric of spacetime with a constant velocity and devoid of any rotational motion. Our analysis reveals a profound insight: the space-time interval, while residing in 4-dimensional space-time, can be conceptually decomposed into three distinct components. This observation serves as the foundation for a novel theoretical model aimed at redefining the visualization of space-time.

Introduction of the Triplane Space-time Framework

Traditional depictions of space-time often involve envisioning it as a seamless continuum in four dimensions. However, our exploration suggests a paradigm shift toward a more intuitive representation: the Triplane Space-time Framework. In this conceptual framework, space-time is envisaged as comprising three intersecting planes, each delineating a spatial dimension along with the temporal dimension. Thus, every plane shares the temporal dimension as a common thread, serving as the intersecting dimension among the three planes. These planes are identified as the $X - t$ plane, the $Y - t$ plane, and the $Z - t$ plane, aligning with the spatial axes X, Y, and Z respectively.

Analogous Framework with Classical Mechanics

Drawing an analogy with classical mechanics, where space is conventionally described as three-dimensional, we adopt a similar approach in our Triplane Space-time Framework. Distances along spatial axes are denoted by Δx, Δy, and Δz, with the total distance represented as Δd:

$$(\Delta d)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \tag{22}$$

While the distances along different axes are observer-dependent, the total distance remains invariant across all observers, illustrating observer independence.

A parallel can be drawn in our Triplane Space-time Framework, where spacetime intervals along the planes are designated as ΔS_x, ΔS_y, and ΔS_z. The total space-time interval, denoted as ΔS, is computed as the sum of the intervals along each plane:

$$(\Delta S)^2 = (\Delta S_x)^2 + \left(\Delta S_y\right)^2 + (\Delta S_z)^2 \tag{23}$$

Similar to classical 3-dimensional space, these intervals along different planes exhibit observer-dependence, while the overall space-time interval maintains uniformity among all observers within the framework of special relativity.

Consistency in Coordinate Assignment and Mathematical Processes

In the transition to a Triplane Space-time Framework for conceptualizing spacetime, it's imperative to underscore the enduring consistency in the assignment of coordinates to events within the framework of special relativity. Despite the shift in conceptualization, the traditional 4-dimensional space-time framework remains intact, ensuring a seamless continuity in theoretical constructs. Events continue to be designated by coordinates such as (t, x, y, z) or in index notation as (x^0, x^1, x^2, x^3), a convention deeply ingrained in the fabric of relativistic physics. This steadfast adherence to established coordinate systems facilitates compatibility and interoperability with existing theoretical frameworks, thereby preserving the integrity of mathematical formalisms and analytical methodologies. Moreover, the preservation of coordinate assignment systems ensures the preservation of mathematical processes inherent to special relativity. Lorentz transformations, interval calculations, and other mathematical operations crucial for elucidating relativistic phenomena remain fundamentally unchanged. This consistency not only streamlines theoretical developments but also reinforces the mathematical rigor essential for the advancement of relativistic physics. Thus, while the conceptual shift towards a Triplane Space-time Framework offers a novel perspective on space-time visualization, the underlying consistency in coordinate assignment serves as a cornerstone for maintaining continuity and coherence in relativistic theory and practice.

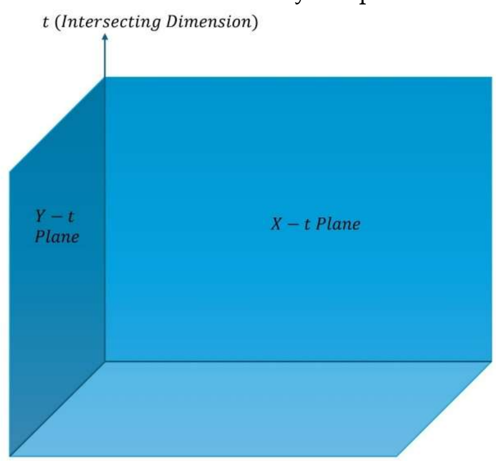

Figure 6. The $X - t$ and $Y - t$ planes and their intersecting $X - Y - t$ 3d space-time.

Nomenclature for 3D Space-time Regions

The regions of 3-dimensional space-time enclosed between consecutive planes are bestowed with distinctive nomenclature:

- The space-time region between the $X - t$ and $Y - t$ planes is designated as the '$X - Y - t$' 3-dimensional space-time.
- Analogously, the region between the $Y - t$ and $Z - t$ planes is identified as the '$Y - Z - t$' 3-dimensional space-time.
- And finally, the space-time region between the $X - t$ and $Z - t$ planes is designated as the '$X - Z - t$' 3-dimensional space-time.

This refined theoretical model not only offers a more intuitive visualization of space-time but also elucidates the intricate interplay between spatial and temporal dimensions within the realm of special relativity.

Implications and Mathematical Formalism

In the preceding section, we introduced the Triplane framework, a conceptual model that redefines the structure of spacetime by conceptualizing it as three orthogonal planes intersecting at the temporal dimension. This framework offers several significant implications. Firstly, it obviates the necessity to explicitly consider a separate temporal direction, as the temporal dimension becomes inherently entwined with the spatial dimensions within each plane. Secondly, it simplifies the description of the spacetime manifold by reducing the number of components needed to characterize it from four to three, aligning with the intuitive spatial intuition of observers. Moreover, the Triplane framework entails that the components used to describe physical phenomena exhibit different transformation behaviours under changes in reference frames. Consequently, it becomes imperative to establish precise mathematical formalisms delineating the transformation rules governing these components across distinct frames of reference. In this section, we embark on precisely this endeavour, elucidating the mathematical underpinnings of component transformations within the Triplane framework, thereby laying the groundwork for a comprehensive understanding of relativistic dynamics within this novel conceptual framework.

In the previous sections, we have only dealt with one observer who observes a particle moving through space with a velocity v. Let's name this observer 'Observer O'. Now we introduce another observer O', who is moving at a velocity of v' in reference to O along the X-axis. We describe the space-time interval components in observer O's reference frame as follows:

$$\Delta S \rightarrow \begin{pmatrix} \Delta S_x \\ \Delta S_y \\ \Delta S_z \end{pmatrix}$$

And we describe the space-time interval components in observer O''s reference frame as follows:

$$\Delta S' \rightarrow \begin{pmatrix} \Delta S_x{}' \\ \Delta S_y{}' \\ \Delta S_z{}' \end{pmatrix}$$

From our previous formulations, we know that:

$$\begin{gathered} (S_x)^2 = (\Delta x)^2 - (c\cos\theta_x\,\Delta t)^2 \\ Or, \quad (S_x)^2 = (v_x\Delta t)^2 - (c\cos\theta_x\,\Delta t)^2 \\ And\ \ (S_x{}')^2 = (\Delta x')^2 - (c\cos\theta_x{}'\,\Delta t')^2 \\ Or, \quad (S_x{}')^2 = (v_x'\Delta t')^2 - (c\cos\theta_x{}'\,\Delta t')^2 \end{gathered} \tag{24}$$

By dividing equation (25) by equation (24) and doing some algebraic calculations, we get:

$$\frac{(S_x{}')^2}{(S_x)^2} = \gamma^2\left(1 - \frac{v'v_x}{c^2}\right)^2\left(\frac{(v_x')^2 - (c\cos\theta_x{}')^2}{(v_x)^2 - (c\cos\theta_x)^2}\right) \tag{25}$$

$$Where\ \gamma = \frac{1}{\sqrt{1 - \left(\frac{v'}{c}\right)^2}} \tag{26}$$

We can find the value of v_x' by applying the relativistic velocity addition formula as shown below:

$$v_x' = \frac{v_x - v'}{1 - \frac{v_x v'}{c^2}} \tag{27}$$

Furthermore, we can find the value of $\cos\theta_x{}'$ by using the formula:

$$\cos\theta_x{}' = \frac{v_x{}'}{\sqrt{(v_x{}')^2 + (v_y{}')^2 + (v_z{}')^2}} \tag{28}$$

Since observer O' has no velocities along the Y and Z axes in reference to observer O, we can say:

$$v_y{}' = v_y\ \ and\ \ v_z{}' = v_z$$

This implies that:

$$\cos\theta_x{}' = \frac{v_x{}'}{\sqrt{(v_x{}')^2 + (v_y)^2 + (v_z)^2}} \tag{29}$$

We can input the value of $v_x{}'$ in this formula to find the value of $\cos\theta_x{}'$. Then we can input the values of these variables in equation (26) to find $(S_x{}')^2$ in terms of $(S_x)^2$.

Now we establish the mathematical relation between $\left(S_y{}'\right)^2$ and $\left(S_y\right)^2$. We know:

$$\left(S_y\right)^2 = (\Delta y)^2 - \left(c\cos\theta_y\,\Delta t\right)^2$$

$$Or, \quad \left(S_y\right)^2 = \left(v_y\Delta t\right)^2 - \left(c\cos\theta_y\,\Delta t\right)^2 \tag{30}$$

And

$$\left(S_y{}'\right)^2 = (\Delta y')^2 - \left(c\cos\theta_y{}'\,\Delta t'\right)^2$$

$$Or, \quad \left(S_y{}'\right)^2 = \left(v_y\Delta t'\right)^2 - \left(c\cos\theta_y{}'\,\Delta t'\right)^2 \tag{31}$$

$$\left[v_y' = v_y\right]$$

By dividing equation (31) by equation (30) and doing some algebraic calculations, we arrive at:

$$\frac{\left(S_y{}'\right)^2}{\left(S_y\right)^2} = \gamma^2\left(1 - \frac{v'v_x}{c^2}\right)^2\left(\frac{\left(v_y\right)^2 - \left(c\cos\theta_y{}'\right)^2}{\left(v_y\right)^2 - \left(c\cos\theta_y\right)^2}\right) \tag{32}$$

$$Where\ \gamma = \frac{1}{\sqrt{1-\left(\frac{v'}{c}\right)^2}}\ \ and\ \cos\theta_y{}' = \frac{v_y}{\sqrt{(v_x{}')^2 + \left(v_y\right)^2 + (v_z)^2}}$$

Following an analogous algebraic calculation, we can establish the mathematical relation between $(S_z{}')^2$ and $(S_z)^2$, which is:

$$\frac{(S_z{}')^2}{(S_z)^2} = \gamma^2\left(1 - \frac{v'v_x}{c^2}\right)^2\left(\frac{(v_z)^2 - (c\cos\theta_z{}')^2}{(v_z)^2 - (c\cos\theta_z)^2}\right) \tag{33}$$

$$Where\ \gamma = \frac{1}{\sqrt{1-\left(\frac{v'}{c}\right)^2}}\ \ and\ \cos\theta_z{}' = \frac{v_z}{\sqrt{(v_x{}')^2 + \left(v_y\right)^2 + (v_z)^2}}$$

Using these transformation formulas, we can transform the components of the space-time interval between different frames of reference.

CONCLUSION

In conclusion, our exploration into the distribution of speed limits and space-time intervals in three-dimensional and four-dimensional spacetime has provided profound insights into the fundamental principles of special relativity. By delving into the spatial constraints imposed on particles moving with constant velocity and negligible motion along specific axes, we have elucidated the intricate relationship between spatial and temporal dimensions. Through rigorous mathematical derivations and theoretical frameworks, we have established a comprehensive understanding of how speed limits and space-time intervals manifest in different dimensions of spacetime.

In the realm of three-dimensional spacetime, our analysis has revealed the nuanced distribution of speed limits along the X and Y axes, dictated by the angles formed by the particle's trajectory. By considering the temporal dilation effects induced by relativistic motion, we have derived precise formulations for space-time intervals along each axis, highlighting the interplay between spatial displacement and temporal duration. Moreover, our examination of the combined space-time interval underscores the holistic nature of relativistic physics, wherein spatial and temporal dimensions intricately intertwine to define the particle's trajectory.

Expanding our investigation to four-dimensional spacetime has further enriched our understanding, introducing the complexities of spatial orientation and speed limits along the Z axis. Through rigorous mathematical formulations and conceptual frameworks, we have extended our analysis to encompass the full spectrum of relativistic dynamics, unveiling the profound implications of acceleration on speed limit distributions.

Our proposition of the Triplane Space-time Framework offers a novel perspective on visualizing spacetime, delineating three intersecting planes to represent spatial dimensions alongside the temporal dimension. This conceptual framework not only facilitates a more intuitive understanding of spacetime but also underscores the enduring consistency in coordinate assignment and mathematical processes within the framework of special relativity.

In essence, our exploration has not only deepened our comprehension of the fundamental tenets of special relativity but has also paved the way for future advancements in theoretical physics. By elucidating the intricate interplay between spatial and temporal dimensions, we have laid the groundwork for a refined theoretical model that offers a comprehensive depiction of spacetime dynamics. Through continued research and exploration, we endeavour to further unravel the mysteries of the universe and unlock new frontiers in relativistic physics.

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