

## Pseudo-Construction Errors in Solving Number Pattern Problems Based on APOS Theory

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### Abstract

The concept of mathematical material is still not fully embedded in students' schemes, this means that errors occur in constructing the concept. This research aims to describe the actual situation regarding pseudo-construction errors that occur at the APOS (Action, Process, Object, and Scheme) stages. This research involved 28 students in class VIII A of SMPN 2 Silo Jember as subjects through problem solving test to get an comprehensive information about occurring errors. Then by the purposive sampling and snowball sampling techniques, two subjects are choosen to explore in depth through interview. The data valiadtion is guarantee through the member check process. Data analysis includie data condensation, data presentation, and concluding. The results of the research are that pseudo-construction errors occur at every stage of APOS which results in the concept not being fully constructed in the student's scheme. As a recommendation, developing a mathematics learning model based on APOS theory to minimize the error is needed such that the concepts will be well and completely constructed in students' schemes.

**Keywords:** Pseudo-construction; number patterns; APOS theory

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### Abstrak

Konsep materi matematika masih belum sepenuhnya tertanam dalam skema siswa. Hal ini menunjukkan bahwa terjadi kesalahan dalam mengkonstruksi konsep. Penelitian ini bertujuan untuk menggambarkan atau mendeskripsikan keadaan yang sebenarnya terkait kesalahan konstruksi semu yang terjadi pada tahapan APOS (Aksi, Proses, Objek, dan Skema). Penelitian ini melibatkan 28 siswa kelas VIII A SMPN 2 Silo Jember sebagai subjek melalui tes pemecahan masalah untuk mendapatkan informasi komprehensif tentang kesalahan yang terjadi. Kemudian dengan teknik purposive sampling dan snowball sampling, dipilih dua subjek untuk dieksplorasi secara mendalam melalui wawancara. Validasi data dijamin melalui proses member check. Analisis data meliputi kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian yaitu kesalahan konstruksi semu terjadi di setiap tahapan APOS yang mengakibatkan konsep tidak terkonstruksi dengan utuh dalam skema siswa. Sebagai rekomendasi, perlu dikembangkan model pembelajaran matematika berbasis teori APOS untuk meminimalkan kesalahan agar konsep-konsep dapat terkonstruksi dengan baik dan lengkap dalam skema siswa.

### INTRODUCTION

Mathematics is a subject that emphasizes understanding concepts (Dinnullah, 2021). A good understanding of mathematical concepts can help students to solve problems (Nurani et al., 2021; Pramesti & Mampouw, 2020; Rahmawati, 2020), but in reality, there are still many concepts that have not been embedded in students' schemes. This can be seen from the results of previous research that the concept has not been embedded in students' schemes (Novianti & Pratama, 2022; Supratman et al., 2022), and can also be seen from the results of preliminary research conducted on 10 students who had studied number pattern material. The answers of 10 out of 9 students were wrong. It can be seen from Figure 1 in the appendix that the multiple choice is correctly selected, but when seen from the student's reasons (because the sequence number pattern does not include a rectangular pattern) there is a possibility that the answer is just a coincidence and the student does not understand the concept, this is likely to happen when constructing the concept -the concept is wrong which results in the concept still not being properly embedded in students' minds (Jazby & Widjaja, 2019; Leron & Hazzan, 2009; Nizaruddin & Kusmaryono, 2023; Pape, 2004). This is in line with the fact that students appear to give the correct an-

swer, but when investigated it turns out to be wrong, which is called a pseudo construction (Subanji, 2015; Subanji & Nusantara, 2016).

Pseudo construction is a quasi construction. This aims to make the process of forming mathematical concepts "appear" by scientific concepts, but after being explored more deeply it turns out that they are not by scientific concepts. Pseudo construction is used by researchers with various terms, namely the pseudo thinking (Vinner, 1997), pseudo thinking co-variational reasoning (Herna et al., 2016; Subanji & Nusantara, 2016), the pseudo analytical and conceptual (Nizaruddin & Kusmaryono, 2023), theory dual process in the context of solving algebra problems (Leron & Hazzan, 2009), and the direct translation approach versus meaning-based approach in the context of solving word problems (Pape, 2004).

Pseudo-thinking includes pseudo-analytical thinking and pseudo-conceptual thinking based on students' understanding of a concept (Vinner, 1997). Pseudo-analytical and pseudo-conceptual mean that the pseudo-analytic thinking process can produce right or wrong answers (Nizaruddin & Kusmaryono, 2023), the characteristics are the absence of control procedures identifying similarities between problems and other problems and then using artificial procedures that are not appropriate to the problem.

Processes and behavior in completing tasks in dual process theory are grouped into two models, namely the first and second system processes, these two processes describe differences in the speed and ease of various things that come to mind (Leron & Hazzan, 2009). The characteristics of the first system process are fast, automatic, effortless, accidental, and flexible. Meanwhile, the characteristics of the second system process are slow, careful, hard work, full of calculation, and relatively flexible.

Based on the opinion above, it can be concluded that the pseudo-construction studied in this research is more closely related to pseudo-thinking compared to the other terms previously mentioned. This happened because researchers wanted to examine how conceptual construction errors occurred with students' answers that "appeared" to be correct (by scientific concepts), but after being explored in more depth they turned out to be wrong (not by scientific concepts).

Concept construction errors can be seen more clearly through the APOS framework (Nisa *et al.*, 2021; Parraguez & Oktaç, 2010; Rofiki *et al.*, 2020; Silalahi, 2017). Concept construction errors that occur need to be handled to minimize concept construction errors, especially pseudo-construction, including being able to utilize mathematical concepts in solving problems. Handling errors in constructing mathematical concepts can utilize APOS theory (Dubinsky, 2001). What kind of pseudo construction error occurs at each stage of APOS has been described by Ni'mah *et al.*, (2018) and Subanji (2015). The pseudo-construction error indicators at the APOS stage are as follows.

Table 1. Pseudo Construction Error Indicators Based on APOS Theory

APOS stage	PseudoConstruction Error
Action	Identifying concepts but not concepts or identifying not concepts but concepts with the help of external stimuli.
Process	Stating a concept even though it is not a concept or stating not a concept even though it is a concept without the help of external stimulus.
Object	Students can explain the steps that have been done correctly but the concept is wrong, or students can explain the steps that have been done but are wrong, but the concept that the student has is correct.
Scheme	Students can solve the problem correctly, but the concept is wrong, or students can solve the problem but the answer is wrong, but the concept within the student is correct.

Source: (Ni'mah *et al.*, 2018; Subanji, 2015)

One of the materials in mathematics that requires the cultivation of good concepts is number patterns (Novianti & Pratama, 2022). Number patterns are important to learn so that inductive thinking skills can be improved (Apiati *et al.*, 2019). Number pattern material is one of the materials which contains many concepts that must be mastered. The many concepts contained in the number pattern material must be constructed properly from each of these concepts. Students often cannot easily identify number patterns because they lack knowledge of appropriate mathematical concepts (Spangenberg & Pithmajor, 2020). The research results of Novianti & Pratama (2022) show that students in the number pattern material were only able to go through the action and process stages. The results of preliminary research conducted by researchers (Figure 1) show that students only happen to be correct in answering, but do not know the concept, so students are classified as making pseudo-construction errors. Based on the previous explanation, the researcher focused on examining pseudo-construction errors in solving

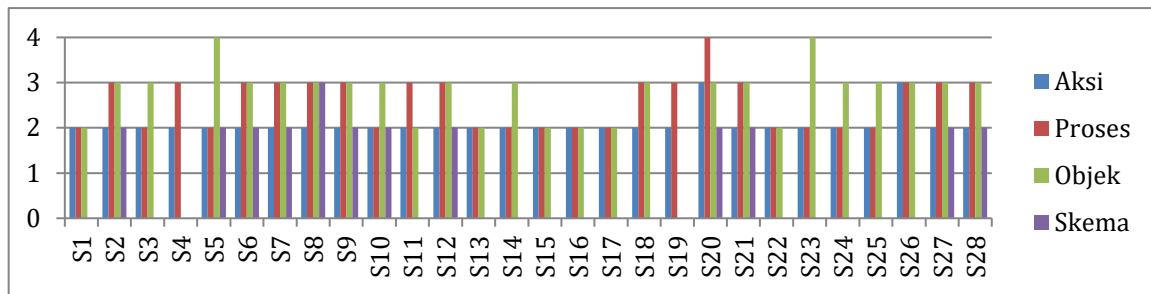


Figure 2. Results of Student Construction Error Tests

number pattern problems based on APOS theory. Meanwhile, previous researchers examined pseudo-construction errors in constructing the concept of number operations and algebraic forms, geometry, and functions (Subanji, 2015).

This research is important to carry out to describe the actual situation related to pseudo-construction errors that occur at the APOS stage. By knowing these pseudo-construction errors then teachers or other researcher can take or develop a learning action to minimize conceptual errors, including being able to utilize mathematical concepts in solving problems. Therefore, researchers are interested to explore the pseudo-construction errors in solving number pattern problems based on APOS theory".

## METHOD

The research was designed as qualitative descriptive research. As qualitative descriptive research design able to describe and deeply explore the pseudo-construction errors in solving number pattern problems based on APOS theory. This research was carried out at SMPN 2 Silo Jember, East Java. The subjects in this research were 28 students in class VIII A of SMPN 2 Silo. The subject selection method is purposive sampling and snowball sampling techniques (Creswell, 2015). Two students will be taken as subjects for more in-depth research (interviews) based on (1) making the most mistakes in constructing concepts at each

stage of APOS, (2) making the most mistakes in constructing mathematical concepts at each stage of APOS, and (3) the subject has good communication skills. It is good to do more in-depth research (conduct interviews). The following figure are the results of tests that were carried out on 28 students in class VIII A of SMPN 2 Silo (see Figure 2).

Figure 2 explains the test results related to concept construction errors made by subjects including pseudo-construction errors, construction holes, incorrect analogical constructions, and incorrect logical constructions at each stage of APOS. Figure 2 shows that subjects S5 and S20 made the most mistakes at each APOS stage. Subject S5 made concept construction errors at the action, process and scheme stages including pseudo-construction errors and construction holes; as well as at the object stage including pseudo-construction errors, construction holes, incorrect analogical constructions, and incorrect logical constructions. Meanwhile, subject S20 made concept construction errors at the action and object stages including pseudo-construction errors and construction holes; the process stage includes false construction errors, construction holes, wrong analogical constructions, and wrong logical constructions; and at the schematic stage including pseudo construction errors and construction holes. Therefore, a more in-depth data checking process (interview) will be carried out by researchers on S5 and S22.

The characteristics of subject S5 according to the mathematics subject teacher for class VIII A SMPN 2 Silo are that the first impression is that the child is quiet, diligent in doing assignments, and likes to write on the board. The characteristics of subject S22 are not much different from the characteristics of subject S5, according to the mathematics subject teacher for class VIII A SMPN 2 Silo, namely during class during mathematics lesson time, namely being diligent, following lessons carefully, diligently doing assignments, likes to write answers on the board, and if there is anything you don't understand, just ask. According to his classmates, the characteristics explained were the same as those explained by the class VIII A mathematics teacher at SMPN 2 Silo. His classmate added that subject S22 was friendlier than subject S5, subject S5 was quieter and sometimes annoying with his occasional pranking behavior in class. Sometimes subjects S5 and S22 sleep in class when they are in a bad mood.

The validity of the data in this research is member check (Creswell, 2015). In member check, the researcher carries out a data-checking process (from test results and interviews) by asking the correctness of the data reduction from S5 and S20 using membercheck list and interview. Data analysis is data condensation, data presentation, and conclusion (Saldana, 2014). Data condensation include arranging the test result data (photos), interview results (making interview transcripts), and member checks to form sentences that are easy for readers to understand. Presentation of data for relevant data from each subject presented in the form of images and coherent narrative text, so that it becomes simple and easier to understand, and a picture of pseudo-construction errors in solving number pattern problems based on

APOS theory will be visible. Drawing conclusions is taken based on the results of tests, interviews, and member checks regarding pseudo-construction errors in solving number pattern problems based on APOS theory.

The research procedures are the stages of preparation, implementation, completion, and conclusion (Lestari & Yudhanegara, 2017). The research procedures in this study are (1) the preparation stage includes making a test instrument including 4 questions and an interview guide, as well as validating 2 lecturers and 1 mathematics subject teacher; (2) the implementation stages include conducting tests on 28 students, selecting subjects who made the most conceptual construction errors at each APOS stage as seen from the test results on 28 students, and conducting interviews on subjects selected to be explored in more depth, namely subjects S5 and S20 with a time range of 1 hour; (3) the completion stage includes checking validity (carrying out member checks on S5 and S20 subjects from test and interview results) and data analysis (from test results, interviews and member checks); and (4) the conclusion drawing stage, namely drawing conclusions from the collected data (test results, interviews and member checks). This research uses 4 questions for each APOS stage. Question number 1 to see pseudo-construction errors at the action stage, question number 2 to see pseudo-construction errors at the process stage, question number 3 to see pseudo-construction errors at the object stage, and questions number 4 to see the pseudo-fault construction at the schematic stage.

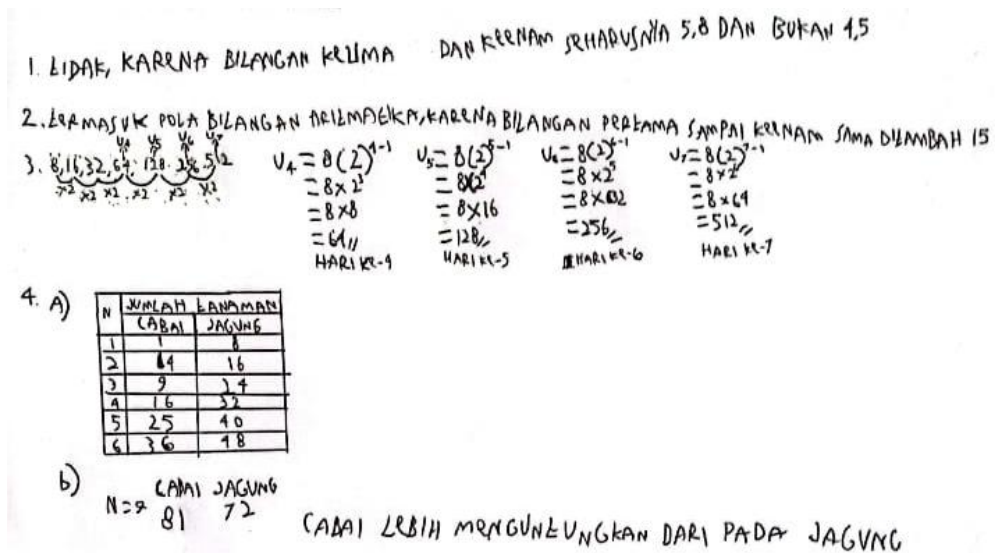


Figure 3. Answer Results S5

## RESULTS AND DISCUSSION

### Results

The results of this research were obtained from test results, interviews, and member checks from S5 and S22 subjects. Following are the results of subject S5's answers.

In the **action stage**, S5 answered question number 1 on the answer sheet, showing that the answer was correct and that the arrangement of the number patterns listed in the question did not include the Fibonacci number pattern because the fifth and sixth numbers should be 5 and 8, not 4 and 5. To find out the apparent construction error experienced by S5, The following is a snippet of the interview.

Po6 : What is the first term of the number pattern formed in the problem?

S5o6 : 1 thump

Po7 : What is the next term?

S5o6 : 2

Based on answer number 1 in Figure 3 and the interview excerpt above, it appears that S5 answered correctly on the test, but during the interview, he said that the first term of the number pattern

in the question was 1 (interview S5o6) and the next term was 2 (interview S5o7). So it appears that the concept in S5's mind is wrong. Therefore, S5 made a concept construction error which was categorized as a pseudo-construction error because it identified the concept but not the concept with external assistance. The results of this analysis can be seen in Figure 5 in the attachment.

At the **process stage**, S5 answered question number 2 on the answer sheet, showing that S5's answer was correct and that the pattern arrangement listed in the question included an arithmetic number pattern because the first to fifth numbers were both added by 15. To find out the apparent construction errors experienced by students, the following is an interview excerpt.

P14 : How do you determine the value of the first term and how much?

S14 : From the picture that is 15.

P15 : How do you determine the value of the next term and how much?

S15 : From the picture that is 30.

Based on answer number 2 in Figure 3 and the interview excerpt above, it appears that S5 answered correctly on the test, but during the interview, he

stated that the value of the first term of the number pattern arrangement was 15 and the next term was 30 (interviews S14 and S15), so it appears that the concept he had in mind S5 is wrong. Therefore, S5 made a concept construction error which was categorized as a pseudo-construction error because he stated a concept even though it was not a concept without the help of an external stimulus. The results of this analysis can be seen in Figure 6 in the attachment.

At the **object stage**, S5 answered question number 3 on the answer sheet, showing that S5 gave two different ways with the same result in answering the question. To find out the pseudo-construction errors experienced by students, the following is an interview excerpt.

- P24 : *How do you determine the value of the first term and how much?*  
 S24 : *The first, second, and third are already there. So I counted the fourth one, at first I checked and was confused, this is what took me a long time. After that, I calculated again and again. I remember that 8 plus 8 equals 16, 16 plus 16 equals 24. So I think this is multiplied by 2 times 2, right?*

Based on answer number 3 in Figure 3 and the interview excerpt above, it appears that S5 wrote two different ways with the same result in answering the question, but during the interview, S5 explained that to solve the problem in question, it was multiplied by two and so on (interview S24), so that shows that the concept he has in mind is wrong. Therefore, S5 made a concept construction error which was categorized as a pseudo-construction error because he was able to explain the steps that had been carried out correctly, but the concept was wrong.

The results of this analysis can be seen in Figure 7 in the attachment.

At the **scheme stage**, S5 answered question number 4 on the answer sheet, showing that S5 was correct in both commands a and b. To find out the apparent construction errors experienced by S5, the following is an interview excerpt.

- P49 : *How do you connect the number patterns that have been arranged so that you can solve problems?*  
 S549 : *This is it, the first one is one, so the chilies are 1,4,9. It's like calculating the first 1,4,9, this is like a square number pattern, so the result is like 1 that adds 3, and 4 is also added.*  
 P50 : *From the arrangement of numbers, chili plants form what number patterns and why?*  
 S550 : *The chili is square*  
 P53 : *What do you know about square number patterns?*  
 S553 : *If the square is like 1 but adds 3, and so on, how can I explain it?*

Based on answer number 4 in Figure 3 and the interview excerpt above, it appears that S5 answered correctly on the test, however during the interview S5 explained that this arrangement was formed because 3 was added (interviews S549 and S553), so it appears that the concept in S5 is wrong. Therefore, S5 made a concept construction error which was categorized as a pseudo-construction error because S5 was able to solve the problem correctly, but the concept was wrong. The results of this analysis can be seen in Figure 8 in the attachment.

The answers to subject S20 will also be presented based on the APOS stages. Following are the results of S20's answers.

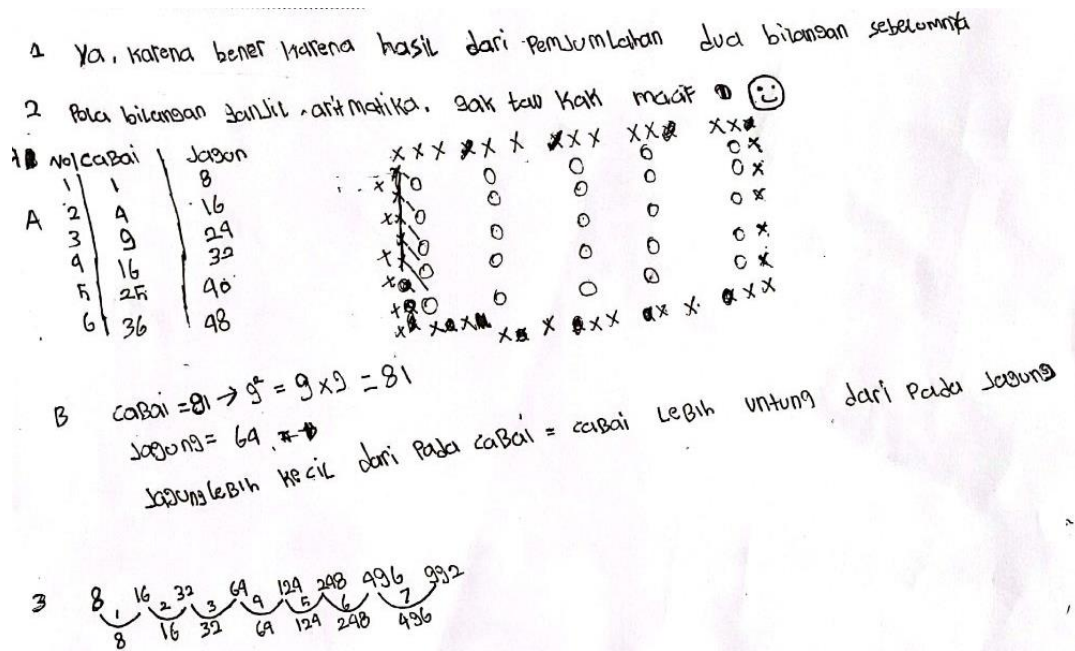


Figure 4. Answer Results S20

At the **action stage**, S20 answered question number 1 on the answer sheet, showing that S20's answer was wrong and that the arrangement of the number patterns listed in the question included the Fibonacci number pattern because it was the result of adding the two previous numbers. To find out the apparent construction errors experienced by the S20, the following is an interview excerpt.

- P05 : Why is that true?
- S2005 : Because from 0,1,1 keep waiting, it will be added to 2, then it will be added to the previous number, 1 plus 2, right?
- P06 : Try paying attention to 0+1=1, 1+1=2, 1+2=3, 2+3=...?
- S2006 : 5 is not 4. Wrong is 4 ma'am
- P16 : So from the arrangement of number patterns that are formed, as mentioned in the question. Is it true that the number pattern arrangement includes the Fibonacci number pattern arrangement and why?
- S2016 : Not Fibonacci because 4 is not the result of adding the previous two terms

Based on answer number 1 in Figure 4 and the interview excerpt above, it appears that S20 answered incorrectly on the test, but when interviewed he said he

could identify the Fibonacci number sequence correctly (interview S2006), so it appears that the concept that S20 had in mind was correct, only that S20 was lacking. Be careful when reading the test questions. Therefore, S20 made a concept construction error which was categorized as a pseudo-construction error because S20 identified not a concept but a concept with external assistance. The results of this analysis can be seen in Figure 5 in the attachment.

At the **process stage**, S20 answered question number 2 on the answer sheet showing that S20 gave two answers "odd number patterns and arithmetic". To find out the apparent construction errors experienced by the S20, the following is an interview excerpt.

- P33 : What is the difference between the two terms?
- S2033 : The difference is 15
- P34 : So what number pattern arrangement is contained in question number two and why?
- S2034 : Arithmetic
- P35 : So what arithmetic number pattern is odd?
- S2035 : Arithmetic

*P33 : What is the difference between the two terms?*

*S2033 : The difference is 15*

Based on answer number 2 in Figure 4 and the interview excerpt above, it appears that S20 wrote two answers on the test, and during the interview, he said that the arrangement in the question was an arrangement of numbers that formed an arithmetic number pattern (interview S2035), so it appears that the concept he had in mind The S20 is correct. Therefore, S20 made a concept construction error which was categorized as a pseudo-construction error because S20 stated that it was not a concept even though it was a concept without the help of an external stimulus. The results of this analysis can be seen in Figure 6 in the attachment.

**At the object stage,** S20 answering question number 3 on the answer sheet shows that S20 provides two different ways with the same result in answering the question. To find out the apparent construction errors experienced by the S20, the following is an interview excerpt.

*P24 : How do you determine the value of the first term and how much?*

*S24 : The first, second, and third are already there. So I counted the fourth one, at first I checked and was confused, this is what took me a long time. After that, I calculated again and again. I remember that 8 plus 8 equals 16, 16 plus 16 equals 24. So I think this is multiplied by 2 times 2, right?*

Based on answer number 3 in Figure 4 and the interview excerpt above, it appears that S20 wrote two different ways with the same result in answering the question, but during the interview, S20 explained that to solve the problem in question, it was multiplied by two and so on (interview S24), so shows that the

concept that the S20 has in mind is wrong. Therefore, S20 made a concept construction error which was categorized as a pseudo-construction error because he was able to explain the steps that had been carried out correctly but the concept was wrong. The results of this analysis can be seen in Figure 7 in the attachment.

**At the scheme stage,** S20 answered question number 3 on the answer sheet, showing that S20 was correct in completing the table and providing a picture of the planting pattern correctly. To find out the apparent construction errors experienced by the S20, the following is an interview excerpt.

*P64 : What do you mean by this picture of a planting pattern (while pointing to the picture on the S2 answer sheet)?*

*S2064 : Initially, I wanted to answer using the picture, but when I compared it with the one in the table the results were different*

*P65 : Why is there a difference between table results and images?*

*S2065 : That's if you don't add one line, if you want to cross it out it won't be good. I'll leave it alone. But I added it and the result was the same as in the table*

*P66 : How can you be right in the fifth term but wrong in the sixth term? How do you understand how to draw it?*

*S2066 : I tried, ma'am if the fifth term was 5 straight, but in the sixth, I was wrong, ma'am.*

Based on answer number 4 in Figure 4 and the interview excerpt above, it appears that S20 answered correctly on the test, however during the interview S20 explained that drawing planting patterns was just trying (interview S2066), so it appears that S20 does not know how to draw planting patterns. Therefore, S20 made a concept construction error which was categorized as pseudo-construction because the student solved the problem correctly, but the concept was wrong.

The results of this analysis can be seen in Figure 8 in the attachment.

## Discussion

This research has described pseudo-construction errors in solving problems based on APOS theory. Based on data analysis, at **the action stage**, students made apparent construction errors because students were able to identify a pattern of Fibonacci numbers with the help of external stimuli in the test, but when explored more deeply it turned out that students only memorized from books without understanding the concept and applying it in their language which resulted in the concept what is in the student's scheme is wrong, namely when the student answers that the first term of the number arrangement is 1 and the second term is 2, even though the first term is 0 and the second term is 1. This is similar to the fact that students tend to memorize the material in the book without understanding the concept (Marion et al., 2015). Therefore, it can be seen that students make pseudo-correct construction errors because students appear to give the correct answer, but when investigated it turns out to be wrong (Ni'mah et al., 2018; Subanji & Nusantara, 2016; Subanji, 2015). This error occurs because students are unable to manipulate mental (physical) objects that have previously been constructed, so it appears that students do not experience an action. Students experience an action when the student focuses their mental processes on efforts to understand a given concept. As a recommendation to students, when understanding material, do not memorize it from books, but rather understand the concepts contained.

Pseudo-construction errors also occur because students incorrectly identify the Fibonacci number pattern, but when

traced it turns out that the student can identify the Fibonacci number pattern with the help of external stimulus, the student explains and understands the concept of the Fibonacci number pattern, so it appears that they have experienced the action stage well (Yuliana & Ratu, 2018), it's just that students are not careful in reading the questions. This is similar to the fact that students make mistakes in solving problems, not because the concept is wrong, but because they are not careful enough (Kuswanti et al., 2018). Therefore, students make false construction errors. The pseudo construction "wrong" is the answer written by the student incorrectly, but after tracing the cause of the error made by carrying out an interview process (reflection) the student thinks it is correct (Ni'mah et al., 2018; Subanji & Nusantara, 2016; Subanji, 2015). These results are consistent in that actions are transformations of previously constructed cognitive objects that students perceive as external or as a series of instructions that students need to perform each operation in an algorithm (Dubinsky, 2001; Maharaj, 2013; Salgado & Trigueros, 2015). As a recommendation to students, understand the questions well so that there are no writing errors in solving the problem.

Pseudo-construction errors at **the process stage** occur because the student states the concept, but when traced the concept is wrong, namely the student states that the first term is 15 in the arrangement of numbers in question, even though the first term is 0 and the second term is 15, so it appears that the concept is in the student's mind. Wrong. Therefore, students make pseudo-correct construction errors because students appear to give the correct answer, but when investigated it turns out to be wrong (Ni'mah et al., 2018; Subanji & Nusantara, 2016; Subanji, 2015). This class also

shows that students have not experienced the process stage because students are not able to express a concept without the help of a stimulus (Dubinsky, 2001), so the concept has not been well embedded in students. The formed process stage shows that students internalize actions (actions) by mentally repeating and reflecting on them to visualize and explain transitions without having to do them openly (Dubinsky, 2001; Maharaj, 2013; Salgado & Trigueros, 2015). Therefore, understanding concepts must be well constructed, when mathematical concepts are well constructed, they can help students to solve problems (Nurani *et al.*, 2021; Pramesti & Mampouw, 2020; Rahmawati, 2020). As a recommendation to students to practice answering questions a lot, this will help students to express a concept without the help of a stimulus.

Pseudo-construction errors also occur because students write two different answers, namely stating odd number patterns and arithmetic, but when explored, students can state and give reasons with confidence that the arrangement of the number patterns in the question is an arithmetic number pattern, so it appears that the concept they have in mind the student is right. Therefore, S22 commits a pseudo-false construction error. The pseudo construction "wrong" is the answer written by the student incorrectly, but after tracing the cause of the error made by carrying out an interview process (reflection) the student thinks it is correct (Ni'mah *et al.*, 2018; Subanji & Nusantara, 2016; Subanji, 2015), with that the students have gone through the process stages well. These results are consistent in that students internalize actions (actions) by mentally repeating and reflecting on them to visualize and explain transitions without having to do them overtly (Dubinsky, 2001; Maharaj,

2013; Salgado & Trigueros, 2015). As a recommendation to students, understand the questions well so that there is no hesitation in writing answers to solving the problem.

Pseudo-construction errors at **the object stage** occur because students explain the steps that have been done correctly but the concept is wrong. Students give two different ways with the same result. This is in line with how students solve problems in various innovative and various ways (Putri *et al.*, 2019; Robinson, 2010; Treffinger *et al.*, 2008). Therefore, students appear to make pseudo-correct construction errors. Pseudo-true construction appears when students appear to give the correct answer, but when investigated it turns out to be wrong (Ni'mah *et al.*, 2018; Subanji & Nusantara, 2016; Subanji, 2015). The occurrence of false construction errors shows that students do not experience the object stage because the object stage is the processes that have been carried out which have been summarized into a cognitive object and can decompose an object back into processes as they originally were when the properties of the object in question will be used (Dubinsky, 2001). As a recommendation to students to ensure that the concepts are understood correctly, that is by asking the teacher or someone who understands better.

Pseudo-construction errors at **the scheme stage** occur because students can solve the problem correctly, but the concept is wrong. Students explain that the arrangement of numbers is formed by adding or spreading and provide answers by drawing planting patterns by trial and error. This shows that students solve problems in various ways (Putri *et al.*, 2019; Robinson, 2010; Treffinger *et al.*, 2008). Therefore, students appear to make pseudo-correct construction errors. This is in line with the explanation that

students appear to give the correct answer, but when investigated it turns out to be wrong or provide clarification on the wrong answer (Ni'mah et al., 2018; Subanji & Nusantara, 2016; Subanji 2015). As a recommendation to teachers, ensure that the concepts given to students are well embedded in the scheme, namely by providing concept construction tests.

Pseudo-construction errors also occur because there are wrong parts (blank) but without affecting the result (correct). This is inversely proportional to the statement that if there is an error in one of the completion steps, it will cause an error in the next step (final result) (Listiana & Sutriyono, 2018). Therefore, students appear to make false construction errors. A false construction error occurred because the answer written by the student was wrong, but after tracing the cause of the error made by carrying out an interview process (reflection), the student thought it was correct (Ni'mah et al., 2018; Subanji & Nusantara, 2016; Subanji, 2015). Students are just not careful in writing their answers, but the results are correct. Students are not fully able to relate one concept to another, namely connecting several concepts including the concept of square number patterns and arithmetic (algebra), planting patterns (geometry), and profit and loss (social arithmetic), students can only connect algebraic concepts with social arithmetic. Therefore, it appears that students have not experienced a schema stage. Students appear to have experienced the scheme stage well, namely when students can relate one concept to another (Mulyono, 2011; Yuliana & Ratu, 2018). Consistently, a schema for a particular mathematical concept is a collection of actions, processes, objects, and perhaps other schemas that are connected by several general principles to form

an individual's frame of mind in solving problems related to the concept being studied (Dubinsky, 2001). As a recommendation for students to better understand the concept rather than just trying it out, that is by practicing questions and asking the teacher about the concepts contained.

An overview of the results of the discussion regarding pseudo-construction errors at the action stage can be seen in Figure 9 in the appendix, the process stage from Figure 10 in the appendix, the object stage from Figure 11 in the appendix, and the schematic stage from Figure 12 in the appendix.

The results of this research are similar to the results of previous research (Anggraini et al., 2018; Herna et al., 2016; Hurst & Hurrell, 2020; Ni'mah et al., 2018; Subanji & Nusantara, 2016) which used pseudo-thinking or pseudo-construction as the basis for its thinking, where students experience pseudo-thinking or pseudo-construction in solving problems, and we agree that pseudo-thinking or pseudo-construction can be transformed into real thinking in students' schemes. Our difference with their research is that theoretically, we have presented a picture related to pseudo-construction based on APOS theory (action, process, object, and schema).

### Implication of Research

The use of APOS theory can influence the construction of concepts that will be embedded in students' schemes. Concept construction errors can be seen more clearly through the APOS framework (Choirun Nisa et al., 2021; Parraguez & Oktaç, 2010; Rofiki et al., 2020; Silalahi, 2017). The results of this research are used as input for students, teachers, prospective teachers, researchers, experts, and so on to minimize the occurrence of

errors in the construction of mathematical concepts, namely pseudo-construction errors so that mathematical concepts are constructed well or intact in the student's scheme.

### Limitation

This research has limitations, the researcher only examines one point of view on errors in the construction of mathematical concepts, namely pseudo-construction, although there are several other points of view on errors in the construction of mathematical concepts which may be a problem in this theme, such as construction holes, incorrect analogical constructions, and incorrect logical construction. One point of view of pseudo-construction errors that the researcher chose was based on the reason that the test results of 28 students of SMPN 2 Silo regarding errors in constructing mathematical concepts based on APOS theory all made pseudo-construction errors, so this research focuses on pseudo-construction errors to be studied in more depth. In addition, the study was limited to just one school.

### CONCLUSION

Apparent construction errors occur at every stage of APOS with various causes. At the action stage, pseudo-construction errors occur because students build concepts by memorizing without understanding the actual concept and because students are not careful in reading the questions. At the process stage, pseudo-construction errors occur due to incorrectly stating the first and second terms of the composition of the number pattern and because students incorrectly state the concept of the number pattern contained in the known number pattern.

At the object stage, pseudo-construction errors occur because students have the wrong concepts in solving the problem. At the scheme stage, pseudo-construction errors occur because students have the wrong concept and are just trying out the method used to solve the problem, even though the final answer is correct. This conclusion shows that the concept is still not well constructed, which results in the concept not being well embedded in the student's schema. Therefore, we recommend providing a learning approach called the ACE learning cycle (Activity, Class discussion, and Exercise) which is a learning approach based on the APOS theory learning model so that concepts are well constructed and intact in the student's scheme.

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## Appendix.

Kerjakanlah soal berikut ini!

Dibawah ini terdapat suatu barisan bilangan yang membentuk pola persegi panjang, kecuali?

- a. 2, 6, 12, 20
- ⓑ 1, 4, 9, 16
- c. 42, 56, 72, 90
- d. 12, 20, 30, 42
- e. 110, 132, 156, 182

Alasannya?

*Karena pola bilangan barisannya tidak termasuk pola  
persegi panjang*

Figure 1. Preliminary Research Results

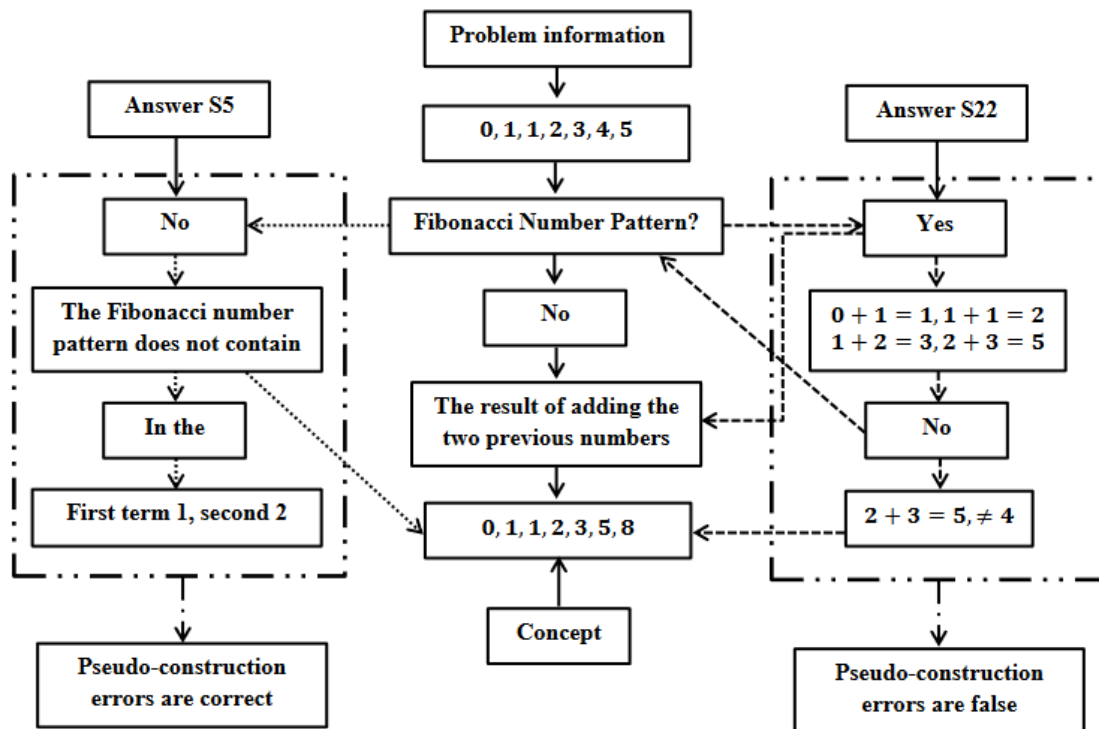


Figure 5. Student Answers at the Action Stage

Note:

————> : Problem and concept information

.....> : Answer S5

-----> : Answer S20

..... ] : Construction hole error

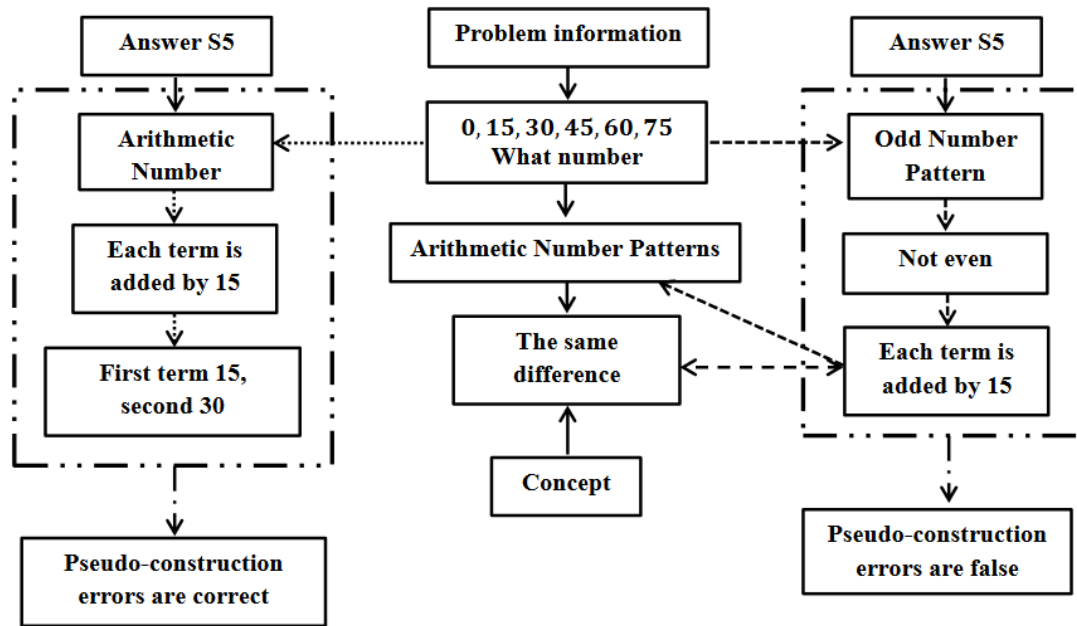


Figure 6. Student Answers at the Process Stage

**Note.**

- : Problem and concept information
- .....→ : Answer S5
- : Answer S20
- - - - - ] : Construction hole error

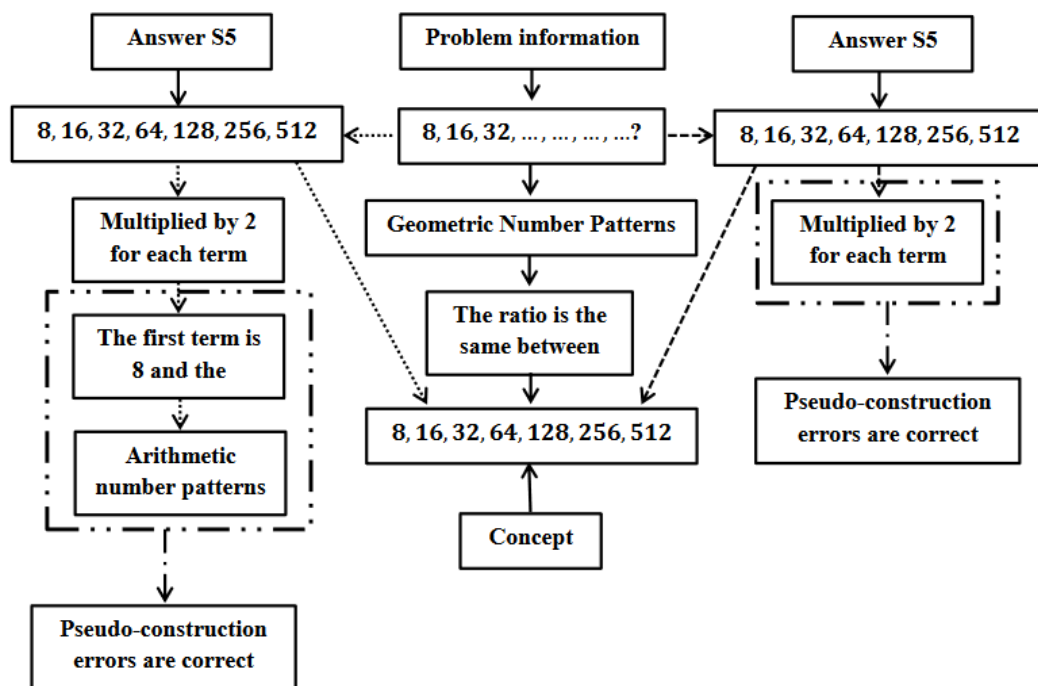


Figure 7. Student Answers at the Object Stage

**Note.**

- : Problem and concept information
- .....→ : Answer S5
- : Answer S20
- - - - - ] : Construction hole error

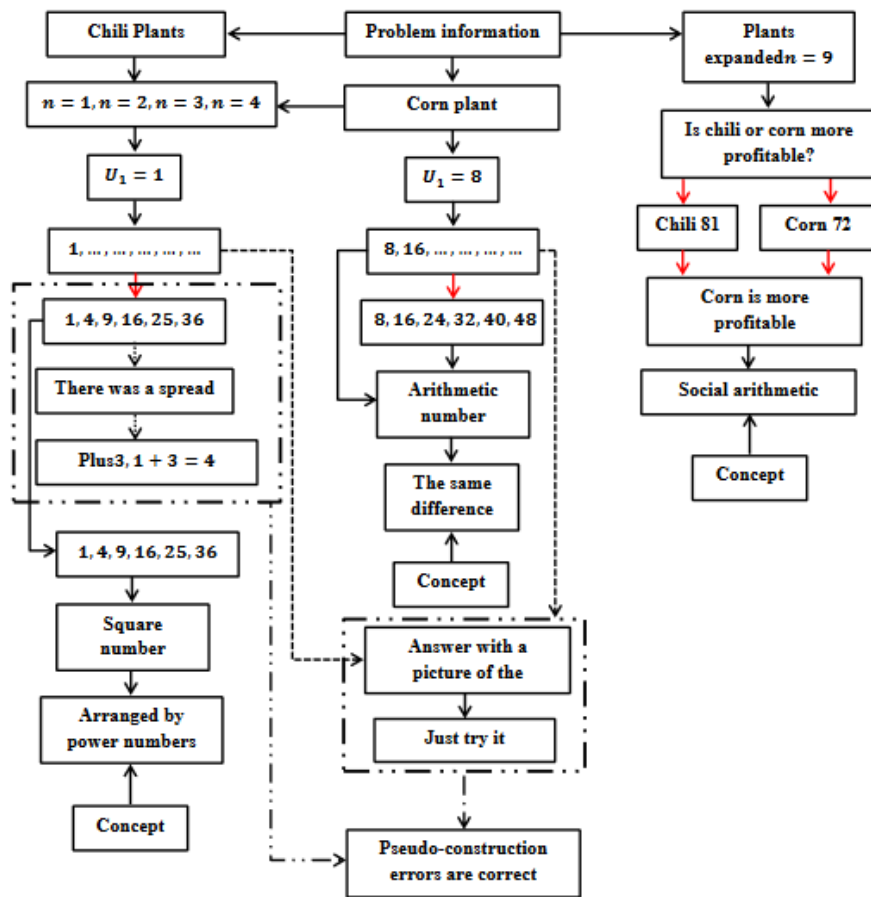


Figure 8. Student Answers at the Scheme Stage

**Note.**

→ : Problem and concept information

→ : Answer S<sub>5</sub> and S<sub>20</sub>

⋮ : Construction hole error

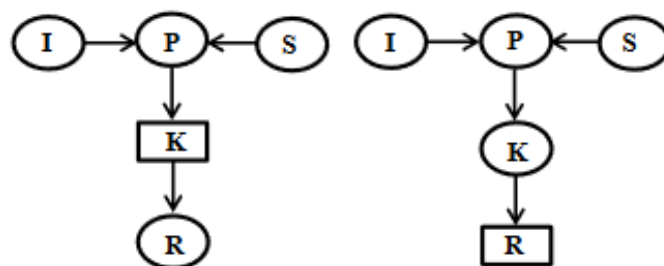


Figure 9. Action Stage

**Information.**

○ : Correct  
□ : Wrong

I: Problem information

P: Perception

S: Stimulus

K: Concept

R: Response

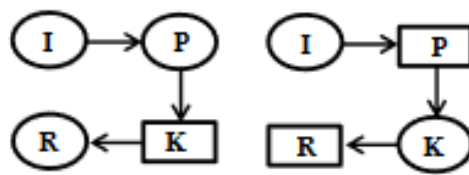


Figure 10. Process Stage

**Information.**

○ : Correct  
 □ : Wrong

I: Problem information | P: Perception | K: Concept | R: Response

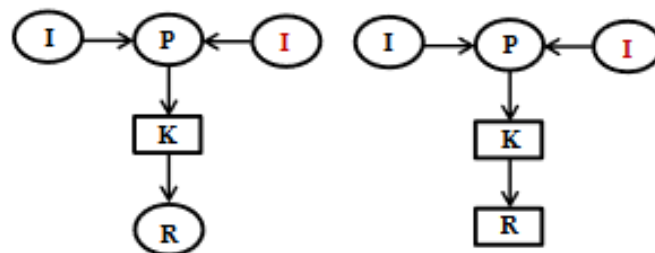


Figure 11. Object Stage

**Information.**

○ : Correct  
 □ : Wrong

I: Problem information | P: Perception | K: Concept | R: Response | I: Stimulus from the problem information itself

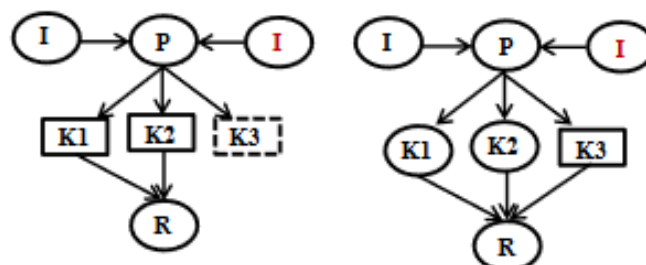


Figure 12. Schematic stage

**Information.**

○ : Correct  
 □ : Wrong  
 □ : Wrong or incomplete

I: Problem information | P: Perception | K: Concept | R: Response | I: Stimulus from the problem information itself | K1: Algebraic concept | K2: The concept of social arithmetic | K3: Geometric concept