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Decomposition Model and Extension of Spring Pendulum Systems in the 21st Century: A Systematic Literature Review

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Abstract

A spring pendulum is a mechanical system that describes the reciprocating motion of a mass suspended on a spring around an equilibrium point. The system consists of a spring with certain elasticity characteristics and a fulcrum to which the spring is attached. When the mass is pulled from its equilibrium position, the spring generates a restoring force proportional to the distance of displacement, inducing the mass to move periodically. Spring pendulum systems have seen frequent development in the design of mechanical systems that require high-precision vibration control. This study examines developments in system expansion and equation decomposition models of spring pendulums by conducting a Systematic Literature Review (SLR) of 36 pertinent articles from Scopus and Google Scholar (2000–2024). The main focus of the research lies on the expansion of the basic spring pendulum system through various modifications, which is the dominant topic in the review. The most popular technique for resolving the system's equations of motion is breaking down the equations using analytical mathematical formulations, particularly the Lagrangian method, which refers to the theoretical derivation of equations of motion through well-known mathematical frameworks like Lagrangian and Hamiltonian mechanics. The findings provide deep insights for the development of mathematical models as well as a comprehensive overview of the development of spring pendulum research. The implications of this research can contribute to innovations in engineering, physics, and other related disciplines.

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INTRODUCTION

The development of research on spring pendulum systems has undergone significant expansion in recent decades, especially in the 21st century. This is due to their vital role in the analysis of nonlinear dynamics, vibration control, and modeling of more complex mechanical systems (Strogatz, 2018). The study of spring pendulums originated from a simple approach in classical mechanics, which later developed rapidly thanks to advances in computational technology and experimental methods (Thornton & Marion, 2004). When examining spring pendulum systems, scientists usually use two different methodological techniques. Theoretical derivations of equations of motion using well-established mathematical frameworks like Lagrangian mechanics, Hamiltonian formulation, and variational principles are referred to as analytical techniques. In order to comprehend system behavior, these methods place a strong emphasis on theoretical analysis and mathematical formulation. On the other hand, empirical methods emphasize force-based analysis based on Newton's laws, experimental measurement, and hands-on system dynamics observation. When the system experiences oscillations with more complicated degrees of freedom, these techniques provide a methodical and logical perspective on the equations of motion (Timoshenko, n.d.). Theoretical developments and real-world applications in several engineering domains are reflected in this evolution.

A fundamental spring pendulum system generally consists of a mass suspended by a spring, which is capable of oscillatory and pendular motion. Spring pendulums have unique characteristics due to the combination of spring restoring forces and gravitational effects that result in complex dynamics, including harmonic oscillations, sub-harmonics, and chaotic behavior (Nayfeh & Mook, 2000; Strogatz, 2018). To understand this complexity, analytical approaches based on Lagrange equations are widely used because they provide a systematic mathematical basis for formulating the equations of motion. In addition, applications of perturbation theory and advanced numerical methods such as the finite element method are also developed to obtain stable and scalable approximation solutions. However, modern research has developed these basic configurations into more complex systems, including coupled systems (Aprilia & Dwandaru, 2023), electrical transmission devices (Zhang et al., 2015), and vibration control mechanisms (Anh et al., 2007). These developments are driven by technological advancements and increasing demands for efficient energy systems and structural control solutions.

Along with the rapid development of models, research on spring pendulums is also increasingly relevant in the design of mechanical systems that require high-precision vibration control. Based on this need, a comprehensive and systematic review is important to identify research gaps and recent methodological trends. Therefore, this article aims to conduct a Systematic Literature Review (SLR) focusing on key aspects that focus on system expansion and decomposition of spring pendulum equation models in the 21st century. Previous reviews have focused on specific aspects such as energy harvesting (Jiang et al., 2020) or nonlinear dynamics, but a holistic analysis of recent developments is lacking. Thus, a more complete picture of the development of spring pendulum research and its implications for innovation in engineering, physics and other related disciplines is expected (Strogatz, 2018). Through this review, we seek to provide researchers and practitioners with a comprehensive understanding of recent developments and identify promising directions for future research in spring pendulum systems. By doing so, it is hoped that a more complete picture of the development of spring pendulum research and its implications for innovation in engineering, physics, and other related disciplines will be obtained.

Expansion of the spring pendulum system

The expansion of spring pendulum systems has been the focus of research in recent decades, mainly to improve the performance of the system or adapt it to more complex applications. A common form of expansion is to add damping elements to the basic system. This damping can be either damping or nonlinear damping (Nayfeh & Mook, 2000). In addition, combinations of spring pendulum and double pendulum systems are also extensively investigated. These systems offer more complex dynamics and are rich in nonlinear phenomena such as chaos, which can be utilized in vibration control.

Another frequent modification is the introduction of nonlinear springs or springs with varying stiffness characteristics (Sypniewska-Kamińska et al., 2018). Nonlinear springs can produce more varied system responses, including high-harmonic and subharmonic oscillations, which are useful in systems that require flexible dynamic responses.

With these various extensions, the spring pendulum remains a relevant model for studying the dynamics of nonlinear systems, both in theoretical and applied contexts. The development of more advanced numerical and analytical techniques allows for a more in-depth study of these systems, with a focus on an increasingly wide range of practical applications in various scientific and engineering fields.

Decomposing the spring pendulum equation

Decomposing the equation of motion model of the spring-pendulum system is a fundamental step in understanding the dynamics of the system. In general, the spring pendulum equation of motion can be derived using mathematical and physical approaches. Mathematically emphasizing more on formulation and theoretical analysis, the Lagrangian method is widely used because it can simplify the analysis of systems with many degrees of freedom (Goldstein et al., 2002). In a simple spring-pendulum system, the kinetic energy comes from the translational motion of the mass, and the potential energy comes from the deformation of the spring and the force of gravity. For more complex systems such as damped spring pendulums or nonlinear springs, decomposing the equations becomes more difficult. For example, viscous damping adds a nonconservative term to the equations of motion, the solution of which requires specialized numerical or analytical approaches (Nayfeh & Mook, 2000).

Conversely, the empirical approach emphasizes experimental analysis and real-world measurement. This method makes more use of force-based strategies that apply Newton's rules. Decomposing the equations of motion of a spring pendulum is important not only for theoretical analysis but also for practical applications such as the development of vibration control systems and optimization of machine performance. Therefore, the development of methods for decomposing the equations of motion continues to be an active area of research, with the aim of improving the accuracy and efficiency of solving increasingly complex systems.

Based on the analysis conducted in this article, some research questions (RQs) that can be asked to deepen the understanding of the double pendulum system are as follows:

RQ1: How has research on spring pendulum systems progressed from 2000 to 2024?

RQ2: How has the categorization and classification of spring pendulum system research progressed in the scientific literature?

RQ3: What is the evolution of methodological approaches in spring pendulum system research?

METHOD

Literature Search

The literature search for article writing was carried out using two databases, namely Scopus and Google Scholar which were accessed through Publish or Perish. Both database sources were chosen because they have a focus on relevant coverage in the field of physics. The search for SLR focused on article titles or keywords containing the term spring pendulum. The literature search in this study adopted the PRISMA protocol to ensure transparency and traceability of the process. The

Scopus database search was conducted using an advance query: TITLE (“spring pendulum”). On this criterion, 83 related articles were obtained. Meanwhile, for the Google Scholar database accessed through publish or perish with the keyword “Spring Pendulum” in the title of the article with a vulnerable time of 2000-2024 in English, 160 articles were obtained. To clarify the related literature search can be seen in table 1.

Table 1. Literature search

Database	String search	Intial result	Final result
Scopus	TITLE ("spring pendulum")	98	83
Google Scholar	("spring pendulum")	197	160

Inclusion and Exclusion Criteria

In this systematic literature review process, setting inclusion and exclusion criteria is an important stage to ensure the quality and relevance of the articles analyzed. The article selection process is systematic and structured to ensure that the literature selected is in line with the research objectives and objectives. These criteria consider various aspects, ranging from publication quality to content relevance, to ensure the review.

Academic standards and research relevance were used to establish inclusion standards. Articles included in this review should be from publications in high-quality journals indexed in databases such as Scopus and Google Scholar, with a focus on articles published between 2000 and 2024. This timeframe was chosen to ensure broad coverage while considering the validity of the results.The selected articles should prioritize the formation of spring pendulum systems as well as the development of model trends in spring pendulum research in their analysis. Articles should also be written in English to ensure that they can be accessed and understood properly. At the same time, exclusion criteria were established to exclude articles that did not meet the necessary quality and relevance criteria. Conference abstracts, unpublished technical reports, or articles published in predatory journals were not included in the review. Publications that did not provide access to the full text or were not well documented were also excluded from the review. To ensure the application of standards.

For effectiveness, the review process will be conducted in two stages. In the first stage, an initial selection is made based on the title and abstracts that clearly do not meet the basic criteria are immediately excluded. In the second stage, the full text of papers that pass the first stage of review will be evaluated. More detailed and stringent standards apply.This process evaluates each article using a quality scale that considers aspects such as methodology, clarity of presentation, validity of results, and importance of contribution.One of the Scopus screening sources was conducted with the advance query TITLE (“spring pendulum”) AND PUBYEAR > 1999 AND PUBYEAR < 2025 AND (LIMIT-TO (LANGUAGE, “English”)) AND (LIMIT-TO (OA, “all”)). From the screening results that have been carried out, 48 articles were found that match the selection criteria.

Eligible

The next eligibility process involves further examination of the articles that have been obtained. The purpose of this check is to ensure that the articles are related to spring pendulums, cover various extensions of the system, and examine the techniques used. The Eligible process, which is an important stage in the systematic literature review.At the beginning of the process, the researcher conducted a thorough search in two major academic databases, Scopus and Google Scholar.The researcher collected an initial set of articles that might meet the research criteria using complex and organized search strings. This process can be viewed through PRISMA, as shown in figure 1.

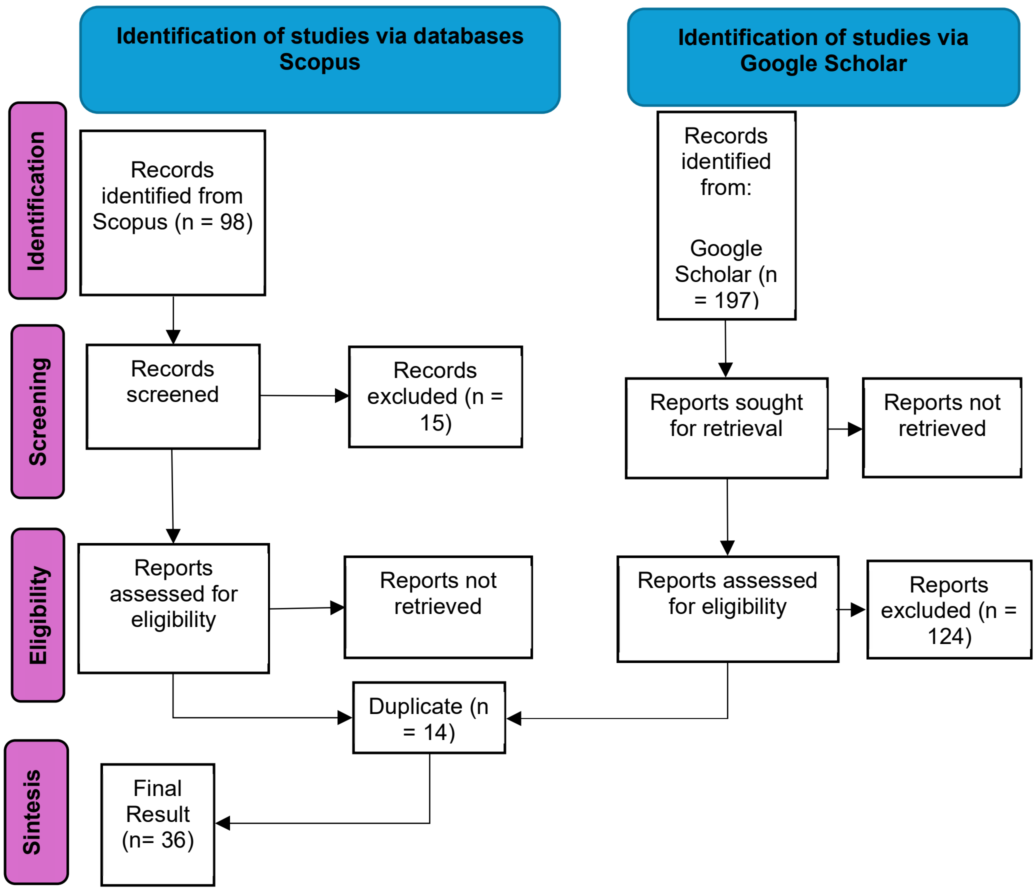

Figure 1. PRISMA Search and selection procedure flow

In the eligibility process of the two source databases, after several series, the final result was 36 articles as further references for the synthesis of the review. The Eligible process in this systematic literature review is not just about selecting articles, but also a method that ensures the integrity and quality of knowledge synthesis. Through a systematic and transparent approach, researchers were able to produce a comprehensive literature review, providing in-depth insight into research developments in the field of spring pendulum systems.

Synthesis

The synthesis process is an important step in a systematic literature review that transforms the selected set of articles into a comprehensive and meaningful study outcome. This phase began with systematic data extraction of the 36 articles, where the researcher systematically collected key information from each included article. Data extraction is not just about gathering information but conducting a thorough search that includes all aspects of the bibliography such as author name, year of publication, journal in which the paper was published, and the organizational context of the study.

The classification is multidimensional and covers the main aspects of system expansion and equation decomposition models on spring pendulums. Each article was examined in detail to identify research patterns, methodological approaches used, and unique contributions to the research field. The synthesis process in this systematic literature review is essentially a systematic attempt to transform a collection of individual articles into a comprehensive and meaningful set of findings. Through a rigorous analytical and synthetic approach, the researchers not only summarize previous research findings but also provide a synthesis of knowledge that paves the way for future scientific research in the field of spring-bulb systems and Lagrangian mechanics

RESULTS AND DISCUSSION

Expansion of the spring pendulum system

Based on a comprehensive analysis of the development of spring pendulum systems from 2000 to 2024, significant progress has been made in various aspects, ranging from system expansion to the development of diverse models. Based on a systematic review of 36 synthesized articles that met the criteria, several important trends and developments were identified that shape the research landscape in this field. As a result of the analysis, several major categories in the development and extension of spring pendulum systems were identified. The results of the systematic analysis of research on spring pendulums revealed 6 major classifications that reflect the development and variation in this field. Among the classifications of articles analyzed, four types of system extension were the dominant topics of discussion. There are 14 articles that examine the extension of the basic spring pendulum system with modifications, and the other three dominant extensions are 5 articles that respectively discuss the extension of the spring pendulum system with damping, the extension of the coupled spring pendulum system, and the spring pendulum system with applications and geometry modifications. A more detailed explanation can be illustrated in Table 2.

Tabel 2. Expansion of the spring pendulum system

Expansion of	Frek	Author
Basic spring pendulum system with modifications	14	(Alasty & Shabani, 2006) ,(Acosta-Zamora et al., 2024), (H. T. J.-S. L. C. X. Xiao, 2019), (J. Xiao et al., 2024), (Gitterman, 2010),(Butikov, 2015), (Awrejcewicz et al., 2016), (Baleanu et al., 2018), (Sypniewska-Kamińska et al., 2018), (Eissa & Sayed, 2008), (Bhattacharyya, 2000), (Pokorny, 2008), (C. de Sousa et al., 2018a), (C. de Sousa et al., 2018b).
Spring pendulum system with damping	5	(Digilov et al., 2005), (Bek et al., 2020), (Bewersdorff & Weiler, 2024), (Wang et al., 2021), (Abohamer et al., 2021).
Coupled & multi-pendulum spring pendulum system	5	(Aprilia & Dwandaru, 2023), (Rini et al., 2023), (Szumiński & Maciejewski, 2024), (Rini & Saefan, 2023), (Rifai et al., 2007).
Application system and geometry modification	5	(T. S. Amer & Bek, 2009), (A. Amer et al., 2023), (Arinstein & Gitterman, 2008), (Sang & Thang, 2008), (Anh et al., 2007).
Engineering and structural application system	3	(Song et al., 2024), (Zhang et al., 2015), (Awrejcewicz, 2014).
Energy application system	4	(He et al., 2022), (Jiang et al., 2020), (T. S. Amer et al., 2024), (Wu et al., 2018).

In this case, the basic spring pendulum system with modifications shows that fundamental research is still an important focus with various variations developed. Not only the basic spring pendulum but also non-linear modifications to rubber material are developed. This system is illustrated in Figure 2.

Figure 2. Basic spring pendulum system

Another dominant aspect is the damping and control system, which is not only a basic damping system but also a modification of the damping system on the transmission tower.It also integrates the use of modern technology with the use of piezoelectricity.Then another dominant category discusses the development of coupled spring systems. The integration of modern technology is reflected in the development of a hybrid system of dampers based on piezoelectric elements, which is a blend of traditional and modern technology.An illustration of this system can be seen in Figure 3.

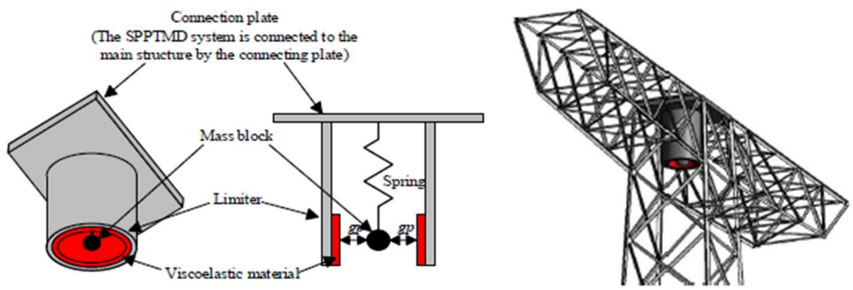

Figure 3. Spring pendulum system on a transmission tower

This pattern of research development shows a significant evolution from basic concepts to more complex and practical applications. The diversification of research focus, from fundamental systems to energy applications, reflects the continued adaptability and relevance of spring pendulum systems in a modern technological context.

Decomposition of the spring pendulum equation

To integrate the expansion of the system that has been developed, it is necessary to develop an appropriate model or method.Therefore, in the existing article, the author has classified the models used with two main approaches, the analytical approach and the empirical approach.In detail, it can be seen in Figure 2.

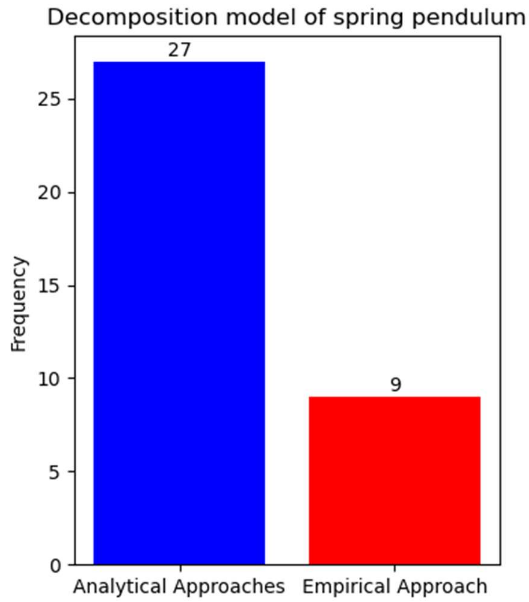

Figure 4. Spring pendulum decompisition models graph

The analytical method places a strong emphasis on theoretical analysis and mathematical formulation. This method predominates in the literature review. The most popular approach turned out to be the Lagrangian formulation. The 27 analytical formulations show that 23 researchers covered more than half of the examined publications and employed Lagrangian formulations. The empirical approach then concentrates on experimental analysis and real-world measurement. This method makes more use of force-based strategies that apply Newton's rules. The 9 empirical approaches show that this approach is used in 6 investigations.

The development of the pendulum system research model shows a spectrum of complexity. Most of the studies in this literature use mathematical models to solve the Equation of Motion. The solution of EOM is dominated by the basic spring pendulum system with modification (Figure 3) using Lagrangian equations. In this system, the EOM is solved by the formula

$$\ddot{r} = (1 + r)\dot{\theta}^2 + g\cos\theta - \omega_r^2 r \quad (1)$$

$$\ddot{\theta} = -\frac{2}{l + r}\dot{r}\dot{\theta} - \omega_\theta^2 \sin\theta \quad (2)$$

With $\omega_r = \sqrt{\frac{k}{m}}$ and $\omega_\theta = \sqrt{\frac{g}{l+r}}$.. The solution of EOM with this formula is found in some literature as written by (Baleanu et al., 2018), (Pokorny, 2008), (H. T. J.-S. L. C. X. Xiao, 2019), (Gitterman, 2010). The use of this equation is one of the simple solutions that often appear.

In addition, there are also some EOMs that are solved with quite complex equations (Alasty & Shabani, 2006), (T. S. Amer & Bek, 2009), (Sypniewska-Kamińska et al., 2018) seperti

$$\ddot{x} + c_1\dot{x} + \omega_1^2 x - (1 + x)\dot{\varphi} + \omega_2^2(1 - \cos\varphi) + (1 + x)^2\ddot{\varphi} + c_2\dot{\varphi} + 2(1 + x)\dot{x}\dot{\varphi} + \omega_2^2(1 + x)\sin\varphi \quad (3)$$

Then, along with the development of mathematical model solving research began to integrate additional factors. For example (Bek et al., 2020), (Eissa & Sayed, 2008), (Wang et al., 2021), (Song et al., 2024)

$$M\left[\ddot{r}-(l+r)\dot{\theta}^2\right]+\left(M_g+F_b\right)(1-\cos\theta)+kr=f_0\cos(\Omega t)-C_1\dot{r}-\frac{1}{2}\rho AC_l(l+r)^2\dot{\theta}^2+\dot{r}^2 \quad (4)$$

$$M[\ddot{\theta}(l+r)^2+(l+r)(2\dot{\theta}\dot{r}+\left(g-\frac{F_b}{M}\right)\sin\theta)]=-\frac{1}{2}\rho AC_D\left[(l+r)^2+\dot{\theta}^2+\dot{r}^2\right]-c_2\dot{\theta} \quad (5)$$

The development of this model significantly expands the ability of researchers to understand more complex spring pendulum systems, such as the use of intrinsic spring pendulums, non-linear spring pendulums and the use of spring pendulums on transmission towers.

This analysis reveals that the development of spring pendulum system research is not only linear in terms of system complexity, but also shows a sophisticated diversification of methodologies. The balance between mathematical and physical approaches, and the evolution from basic systems to practical applications, reflect the maturity of this field in facing the challenges of modern technology. The development of models to solve EOM from simple to complex models reflects the intellectual journey to understand spring pendulum systems. Each level of complexity expands the understanding of how spring pendulums continue to evolve. Seen from the transition from linear to non-linear models, it marks a significant advance in mathematical approaches that allow researchers to predict and further explore previously unimaginable behavior. This study makes it clear that understanding spring pendulum systems through accurate mathematical approaches is not only an academic necessity, but also an important step in creating innovative solutions to challenges in various engineering applications. Implications include the development of more accurate predictive models and better integration into future robotic systems and control technologies.

CONCLUSION

This study presents significant developments in spring-pendulum systems from 2000 to 2024 and reflects the complex developments in the field of nonlinear mechanics and dynamics. A systematic review of 36 research papers reveals that the extensions of the system can be divided into four main classifications. Applications involving basic modifications of spring pendulums, damping systems, coupling systems, and geometry changes. This work marks an important transition from linear models to nonlinear models, which expands researchers' ability to understand the behavior of increasingly complex spring systems.

The core of this work is analytical methods, particularly Lagrangian formulations, which are used in 23 of the 27 analytical formulations. This illustrates how analytical formulation techniques are superior for examining the dynamics of spring pendulums. The creation of these models has significant practical ramifications in addition to academic ones, such as the creation of novel solutions to problems in a range of engineering applications, such as robotic systems and emerging control technologies.

The results of this study confirm that research on spring-pendulum systems is not only focused on fundamental aspects but also aimed at practical applications in vibration control, energy harvesting, and structural dynamics. For future research, the development of more realistic nonlinear models remains an important challenge, especially to understand the dynamic behavior under different environmental conditions. In addition, the integration of modern technologies such as piezoelectricity, artificial intelligence, and adaptive sensing can provide a revolutionary step to improve the efficiency and stability of the system. Overall, this study provides comprehensive insights into the development of spring-pendulum systems and opens up opportunities for further innovation in engineering, robotics, and machine control systems. By understanding the development trends of models and methods used, future research will be more effective in developing more sophisticated and efficient spring pendulum applications.

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