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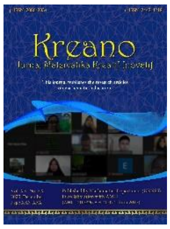

Elementary School Students' Conceptual and Procedural Knowledge in Solving Fraction Problems

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Abstract

Conceptual and procedural knowledge is fundamental for students to understand and solve fraction problems comprehensively. However, empirical studies indicate that elementary school students still do not have adequate conceptual and procedural knowledge in solving fraction problems. This study aimed to analyze the forms of conceptual and procedural knowledge of elementary school students in solving fraction problems. This study used a qualitative method involving 86 participants from grades four to six in one of the public elementary schools in Sidoarjo. Seven subjects were selected to represent the forms based on each aspect of conceptual and procedural knowledge of fractions. The data analysis used was descriptive analysis with data collection methods using tests, interviews, and documentation. Empirical research showed that students can successfully use conceptual and procedural knowledge of fractions in certain forms. There are three forms of conceptual knowledge of fractions: comparing, applying, and visualizing fractions. Meanwhile, there are four forms of procedural knowledge of fractions: explaining procedures, converting fractions, adding or subtracting fractions, and simplifying fractions. The results of this study have implications for educators or academics to emphasize learning by integrating forms of conceptual and procedural knowledge so that students avoid failure in solving fraction problems.

Keywords: *Concepts; Procedures; Fractions*

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Abstrak

Pengetahuan konseptual dan prosedural merupakan pengetahuan mendasar agar siswa dapat memahami dan memecahkan masalah pecahan secara komprehensif. Namun, berdasarkan studi empiris diindikasikan bahwa siswa sekolah dasar masih belum memiliki pengetahuan konseptual dan prosedural yang memadai dalam memecahkan masalah pecahan. Tujuan penelitian ini untuk menganalisis bentuk-bentuk pengetahuan konseptual dan prosedural siswa sekolah dasar dalam memecahkan masalah pecahan. Penelitian ini menggunakan metode kualitatif dengan melibatkan 86 partisipan siswa kelas empat sampai enam di salah satu sekolah dasar negeri di Sidoarjo. Tujuh subjek dipilih untuk mewakili bentuk-bentuk berdasarkan setiap aspek pengetahuan konseptual dan prosedural pecahan. Analisis data yang digunakan adalah analisis deskriptif dengan metode pengumpulan data menggunakan tes, wawancara dan dokumentasi. Hasil penelitian secara empiris menunjukkan siswa berhasil menggunakan pengetahuan konseptual dan prosedural pecahan dalam bentuk-bentuk tertentu. Terdapat tiga bentuk pengetahuan konseptual pecahan, yaitu: membandingkan pecahan, menerapkan pecahan, dan memvisualisasikan pecahan. Sementara, terdapat empat bentuk pengetahuan prosedural pecahan, yaitu: menjelaskan prosedur, mengkonversi pecahan, menjumlahkan atau mengurangkan pecahan, dan menyederhanakan pecahan. Hasil penelitian ini berimplikasi bagi para pendidik atau akademisi untuk menekankan pembelajaran dengan mengintegrasikan bentuk-bentuk pengetahuan konseptual dan prosedural, agar siswa terhindar dari ketidakberhasilan dalam memecahkan masalah pecahan.

INTRODUCTION

Fractions is an important topic because it is a prerequisite for students' success in understanding further topics that are more complex (Dogan-Coskun, 2019; Flores et al., 2020; Karika & Csikos, 2022; Laidin & Tengah, 2021; Zhang et al., 2020). Bennett et al. (2012) mentioned further topics based on fraction knowledge, including decimal numbers, rational and irrational numbers, and real numbers. However, students' understanding of fraction material is not always optimal. Students often have difficulty in distinguishing fractions from whole numbers, including students being unable to represent numerators and denominators as parts of a whole (Deringol, 2019; Dogan-Coskun, 2019; Durkin & Rittle-Johnson, 2015; Simon et al., 2018). In this case, Lenz et al. (2020) indicated that students who understand fractions if they have sufficient conceptual and procedural knowledge.

More specifically, conceptual knowledge of fractions is used to form a basic conceptual understanding of the meaning of fractions, how fractions are represented, and how fractions are used in mathematical situations (Braithwaite & Sprague, 2021; Canobi, 2009; Maulina et al., 2020). Meanwhile, procedural knowledge is used to operate and apply these concepts to fractions, which includes skills in calculating and manipulating fractions according to the correct rules and steps (Baroody et al., 2007; Braithwaite & Sprague, 2021; Star & Stylianides, 2013).

Experts state that adequate conceptual and procedural knowledge is most needed for elementary school students (Abbas et al., 2022; Nahdi & Jatisunda, 2020; Wiest & Amankonah, 2021). In this case, for elementary school students, conceptual and procedural knowledge about fractions is needed to help determine strategies, provide logical answers, and detect errors in solving problems (Braithwaite & Sprague, 2021; Manandhar et al., 2022; Rittle-Johnson, 2017; Saban et al., 2021). Furthermore, according to Abbas et al. (2022), elementary school students need adequate conceptual and procedural knowledge to solve complex fraction problems. Therefore, conceptual and procedural knowledge becomes the foundation of elementary school students' mathematical competence which has an impact on the success of subsequent mathematics learning achievements (Nahdi & Jatisunda, 2020; Rittle-Johnson et al., 2015).

Specifically, conceptual and procedural knowledge about fractions is needed for elementary school students to solve basic fraction problems successfully. Suppose students are given a problem adapted from Pirie & Kieren (1989): "Prove and give the steps to show which of the larger pieces of cake that person A and B share with each person. If person A shares 1 cake with 3 people, while person B shares 3 cakes with 7 people". In this problem, students must intuitively use procedural knowledge and conceptual knowledge by drawing fractions of 1 from 3 and 3 from 7.

Based on preliminary studies at Pucang Sidoarjo State Elementary School (SDN) in September 2023, some preliminary findings were obtained namely, some students: (1) did not fully visually represent part of a whole simple fraction; (2) were accustomed to representing fractions on a number line, rather than a picture or visually; (3) only focused on procedural counting; (4) could not explain the meaning of fractions using their own language; (5) could not explain the relationship between mixed fractions to ordinary fractions, or vice versa. In this case, experts mention the difficulties experienced by students in understanding and solving fraction problems because students lack adequate conceptual and procedural knowledge of fractions (Braithwaite & Sprague, 2021; Hussein, 2022; Morano & Riccomini, 2020; Simon, 2019).

Meanwhile, previous studies have not specifically discussed the forms of conceptual and procedural knowledge of elementary school students in solving fraction problems. Lenz et al. (2020) examined conceptual and procedural knowledge fractions, but not qualitatively or with elementary school students. Laily et al. (2020) examine conceptual and procedural knowledge profiles in fractions-related geometry problems. Phuong (2020) examines students' conceptual and procedural knowledge in solving math problems in general. Manandhar et al. (2022) examined conceptual and procedural knowledge of algebra. Idrus et al. (2022) examine elementary school students' conceptual knowledge of area measurement. Tesfaye et al. (2020) examined the conceptual and procedural knowledge of number pattern concepts.

Thus, to obtain solutions so students can successfully use conceptual and procedural knowledge in solving fraction problems, it is necessary to conduct research to deeply analyze the forms of conceptual and procedural knowledge elementary school students use in solving fraction problems. Hurrell (2021) revealed that research on conceptual and procedural knowledge analysis provides significant material to improve student learning performance and the quality of teacher teaching. Therefore, the results of this study are expected to contribute knowledge to recognize the forms of success of elementary school students so that students avoid unsuccessful use of conceptual and procedural knowledge fractions during learning. Hence, teachers, practitioners, and researchers can evaluate and provide more appropriate learning treatments to use conceptual and procedural knowledge on the topic of fractions or other topics in general more comprehensively.

METHOD

The method used in this research is qualitative with a *case study* approach. Qualitative research is a research procedure that uses descriptive data in the form of written or spoken words obtained from sources, while the case study approach is a process of collecting data and information in depth (Creswell, 2012). In this case, a case study is used to deeply analyze the phenomenon of conceptual and procedural knowledge of elementary school students in solving fraction problems.

This research was conducted at SDN Pucang Sidoarjo. There were 86 participants, consisting of 26 fourth graders, 30 fifth graders, and 30 sixth graders. Determining the subject uses purposive methods based on consideration and focuses on particular characteristics relevant to the research (Creswell, 2012). In this case, purposive determination of subjects was carried out by taking one student who successfully used conceptual and procedural knowledge fractions. One student was taken to represent each form of conceptual and procedural knowledge of fractions, including 3 aspects of conceptual knowledge, namely: C1 (fraction comparison), C2 (fraction application), C3 (fraction visualization), and procedural knowledge, namely: P1 (procedure verbalization), P2 (fraction conversion), P3 (fraction addition or subtraction), P4 (fraction simplification). Furthermore, the researcher coded the subject's initials, resulting in S1 (subject 1)-S7 (subject 7).

This research instrument consists of conceptual and procedural fraction knowledge tests and interviews. The test aims to obtain written data for constructing and justifying students' conceptual and procedural knowledge in solving fraction problems. The instrument was developed based on conceptual and procedural knowledge indicators from Lenz et al. (2020). The instrument developed has 7 problems, including 1 problem representing aspects C1-C3 and P1-P4, as shown in Table 1. The conceptual and procedural knowledge test instruments are presented in Table 2. Meanwhile, the interview contains a series of oral questions regarding the conceptual and procedural knowledge possessed by subjects, which aims to deepen the results of the test instruments that have been carried out.

The research stages were carried out through several steps: reviewing the literature, developing and validating research instruments, giving tests to all students, conducting interviews on selected subjects, triangulating data, and analyzing data. The overall stages of the research are presented in Figure 1.

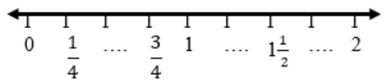
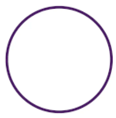
The credibility of this research data uses triangulation, a method used to verify data from various sources using various methods at different times (Creswell, 2012). The type of triangulation used is a technique triangulation. In this case, researchers analyzed or extracted data obtained during observations of test completion, test results, and interview highlights based on C1-C3 and P1-P4.

Table 1. Fraction Conceptual and Procedural Knowledge Indicators

Knowledge	Aspects	Knowledge Indicators	Code
Concept (C)	Fraction comparison	Verbalizing fraction concept	C1
	Fraction application	Applying the concept of fractions on a number line	C2
	Fraction visualization	Visualizing fractions in the form of pictures or graphs	C3
Procedure (P)	Verbalization of procedures	Verbalize procedures in solving fraction problems.	P1
	Fraction conversion	Converting fractions from one form to another	P2
	Fraction addition or subtraction	Performing addition or subtraction of fractions	P3
	Fraction simplification	Expanding and simplifying fractions	P4

(Lenz et al., 2020)

Table 2. Research Instruments

Number	Indicator Codes	Problems
1	C1	The sum of $\frac{1}{2}+\frac{1}{4}$ is greater than $\frac{1}{2}+\frac{1}{2}$. This is because 4 in the first addition is greater than 2 in the second addition. Explain why this is not correct.
2	C2	Describe the correct location to put the part that reflects the sum $1+\frac{1}{4}$ on the number line beside 
3	C3	Draw the sum of $\frac{2}{8}+\frac{4}{8}$ from the figures below! 
4	P1	Write the addition steps of fractions with different denominators!
5	P2	Change the fraction $3\frac{1}{4}$ into fraction addition form!
6	P3	Sum the following fractions $\frac{1}{2}+\frac{3}{5}$!
7	P4	$\frac{14}{5}+\frac{12}{5}$ has a simple form of addition of fractions $2\frac{4}{5}+2\frac{2}{5}$ which is a mixed fraction. Find another form of $\frac{14}{5}+\frac{12}{5}$ by completing the blanks in the following fractions $1\frac{....}{....}+1\frac{....}{....}$

The data analysis techniques include data reduction, data presentation, and conclusion drawing. At the data reduction stage, researchers focused on raw written data and interview quotes to match the conceptual and procedural knowledge indicators, namely C1-C3 and P1-P4. At the data presentation stage, the researcher presents a visualization of the snippets of the written test results and the corresponding interview excerpts. At the conclusion drawing stage, the researcher extracts the similarity of conceptual and procedural knowledge obtained during observation, test results, and interview results based on C1-C3 and P1-P4.

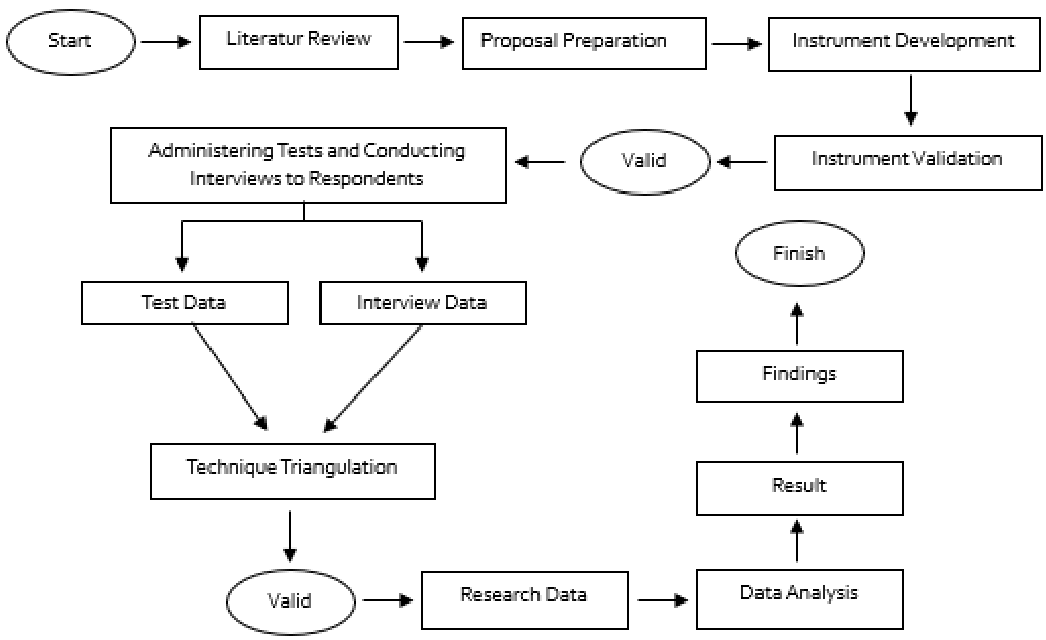


Figure 1. Research Stages

RESULTS AND DISCUSSION

Results

Referring to the conceptual and procedural knowledge indicators from Lenz et al. (2020), the data grouping is presented in Figure 2.

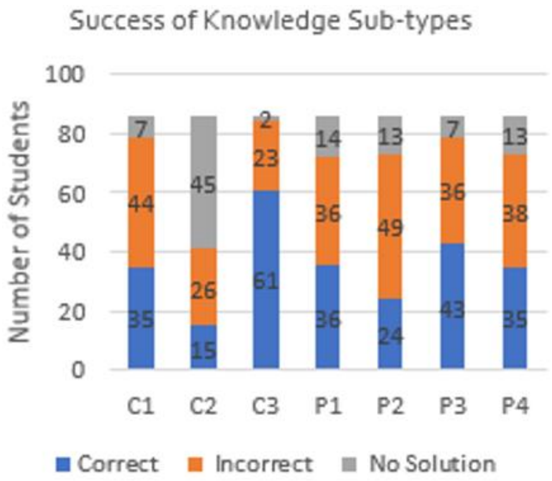


Figure 2. Data Grouping

Based on the data in Figure 2, there are differences in the level of achievement in each aspect of the problem. A brief overview of students' success in solving conceptual and procedural fraction problems is presented in Table 3.

Table 3. Recapitulation of Students' Conceptual and Procedural Knowledge

Knowledge Aspects	n	%	Subject
C1	35	40, 69%	S1
C2	15	17, 44 %	S2
C3	61	70, 93 %	S3
P1	36	41, 86 %	S4
P2	24	27, 90 %	S5
P3	43	50%	S6
P4	35	40, 69%	S7

Description :
n = Number of students who answered correctly
S1-S7 = Research subjects

Table 3 shows students' success in solving conceptual (C1-C3) and procedural (P1-P4) fraction problems. There were 35 students in C1, 15 students in C2, 61 students in C3, 36 students in P1, 24 students in P2, 43 students in P3, and 35 students in P4 who answered correctly. Only 1 student obtained the highest conceptual and procedural level for each form of knowledge. In this case, S1-S3 represents the success of conceptual knowledge forms C1-C3. Meanwhile, S4-S7 each represents the success of procedural knowledge forms P1-P4.

Conceptual Knowledge of C1

Figure 3 shows the results of student work coded with S1.

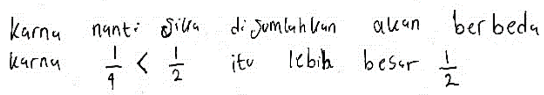


Figure 3. S1's Answer on C1

A total of 35 (40.69%) students answered correctly, as S1 in Figure 3. S1 can be said to have conceptual knowledge because it fulfills the indicators on C1. S1 successfully compared the fraction $\frac{1}{4}$ with $\frac{1}{2}$. S1 stated the concept of fractions correctly, namely, the larger the denominator of the fraction, the smaller the fraction value. S1 compares numerical values of fractions.

Researcher	:	*Are you sure $\frac{1}{2}$ is bigger than $\frac{1}{4}$?*
S1	:	*Sure.*
Researcher	:	*Isn't 4 bigger than 2?*
S1	:	*But, $\frac{1}{4}$ does 4 while dividing $1\frac{1}{2}$ is 1 divided by 2. More people get $\frac{1}{2}$ cake than $\frac{1}{4}$ cake.*

Based on the interview results, S1 understands that fractions are a single number, not as two separate integers. This reflects a strong understanding that fractions are a way of describing a part of a whole. S1 has shown a good understanding of this concept and can apply it well in mathematical problem-solving.

Conceptual Knowledge of C2

Figure 4 shows the results of student work coded with S2.

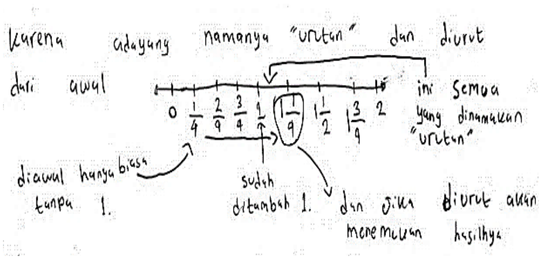


Figure 4. S2's Answer on C2

A total of 15 (17.44%) students answered correctly as S2 in Figure 4. However, only S2 answered using a simple chart. S2 showed conceptual solid knowledge of fractions as evidenced by successfully placing the fraction 1 + $\frac{1}{4}$. In this case, S2 has a high level of conceptual knowledge because he can apply the concept of fractions to the number line.

Researcher	:	*Are you sure the fraction $\frac{1}{4}$ is there?*
S2	:	*Yes, sure.*
Researcher	:	*Try to explain why it is located there.*
S2	:	*I see that after 0, it is $\frac{1}{4}$, after that is an empty area, then $\frac{3}{4}$ and 1. I know that the empty area is $\frac{2}{4}$. So after 1, come back $1\frac{1}{4}$ then $1\frac{1}{2}$, $1\frac{3}{4}$ and 2.*
Researcher	:	*I see.*

Based on the interview results, S2 works on problem C2 with the help of whole numbers 0 and 1. S2 examined the whole number part to evaluate the fraction comparison on the number line. In addition, S2 can connect the idea of fractions with equal parts of the whole. S2 can identify that fraction $\frac{2}{4}$ has the same value as a fraction $\frac{1}{2}$. Although the number line task on fractions is difficult, it can effectively understand fractions because it matches the desired mental representation and utilizes pre-existing spatial-numerical knowledge.

Conceptual Knowledge of C3

Figure 5 shows the results of student work coded with S3.

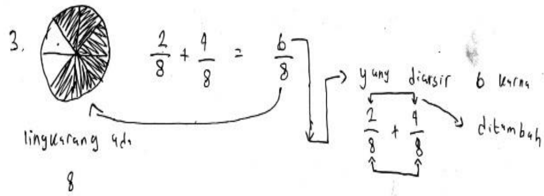


Figure 5. S3's Answer on C3

A total of 61 (70.93%) students answered correctly as S3 in Figure 5. However, some students were unable to answer the C3 questions correctly. S3 succeeded in visualizing the addition of fractions $\frac{2}{8}+\frac{4}{8}$.

Researcher	:	*Why is the shading like that?*
S3	:	*Yes, because $\frac{2}{8}+\frac{4}{8}$ equals $\frac{6}{8}$, so the shading is 6 parts out of 8.*
Researcher	:	*If there is no + sign. How do you do it?*
S3	:	*Yes, draw $\frac{2}{8}$ and $\frac{4}{8}$.*
Researcher	:	*Is the drawing the same as $\frac{2}{8}+\frac{4}{8}$?*
S3	:	*Same.*
Researcher	:	*Are you sure?*
S3	:	*Sure.*

Based on the interview results, In working on problem C3, S3 focuses on the addition problem so that S3 operates first before visualizing the fraction. Even so, S3 understood the concept in question C3. S3 did not feel confused if asked to visualize directly the fractions of $\frac{2}{8}$ and $\frac{4}{8}$ on the pie chart. Correct conceptual knowledge helps S3 visualize fractions and relate them to whole numbers. As S3 said in the interview that $\frac{6}{8}$ or 6 of 8 parts. This indicates good conceptual knowledge. S3 understands that a fraction represents a part of a number.

Procedural Knowledge of P1

Figure 6 shows the results of student work coded with S4.

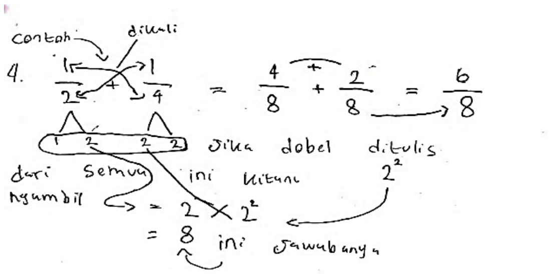


Figure 6. S4's Answer on P1

A total of 36 (41.86%) students answered correctly, such as S4 in Figure 6. However, only S4 answered with an example of fraction addition $\frac{1}{2}+\frac{1}{4}$. S4 used the factorization method to get the same denominator, which is the numeral 8.

Researcher	:	*Where did you get the denominator 8 from?*
S4	:	*By factor trees 2 and 4.*
Researcher	:	*After that, how can it be $\frac{6}{8}$?*
S4	:	*After the denominator is equal, divide the unequal denominator, then multiply the numerator.*
S4	:	*How's that?*
Researcher	:	*8 is divided by 2 and then multiplied by 1 to make $\frac{4}{8}$. 8 is divided by 4 and then multiplied by 1 to get $\frac{2}{8}$. So $\frac{4}{8}+\frac{2}{8}$ equals $\frac{6}{8}$.*

Based on the interview results, S4 explained how to add fractions with different denominators, and both denominators must be equalized first by using factorization. S4 explained the steps in detail, including dividing unequal denominators, using numerators to sum, and using factor trees to illustrate the process. S4 showed strong knowledge of fraction addition operations and identification of common denominators. This indicates that S4 has good knowledge of mathematical concepts, especially on fraction material.

Procedural Knowledge of P2

Figure 7 shows the results of student work coded with S5.

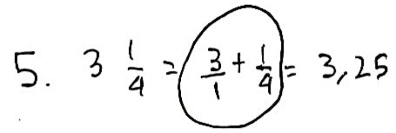


Figure 7. S5's Answer on P2

A total of 24 (27.90%) students answered correctly, like S5 in Figure 7. Problem P2 is converting fractions into addition forms. Many students answered by finding the fraction that produces $3\,\frac{1}{4}$. S5 knows that the fraction $3\,\frac{1}{4}$ consists of 3 as a whole number and $\frac{1}{4}$ as a fraction. This knowledge helps S5 in solving problem P2.

Researcher	:	*Why is the sum $\frac{3}{1}+\frac{1}{4}$?*
S5	:	*Since it's 3, I changed it to $\frac{3}{1}$ to make it a fraction.*
Researcher	:	*Do you think 3 is the same as $\frac{3}{1}$?*
S5	:	*Yes, 3 divided by 1 is 3.*
Researcher	:	*Then why should it be changed to $\frac{3}{1}$?*
S5	:	*So that, they can be summed.*

Based on the interview results, S5 understands how to add whole numbers with fractions. It is necessary to convert whole numbers into fractions. In this case, S5 converted the whole number 3 into a fraction $\frac{3}{1}$. S5 also showed strong knowledge of the concept of fractions, S5 stated that 3 equals $\frac{3}{1}$ because "3 divided by 1 is 3". This indicates that S5 has strong conceptual and procedural knowledge in solving fraction problems.

Procedural Knowledge of P3

Figure 8 shows the results of student work coded with S6.

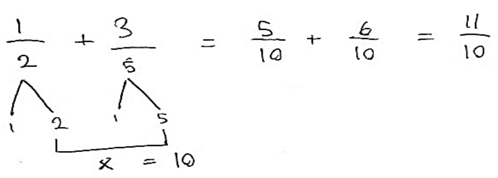


Figure 8. S6's Answer on P3

A total of 43 (50%) students answered correctly, like S6 in Figure 8. S6 solved problem P3 according to the correct procedure. Problem P3 is closely tied to problem P1. In this case, students who cannot verbalize the procedure for adding fractions with different denominators in P1 certainly cannot answer problem P3.

Researcher	:	*Where did you get the number 10?*
S6	:	*From that, use the downline, which is the name I forgot.*
Researcher	:	*Tree factor?*
S6	:	*That's right!*
Researcher	:	*Why use a factor tree?*
S6	:	*Looking for a common denominator, after that, calculate.*
Researcher	:	*I see.*

Based on the interview results, S6 solved problem P3 by equalizing the denominator before operating the addition of fractions. S6 used the factor tree to find the common denominator before operating it. S6 identified the number 10 as the result of finding the least common multiple (LCM) of the two denominators of the fraction. This indicates that S6 has the correct procedural knowledge on how to solve fraction addition problems with different denominators. S6 also showed good knowledge of the least common multiple (LCM) concept and how it is used to equalize denominators.

Procedural Knowledge of P4

Figure 9 shows the results of student work coded with S7.

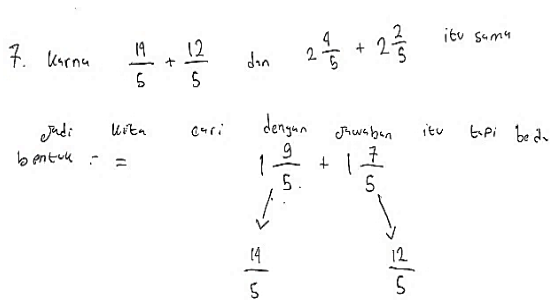


Figure 9. S7's Answer on P4

A total of 35 (40.69%) students answered correctly, like S7 in Figure 9. In solving P4 problems, students must understand the procedure for converting mixed fractions to simple fractions, e.g., $1\frac{9}{5}$ into $\frac{14}{5}$.

Researcher	:	*Why can you answer* $1\frac{9}{5}$ *and* $1\frac{7}{5}$*?*
S7	:	$2\frac{4}{5}+2\frac{2}{5}$ *is another form of* $\frac{14}{5}$ *and* $\frac{12}{5}$.
Researcher	:	*Then?*
S7	:	*The question is already filled with 1 and 1. so the denominator is filled with 5, and the numerator is 9 and 7.*
Researcher	:	*Where do 9 and 7 come from?*
S7	:	*That's 5 times 1 equals 5, so to make it 14 means less than 9.*
Researcher	:	*Which 7?*
S7	:	*That's the same way.*

Based on the interview results, S7 understands how to convert mixed fractions into regular ones. S7 realized that $1\frac{9}{5}$ is another form of $\frac{14}{5}$ and $1\ \frac{7}{5}$ is another form of $\frac{12}{5}$. This shows that S7 understands the basic concept of managing mixed fractions. S7 described the conversion process well. S7 multiplied the whole number found in the mixed fraction (1 and 1) by the denominator, then subtracted the result from the original numerator. S7 described the conversion process well. S7 multiplied the whole number found in the mixed fraction (1 and 1) by the denominator, then subtracted the result from the original numerator. S7 demonstrated a strong understanding of converting mixed fractions to ordinary fractions and the ability to answer question P4 correctly by understanding the mathematical concepts involved. S7's ability to expand and simplify fractions with conversion procedures reflects a good understanding of mixed fractions and fraction operations.

Discussion

The research findings empirically produce three forms of conceptual knowledge and four forms of procedural knowledge by elementary school students in solving fraction problems to measure conceptual and procedural knowledge of fractions comprehensively. The forms of conceptual knowledge include comparing fractions, applying fractions, and visualizing fractions. Meanwhile, forms of procedural knowledge include explaining procedures, converting fractions, adding or subtracting fractions, and simplifying fractions. This finding is similar to that of Lenz et al. (2020), who stated that at least seven aspects of conceptual and procedural knowledge contribute to solving fraction problems. Meanwhile, Nahdi & Jatisunda (2020) formulated five forms to measure conceptual and procedural knowledge of fractions, including the definition of fractions, comparison of fractions, concepts of addition, subtraction, and multiplication with fractions, representation, and rules of operation.

Another finding in this study is that the forms of conceptual and procedural knowledge contribute to each other and are holistically related to be used to solve fractional problems. This is different from the findings of Lenz et al. (2020) in that aspects of conceptual and procedural knowledge of fractions were found separately as success factors in solving fraction problems. Al-Mutawah et al. (2019) support this researcher's findings, stating that conceptual and procedural knowledge positively correlates with mathematical problem-solving. Nahdi & Jatisunda (2020) also said that students with conceptual and procedural knowledge together (holistic) can develop good knowledge in mathematics. Therefore, forms of conceptual and procedural knowledge play an important role in students' ability to solve fractional problems. In this case, students with thorough conceptual and procedural knowledge of fractions tend to successfully solve problems flexibly and comprehensively.

Regarding how fractional forms of conceptual knowledge can be successful? The success of comparing fractions is determined by the knowledge of the basic concept of fractions, namely understanding that fractions are a single number, not two separate whole numbers. Braithwaite & Sprague (2021) explain that if students view fractions as non-unitary numbers, then this reflects a strong conceptual foundation. Meanwhile, the knowledge of the fraction equivalence concept determines the success of applying observed fractions based on number line representation. In detail, Hoon et al. (2021) revealed that success in applying fractions on a number line is determined by (1) finding intervals in fractions on a number line, (2) applying the concepts of decimals and exchange with fractions, (3) comparing the equivalence values of fractions. The last form of conceptual knowledge of fractions is visualizing fractions. Visualizing fractions shows students' basic fraction knowledge regarding part to whole. Bennett et al. (2012) explained that the concept of part to whole must be visualized by elementary school students to reflect intuitive understanding.

On the other hand, how can fractional forms of procedural knowledge be successful? In explaining procedures, students illustrate the procedures that occur, for example, using a chart. According to

Gembong (2020), students use sketches to connect the steps of solving fraction problems. In the aspect of converting fractions, students convert mixed fractions to fraction addition or decimal fractions. According to Amir et al. (2021), students have a relational understanding of the relationship of fraction forms to the operations that connect them. In the form of adding or subtracting fractions, students equalize the denominator by finding the smallest common multiple and then operate the numerator. Gembong (2020) explains that the first step as a procedure usually taken by students to make it easier to operate addition or subtraction of fractions is to equalize the denominator by finding the common multiples. Hwang et al. (2019) called this way of equating denominators a decomposition strategy. The last form of procedural knowledge is simplifying fractions. In this case, students simplify mixed fractions to other simpler mixed fractions. Amir & Wardana (2018) stated that students who successfully convert mixed fractions to other simpler mixed fractions are because students can think openly and flexibly.

The successful formation of conceptual and procedural knowledge of fractions can also be further elaborated based on the APOS mental structure theory: action, process, object, and schema. Arnon et al. (2014) explain that in action, a person gives a reaction to external stimuli. In the process, one can repeat and reflect on the entire mental sequence that has been done. In object, one realizes that the actions performed are part of the transformation of knowledge. A schema forms a person's knowledge into a coherent framework for use in other situations. In other words, when elementary school students are first confronted with fraction problems, they take action by linking their conceptual and procedural knowledge about fractions and then looking for a solution strategy. Students try to apply the steps of the solution strategy by using procedural knowledge. In this case, if the problem requires conceptual understanding (for example, visualizing fractions), students use conceptual knowledge. Students who successfully solve different fraction problems show that they have flexible conceptual and procedural knowledge. Students will comprehensively own this conceptual and procedural knowledge if they can use their knowledge on other fraction problems that are more varied.

Implication of Research

The results of this study have important implications in the context of mathematics learning, especially in understanding and overcoming fractions. Educators or academics must emphasize learning by integrating forms of conceptual and procedural knowledge so students are expected to avoid unsuccessful problem-solving fractions.

Limitation

This study analyzes the forms of conceptual and procedural knowledge of fractions in only a few elementary school students at different levels. Thus, the forms of conceptual and procedural knowledge of fractions that have been found need to be further justified by involving other research sites or more external participants to form a theory of forms of conceptual and procedural knowledge in solving fraction problems that are grounded in theory.

CONCLUSION

Elementary school students have seven forms of conceptual and procedural knowledge in solving fractions: comparing fractions, applying fractions, visualizing fractions, explaining procedures, converting fractions, adding or subtracting fractions, and simplifying fractions. This result emphasizes the importance of holistic integration between conceptual and procedural knowledge as a crucial factor in solving fraction problems successfully. Overall, the holistic integration of conceptual and procedural knowledge is an essential key to the success of elementary school students or students in general when it comes to solving fractions.

REFERENCES

Abbas, N. A., Abdullah, N. A., Shahrill, M., & Tengah, K. A. (2022). *Primary school pupils' performance on the addition of fractions: Conceptual and procedural knowledge*. *16*(2), 227–238. https://doi.org/10.22342/jpm.16.2.17811.227-238

Al-Mutawah, M. A., Thomas, R., Eid, A., Mahmoud, E. Y., & Fateel, M. J. (2019). Conceptual understanding, procedural knowledge and problem-solving skills in mathematics: High school graduates work analysis and standpoints. *International Journal of Education and Practice*, *7*(3), 258–273. https://doi.org/10.18488/journal.61.2019.73.258.273

Amir, M. F., & Wardana, M. D. K. (2018). Pengembangan domino pecahan berbasis open ended untuk meningkatkan kemampuan berpikir kreatif siswa SD. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *6*(2), 178-188. http://dx.doi.org/10.24127/ajpm.v6i2.1015

Amir, M. F., Wardana, M. D. K., & Usfuriah, D. (2021). Visual and symbolic representation forming: a Case of relational understanding on elementary student. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(4), 2014-2028. https://doi.org/10.24127/ajpm.v10i4.4361

Arnon, I., Cottrill, J., & Dubinsky, E. (2014). *APOS Theory: A framework for research and curriculum development in mathematics education*. New York, NY: SpringerLink. https://doi.org/10.1007/978-1-4614-7966-6

Baroody, A. J., Feil, Y., & Johnson, A. R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. In *Source: Journal for Research in Mathematics Education*, *38*(2), 115-131. https://doi.org/10.2307/30034952

Bennett, A. B., Button, L. J., & Nelson, L. T. (2012). *Mathematics for elementary teachers : A conceptual approach, ninth edition*.

Braithwaite, D. W., & Sprague, L. (2021). Conceptual knowledge, procedural knowledge, and metacognition in routine and nonroutine problem solving. *Cognitive Science*, *45(10), e13048* https://doi.org/10.1111/cogs.13048

Canobi, K. H. (2009). Concept-procedure interactions in children's addition and subtraction. *Journal of Experimental Child Psychology*, *102*(2), 131–149. https://doi.org/10.1016/j.jecp.2008.07.008

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research, fourth edition.* Boston, MA: Pearson Education.

Deringol, Y. (2019). Misconceptions of primary school students about the subject of fractions. *International Journal of Evaluation and Research in Education*, *8*(1), 29–38. https://doi.org/10.11591/ijere.v8i1.16290

Dogan-Coskun, S. (2019). The analysis of the problems posed by pre-service elementary teachers for the addition of fractions. *International Journal of Instruction*, *12*(1), 1517–1532. https://doi.org/10.29333/iji.2019.12197a

Durkin, K., & Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction*, *37*, 21–29. https://doi.org/10.1016/j.learninstruc.2014.08.003

Fauzi, I., & Suryadi, D. (2020). Learning obstacle the addition and subtraction of fraction in grade 5 elementary schools. In *Jurnal Kajian Pendidikan Islam* 12(1), 51-68. https://doi.org/10.18326/mdr.v12i1.50-67

Flores, M. M., Hinton, V. M., & Meyer, J. M. (2020). Teaching fraction concepts using the concrete-representational-abstract sequence. *Remedial and Special Education*, *41*(3), 165–175. https://doi.org/10.1177/0741932518795477

Gembong, S. (2020). The scheme construction to solve the adding fractions problems using images conducted by elementary students. *Journal of Physics: Conference Series*, *1567*(4). https://doi.org/10.1088/1742-6596/1567/4/042030

Hoon, T. S., Narayanan, G., & Singh, P. (2021). Applying fractional strategies on number line among primary school students. *Pertanika Journal of Social Sciences and Humanities*, *29*(1), 127–139.

Hurrell, D. (2021). Conceptual knowledge or procedural knowledge or conceptual knowledge and procedural knowledge: Why the conjunction is important to teachers. *Australian Journal of Teacher Education*, *46*(2), 57–71. https://doi.org/10.14221/ajte.2021v46n2.4

Hussein, Y. F. (2022). Conceptual knowlegde and its importance in teaching mathematics. *Middle Eastern Journal of Research in Education and Social Sciences*, *3*(1), 50–65. https://doi.org/ 10.47631/mejress.v3i1.445

Hwang, J., Riccomini, P. J., Hwang, S. Y., & Morano, S. (2019). A systematic analysis of experimental studies targeting fractions for students with mathematics difficulties. *Learning Disabilities Research and Practice*, *34*(1), 47–61. https://doi.org/10.1111/ldrp.12187

Idrus, H., Rahim, S. S. A., & Zulnaidi, H. (2022). Conceptual knowledge in area measurement for primary school students: A systematic review. *STEM Education*, *2*(1), 47–58. https://doi.org/10.3934/steme.2022003

Karamarkovich, S. M., & Rutherford, T. (2019). Fraction errors in a digital mathematics environment: Latent class and transition analysis. *Journal of Numerical Cognition*, *5*(2), 158–188. https://doi.org/10.5964/jnc.v5i2.150

Karika, T., & Csikos, C. (2022). A test for understanding simple fractions among 5th grade students at the beginning of lower secondary education. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(2), 1–14. https://doi.org/10.29333/EJMSTE/11654

Laidin, D. R., & Tengah, K. A. (2021). Applying butterfly method in the learning of addition and subtraction of fractions. *Jurnal Pendidikan Matematika*, *15*(2), 161–174. https://doi.org/10.22342/jpm.15.2.13934.161-174

Laily, N. I., Sunardi, Yuliati, N., Imamah, E. N., & Sari, M. P. (2020). The profile of students conceptual understanding and procedural knowledge in solving geometry problems based on van hiele levels. *Journal of Physics: Conference Series*, *1538*(1). https://doi.org/10.1088/1742-6596/1538/1/012080

Lenz, K., Dreher, A., Holzäpfel, L., & Wittmann, G. (2020). Are conceptual knowledge and procedural knowledge empirically separable? The case of fractions. *British Journal of Educational Psychology*, *90*(3), 809–829. https://doi.org/10.1111/bjep.12333

Manandhar, N. K., Pant, B. P., & Dawadi, S. D. (2022). Conceptual and procedural knowledge of students of Nepal in algebra: A mixed method study. *Contemporary Mathematics and Science Education*, *3*(1), ep22005. https://doi.org/10.30935/conmaths/11723

Maulina, R., Zubainur, C. M., & Bahrun. (2020). Conceptual and procedural knowledge of junior high school students through realistic mathematics education (RME) approach. *Journal of Physics: Conference Series*, *1460*(1). https://doi.org/10.1088/1742-6596/1460/1/012017

Morano, S., & Riccomini, P. J. (2020). Demonstrating conceptual understanding of fraction arithmetic: An analysis of pre-service special and general educators' visual representations. *Teacher Education and Special Education*, *43*(4), 314–331. https://doi.org/10.1177/0888406419880540

Nahdi, & Jatisunda. (2020). Conceptual understanding and procedural knowledge: A case study on learning mathematics of fractional material in elementary school. *Journal of Physics: Conference Series*, *1477*(4). https://doi.org/10.1088/1742-6596/1477/4/042037

Phuong, H. T. M. (2020). Measuring conceptual understanding, procedural fluency and integrating procedural and conceptual knowledge in mathematical problem solving. *International Journal of Scientific Research and Management*, *8*(05), 1334–1350. https://doi.org/10.18535/ijsrm/v8i05.el02

Pirie, S., & Kieren, T. (1989). A recursive theory of mathematical understanding. In *For the Learning of Mathematics* 9(3), 7–11. Motreal, Canada: FLM Publishing Association.

Rittle-Johnson, B. (2017). Developing mathematics knowledge. *Child Development Perspectives*, *11*(3), 184–190. https://doi.org/10.1111/cdep.12229

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, *27*(4), 587–597. https://doi.org/10.1007/s10648-015-9302-x

Saban, W., Raz, G., Grabner, R. H., Gabay, S., & Kadosh, R. C. (2021). Primitive visual channels have a causal role in cognitive transfer. *Scientific Reports*, *11*(1), 1–9. https://doi.org/10.1038/s41598-021-88271-y

Simon, M. A. (2019). *Developing a concept of multiplication of fractions: Building on constructivist and sociocultural theory*. 193–212. https://doi.org/10.1007/978-3-030-00491-0_9

Simon, M. A., Placa, N., Avitzur, A., & Kara, M. (2018). Promoting a concept of fraction-as-measure: A study of the learning through activity research program. *Journal of Mathematical Behavior*, *52*(February), 122–133. https://doi.org/10.1016/j.jmathb.2018.03.004

Star, J. R., & Stylianides, G. J. (2013). Procedural and conceptual knowledge: exploring the gap between knowledge type and knowledge quality. *Canadian Journal of Science, Mathematics and Technology Education*, *13*(2), 169–181. https://doi.org/10.1080/14926156.2013.784828

Tesfaye, A., Arefayne, N., Micael, K., Education, M., & Education, M. (2020). Early grade children procedural and conceptual knowledge in number pattern concept at Halaba. *Developing Country Studies*, *10*(12), 1–8. https://doi.org/10.7176/dcs/10-12-01

Wiest, L. R., & Amankonah, F. O. (2021). Conceptual versus procedural approaches to ordering fractions. *European Journal of Science and Mathematics Education*, *7*(1), 61–72. https://doi.org/10.30935/scimath/9534

Zhang, L., Shang, J., Pelton, T., & Pelton, L. F. (2020). Supporting primary students' learning of fraction conceptual knowledge through digital games. *Journal of Computer Assisted Learning*, *36*(4), 540–548. https://doi.org/10.1111/jcal.12422