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A Critical Examination on the Lorentz Contraction of a Rod

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Abstract

When studying special relativity, we usually find Lorentz contraction as one of the most clear and straightforward consequences of the Lorentz Transformation. The derivation of Lorentz contraction from Lorentz Transformation is apparently so simple that we usually overlook an important ideal assumption. A body which is Lorentz contracted is assumed to exist in infinite time in its rest frame of reference. In this paper we will derive Lorentz contraction without that assumption. We will derive Lorentz contraction (using spacetime diagram) of a rod with finite lifetime in its rest frame of reference. The result is such that the famous Lorentz contraction formula is only valid in certain condition concerning the lifetime of the rod.

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INTRODUCTION

Two of the most famous consequences of special relativity are the so called Lorentz contraction and time dilation. We can regard Lorentz contraction as mainly involving space whereas time dilation as involving time. Phenomena involving time are usually more subtle than that of space so the discussion of Lorentz contraction must be simpler.

In this paper we want to point out that in the apparently simple derivation of Lorentz contraction there is an important ideal assumption that has not been adequately emphasized in the usual derivation (Mermin, 1968; Pauli, 1981; Synge, 1956). This ideal assumption is that the body under consideration is assumed to exist at all times in its rest frame of reference. This assumption is not realistic because we certainly know that all bodies must have only a finite or limited time of existence. This ideal assumption carries over into the application of the Lorentz contraction (Kampen, 2008; Redzic, 2004) If we look at the list of particles, for example, (Griffith, 2004) , then it appears that only protons and electrons are stable (the age is infinite) whereas almost all other particles have finite ages. So, in general we can say that the elementary particles have finite ages (Bartoli et al., 1970, 1971; Goldhaber et al., 1976; Hartill et al., 1969; Litke et al., 1973; Tarnopolsky et al., 1974). This fact even goes to undergraduate physics experiments (Coan, Liu, & Ye, 2006; Easwar & MacIntire, 1991; Frisch & Smith, 1963).

Protons and electrons are the main constituents of objects around us so the objects seem to be stable. However, intuitively we certainly do not imagine that the objects around us have infinite ages. Something will certainly cause things around us to have finite ages. It can be said that in general things around us should have finite ages (Ayers et al., 1971; Boyarski, Loh, Niemela, & Ritson, 1962; Durbin, Loar, & Havens, 1952; Eckhause, Harris, & Shuler, 1965; Lobkowicz et al., 1969; Nordberg, Lobkowicz, & Burman, 1967; Ott & Pritchard, 1971) If we use this more realistic and general assumption in the derivation of the phenomenon of Lorentz contraction the resulting conclusion is that the famous Lorentz contraction formula is valid only in certain conditions. We propose that this restriction of the validity of the usual Lorentz contraction formula merits serious consideration.

METHOD

In this study, to investigate the effect of an object's lifetime on its length contraction due to relativistic effects, we will compare two simple cases. The first case involves the length contraction of a rod with an infinite lifetime, while the second case examines the length contraction of a rod with a finite lifetime. Using spacetime diagrams associated with Lorentz transformations for both cases, we will demonstrate the differing situations in each case

RESULT AND DISCUSSION

Lorentz Contraction of a Rod with Infinite Lifetime

Let us consider two inertial frames of reference K and K' with both the x- and x'-axes being coincident. K' is moving with velocity V with respect to K in the direction of positive x-axis of K. In each frame of reference, there are clocks attached at points of space that have been synchronized internally with each other in each frame. The clocks at the origins O and O' of both frames of reference are also synchronized such that when O and O' are coincident the time shown on both clocks are $t = t' = 0$.

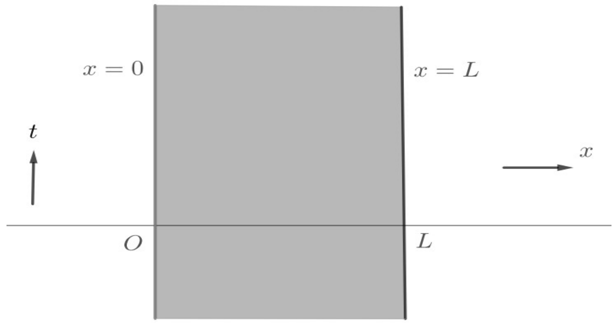

Figure 1. The set of events (shown by the shaded area) of the rod in K

A point event is an event that happens at exactly one certain point of space and at one certain instant of time. A point event can be expressed quantitatively as (x, t) in K and also as (x', t') in K'. There is just one unique physical event in spacetime but there are infinitely many quantitative expressions according to the infinitely many frames of reference which can be used to describe the event. The essence of relativity theory is how those quantitative descriptions of the event are related in such a way that we eventually will get the covariant description of all the laws of physics.

The expressions of the point event (x, t) and (x', t') are related by Lorentz Transformation

$$x' = \Gamma(x - Vt) \tag{1}$$

$$t' = \Gamma\left(t - \frac{Vx}{c^2}\right) \tag{2}$$

with $\Gamma = 1/\sqrt{(1 - V^2/c^2)}$. Let us consider a rod of length L lying at rest along the x-axis of K with one end at $x = 0$ and the other end at $x = L$. This is an event in K, not a point event, but a compound event consisting of infinitely many point events. We have to specify the time of the event. In the usual treatment of Lorentz contraction one makes an important assumption but not stated explicitly, namely that the rod exists at all times in K. In this case, the event of the rod in K can be expressed mathematically by

$$0 \leq x \leq L \tag{3}$$

We can use a spacetime diagram to describe eq.(3) as in Figure 1 in which the set of events is shown by the shaded area; the boundary of the area are the lines $x = 0$ and $x = L$. We see no restriction on the time of the events.

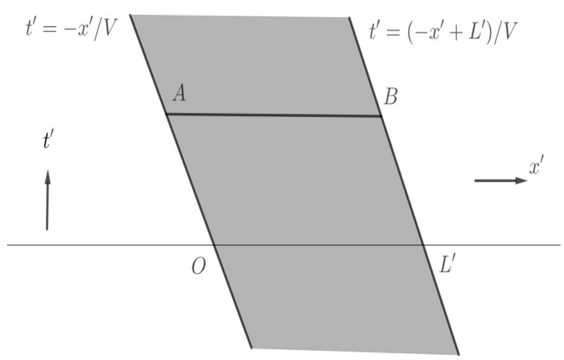

Figure 2. Spacetime events of the rod in K' shown by the shaded area.

Using eq.((1) and eq.(2) we can express this event in K' by substituting

$$x = \Gamma(x' + Vt') \tag{4}$$

into eq.(3) and we will get

$$0 \leq \Gamma(x' + Vt') \leq L \tag{5}$$

This can be described by **Error! Reference source not found.**. The boundaries of the shaded area are the lines

$$g_1 : \Gamma(x' + Vt') = 0 \; or \; t' = -x'/V \tag{6}$$

and

$$g_2 : \Gamma(x' + Vt') = L \text{ or } t' = (-x' + L')/V \quad (7)$$

with $L' = L/\Gamma$. The length of the rod as seen in K' is that with simultaneous t' such as AB in **Error! Reference source not found.**. Because g_1 is parallel with g_2 the length of the rod is constant with t'. When $t' = 0$ then $x' = 0$ for g_1 and $x' = L'$ for g_2. The length of the rod becomes $L' = L/\Gamma$ in K'.

Lorentz Contraction of a Rod with Finite Lifetime

We have the case of a rod with a finite lifetime which can be stated mathematically by

$$0 \le t \le T \quad (8)$$

and can be described diagrammatically by Figure 2.

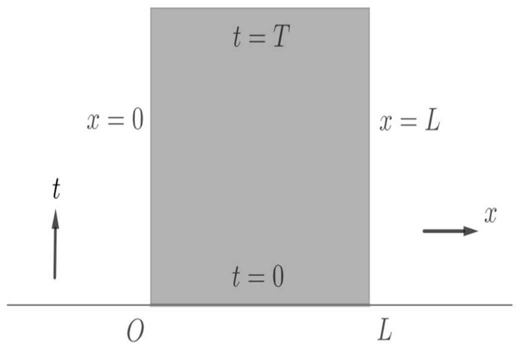

Figure 2. Spacetime events of the rod (with finite lifetime) in K shown in a shaded area.

In K' we can describe the situation by using

$$t = \Gamma(t' + Vx'/c^2) \quad (9)$$

which can be derived from eqs.(1) and eq.(2). We substitute eq.(7) into eq.(6) and get

$$0 \le \Gamma(t' + Vx'/c^2) \le T \quad (10)$$

The set of events can be described diagrammatically by Figure 3 and be shown in a shaded area. The boundaries of the shaded area can be expressed mathematically as the lines g_1, g_2 and

$$g_3 : \Gamma(t' + Vx'/c^2) = 0 \text{ or } t' = -Vx'/c^2 \quad (11)$$

and

$$g_4 : \Gamma(t' + Vx'/c^2) = T \text{ or } t' = -Vx'/c^2 + T' \quad (12)$$

with $T' = T/\Gamma$.

The point of intersection of g_4 with x'-axis which is $cT/(\beta\Gamma)$ (with $\beta = V/c$) is made greater than $L' = L/\Gamma$ with a choice of $T > \frac{L}{c}\beta$. In K' the rod makes an appearance at point A, which is the intersection of g_2 and g_3 which is at $t' = -\frac{L}{c}\beta\Gamma$. The rod then grows in length until at $t' = 0$ it reaches a length of L'. In this first phase, the front end of the rod moves with superluminal speed of c^2/V which can be determined from the reciprocal of the slope of the line g_3; whereas the back end moves with normal speed V to the left.

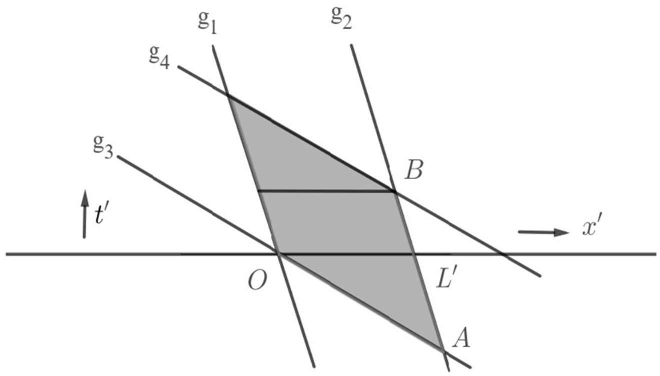

Figure 3. Spacetime events of the rod (with finite lifetime) in K' shown in a shaded area.

In the second phase, the rod moves from point O (at $t' = 0$) to point B which is the intersection of g_2 and g_4 at $t' = T\Gamma - \frac{L}{c}\Gamma\beta$ with usual contracted length L' and normal speed V. So the second normal phase lasts for $T\Gamma - \frac{L}{c}\Gamma\beta$.

The third phase is from point B until point C which is the intersection of g_1 and g_4 at $t' = T\Gamma$. Here the front end of the rod moves normally with speed V whereas the back end moves superluminally with speed c^2/V until the rod disappeared at point C.

So the rod is undergoing three phases. In the first phase which lasts for $\frac{L}{c}\beta\Gamma$ the rod grows in length from zero to the usual Lorentz contracted length L/Γ. In the first phase, the front end of the rod moves superluminally with speed c^2/V. In the second phase which lasts for $T\Gamma - \frac{L}{c}\Gamma\beta$ the rod moves normally with speed V and with the normal Lorentz contracted length L/Γ. In the third phase which lasts for $\frac{L}{c}\beta\Gamma$ like the first phase the rod shrinks its length from the Lorentz contracted length until zero with the back end of the rod moving with superluminal speed c^2/V whereas the front end moves with the normal speed V. This is for the choice of $T > \frac{L}{c}\beta$.

The rod has its Lorentz contracted length from $t' = 0$ until $t' = T\Gamma - \frac{L}{c}\Gamma\beta$ which means that as long as $T > \frac{L}{c}\beta$ then we get the usual phenomenon. When $T = \frac{L}{c}\beta$ we only have the usual length contraction for one moment of time which is at $t' = 0$.

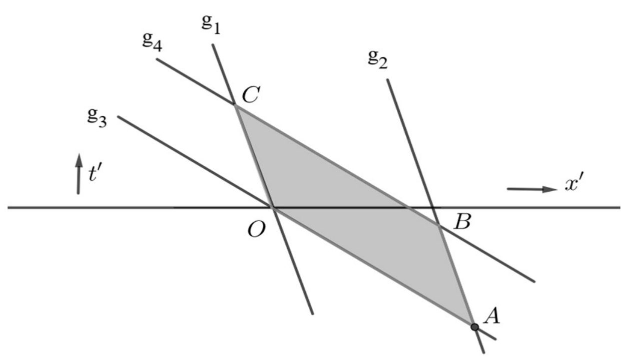

Figure 4. Spacetime events of the rod for $T < \frac{L}{c}\beta$ in K' shown in shaded area.

When $T < \frac{L}{c}\beta$ then the first and third phases last for $T\Gamma$ as can be seen in Figure 4. The second phase which lasts for $\frac{L}{c}\Gamma\beta - T\Gamma$ the rod has a length which is smaller than the usual Lorentz contracted length. The length can be determined from the intersection of $g4$ with the x'-axis which is at $x' = cT/(\Gamma\beta)$ which is smaller than L/Γ due to $T < \frac{L}{c}\beta$. The curious fact is that the rod moves with superluminal speed of c^2/V.

The superluminal speed which we see in K' is due to the fact that in K the rod pops into existence instantaneously at $t = 0$ and ceases to exist at $t = T$ also instantaneously. The more fundamental importance is the fact that if the rod exists only for a short enough time then we get the contracted rod with a length smaller than the usual Lorentz contraction. It means that the usual Lorentz contraction formula isn't valid anymore. The contraction depends on the time of existence of the rod.

CONCLUSION

Here we make a recap of the previous discussion. In K we see a rod lying on the x-axis (an interval $0 \leq x \leq L$) for a finite lifetime (an interval $0 \leq t \leq T$). In K' which is moving along the x-axis with velocity V, we see two different cases depending on the relation between T and L. For $Tc > \beta L$ then we see the usual Lorentz contraction but for only an interval of time $\Delta t' = \Gamma(T - \beta L/c)$ which becomes zero if $Tc = \beta L$. The second case is for $Tc < \beta L$. In K' we see a rod with the length of $cT/\Gamma\beta$ which is shorter than the usual contracted length L/Γ moving with superluminal speed c^2/V. In all practical cases, we always have $T > \beta L/c$ because of the smallness of the value of $\beta L/c$ so that we get the usual Lorentz contraction. We nevertheless feel that the case of $T < \beta L/c$ merits consideration at least for theoretical reasons. Let say, for example, suppose we have 10^3 m (astronomical size) rod-like structure made from muon. It is moving with 0.9 velocity of light. Then according to the formula we get $0.9 \times 10^3 / 3 \times 10^8 = 0.3 \times 10^{-5}$. Then the time of existence of muon (2.197×10^{-6}) is smaller than that. The length will be smaller than Lorentz contraction formula.

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