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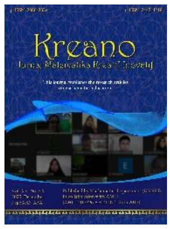

A Didactical Design Research: Knowledge Acquisition of Mathematics Teacher Prospective Students on Multiple Integral Concepts

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Abstract

This research aims to determine prospective mathematics teacher students' learning obstacles and knowledge acquisition process on the concept of fold integrals and their applications. In the acquisition process, students extract, structure, and organize knowledge both obtained from reading results and information received during the learning process in class; this is studied comprehensively. This research is Didactical Design Research. The research participants were 46 fourth-semester students in a study program producing prospective mathematics teachers. Research data comes from tests, observations, interviews, and documentation. The stages of data analysis are identification, clarification, reduction, and verification, which are then presented in a narrative descriptive manner. This research is essential because apart from identifying learning obstacles, various types of learning obstacles and the process of knowledge acquisition on related concepts and their multiple causes are also analyzed. The results obtained were (1) Students experienced ontogenic learning obstacles of psychological, instrumental, and conceptual types in the fold integral concept; apart from that, epistemological learning obstacles were also identified; (2) the process of acquiring knowledge related to the concept of double and triple integrals and their applications is not yet running perfectly, hampered by prerequisite knowledge, lack of technical ability, weak mastery of concepts, and limited understanding related to student habits. Further studies need to be carried out regarding the diffusion process by lecturers to facilitate the knowledge acquisition process so that students can construct the fold integral concept well.

Keywords: Didactical design research, Acquisition of knowledge, Learning obstacles, Multiple integrals.

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Abstrak

Penelitian ini bertujuan mengetahui hambatan belajar dan proses akuisisi pengetahuan mahasiswa calon guru matematika pada konsep integral lipat beserta aplikasinya. Pada proses akuisisi, mahasiswa melakukan ekstraksi, strukturisasi, dan pengorganisasian pengetahuan baik yang diperoleh dari hasil baca maupun informasi yang diterima selama proses belajar di kelas, hal ini dikaji secara kpmprehensif. Penelitian ini merupakan penelitian Didactical Design Research. Partisipan penelitian sebanyak 46 mahasiswa semester empat pada program studi penghasil calon guru matematika. Data penelitian berasal dari tes, observasi, wawancara, dan dokumentasi. Tahapan analisis data yaitu identifikasi, klarifikasi, reduksi, dan verifikasi, yang kemudian disajikan secara deskriptif naratif. Penelitian ini penting dilakukan, karena selain diidentifikasi hambatan belajar, juga dianalisa berbagai tipe hambatan belajar beserta proses akuisisi pengetahuan pada konsep terkait beserta berbagai penyebanya. Hasil yang diperoleh adalah (1) Mahasiswa mengalami hambatan belajar ontogenik tipe psikologis, instrumental, dan konseptual pada konsep integral lipat, selain itu teridentifikasi juga hambatan belajar epistemologis; (2) proses akuisisi pengetahuan terkait konsep integral lipat dua dan lipat tiga beserta aplikasinya belum berjalan sempurna, terkendala oleh pengetahuan prasyarat, kurangnya kemampuan teknis, lemahnya penguasaan konsep, dan terbatasnya pemahaman terkait dengan kebiasaan mahasiswa. Perlu dilakukan kajian lebih lanjut terkait proses difusi yang dilakukan dosen untuk memfasilitasi proses akuisisi pengetahuan, sehingga mahasiswa dapat mengkonstruksi konsep integral lipat dengan baik.

INTRODUCTION

Multiple integrals are one of the concepts in the Multivariable Calculus course, a course in the study program that produces prospective math teachers. In understanding the concept of multiple integrals, several concepts are prerequisites in multivariable calculus courses and other courses. Because of its position, there are still various problems in understanding the concept of integral multiple, both from the perspective of students, lecturers, or the curriculum.

In the secondary education curriculum, integral concepts and their applications are given at the high school level. Although at the application level, integrals are used to determine the area, volume of rotating objects, and length of curves (Susilo et al., 2019), at the high school level, the applications studied are limited to determining the area. Matondang, Saragih, & Maharani (2023) in their research concluded that the integral concept is still considered difficult by class XII students; on the other hand, Integral is given as an effort to equip high school students to be able to continue the material in college in calculus courses (Nursyahidah & Albab, 2017).

Fold integrals, as an extension of the concept of integral functions of one variable, certainly have a higher level of difficulty. At the application level, fold integrals can be used to determine area areas, volumes of solid objects, curve lengths, surface areas, and several applications outside the field of mathematics. When studying the concept of fold integrals, a good mastery of several things is required: 1) the nature and characteristics of the Cartesian coordinate system and polar coordinates; 2) the meaning of the function of two variables; 3) the surface shape of the function of two variables in R3; 4) the most appropriate integration method adapted to the function and form of the integration area; 5) various integration techniques. In the fold integral concept, several thinking skills are trained, including 1) objective thinking, in this case, students are required to solve problems based on supporting facts; 2) critical thinking, in this case, students are required to know the advantages and disadvantages of an integration technique adapted to the form of function and area of integration; 3) creative thinking, in this case, students are required to be skilled at solving problems in several possible ways;

4) think systematically, in this case students are required to solve problems systematically to choose the appropriate integration technique; 5) think logically, in this case students have the right reasons for choosing the integration technique that is most effective in solving a case. Based on this, prospective mathematics teachers must study and master the concept of fold integrals. Even though the idea of fold integrals is not taught at the high school level, the experience and skills gained by prospective mathematics teachers will equip them to develop professional competence as mathematics teachers. Ultimately, they can utilize these skills to help students construct knowledge of other concepts.

Several studies have been conducted and answered problems related to various difficulties experienced by students and multiple causes. Some student teacher candidates are still experiencing various problems in understanding the multiple integral concepts; this was revealed from Özbey & Şenol (2023) research. Students' ability to solve multiple integral problems is often influenced by an understanding of the shape of the curve in the field z=0 as a graphical representation of the function, which is the area of origin of the multiple integrals. In addition to recognizing the shape of the curve in the $z = 0$ plane, an understanding of the graph of the function of two variables and contour maps is also needed. The integration area can be identified from the shape of the curve in the field z = 0, and the integration model selection is easier to use Cartesian coordinates or polar coordinates.

The fact shows that students still experience difficulties sketching graphs of two variables' functions, as has been concluded from several research results. Mathematics teacher candidates still have problems drawing graphs of functions, both graphs of functions f(x,y) in Cartesian coordinates (Simorangkir & Sinaga, 2022; Utari, Septy & Hutauruk, 2021) as well as graphical sketches of functions in polar coordinates (Apriandi & Krisdiana, 2016). The results of this study are in line with the research of Kashefi, Ismail & Yusof (2010), who concluded that the most considerable difficulty experienced by students in multivariable calculus courses was sketching graphs of functions of two variables in 3 dimensions.

Students who cannot make or recognize the shape of a curve in the plane $z = 0$ or a graph of a function of two variables will result in various problems related to multiple integrals, as has been shown by various research results. Mathematics teacher candidates still experience difficulties in determining where integrals come from (Apriandi & Krisdiana, 2016). Besides that, student teacher candidates also still experience problems in determining integration limits; boundary determination errors are also found when variables change even though the initial integration limits are correct (Apriandi & Krisdiana, 2016; Simorangkir & Sinaga, 2022; Siti, 2015; Fahrudin, 2018).

Many cases in double integrals are difficult for students to solve in Cartesian coordinates, but they are easy to solve using polar coordinates. A transformation must be performed when it is decided to use the polar coordinate. However, students still consider transforming from Cartesian coordinates to polar coordinates difficult. This statement is in line with the results of research by Apriandi & Krisdiana (2016) and a study by Siti (2015), which concluded that students still had difficulty writing the form of integration in polar coordinates; this difficulty is an indication of not understanding variable transformations. from Cartesian to polar.

When students understand the shape of the curve in the plane $z = 0$, the form of the graph of the function of two variables, and the selection of the integration model, it does not mean there are no problems. The problems encountered include calculation errors, errors in selecting settlement procedures, etc. This statement is reinforced by the research results of Apriandi, D., & Krisdiana, I. (2016), and Fahrudin, F. A. (2018), concluding that students still have difficulty doing calculations or operations on integrals. Similar research, Simorangkir & Sinaga (2022), Utari Septy & Hutauruk (2021), Siti (2015), Fahrudin (2018) also concluded that students still have difficulty in selecting and using appropriate procedures or systematic problem-solving. Meanwhile, in their research, Ningtyas, Fuad & Lukito (2019) concluded that prospective mathematics teachers can present their ideas in a multi-representative manner but are not coherent and numerically imprecise.

The level of difficulty experienced by students for each sub-topic on double integrals varies, marked by the percentage of students who have difficulty solving problems related to multiple integrals. Judging from the level of student difficulty, Muchlis (2017) in his research concluded that the percentage of students' difficulty level in the concept of multiple integrals increased from the smallest to the highest, starting from determining the double integral in a rectangle, calculating fold integrals in polar coordinates; choose the surface area; determine the double integral in polar coordinates; and the double integral application.

Various things caused the difficulties experienced by students, i.e., not understanding how to determine integral boundaries, not understanding integration techniques, not understanding polar functions, being unable to describe trigonometry functions in polar coordinates, being unable to draw coordinate points when presented in variable form and determine their equations (Muchlis, 2017). Difficulties are also caused by the low initial ability of students plus advanced calculus material that is abstract and difficult (Takaendengan, Asriadi & Takaendengan, 2022). Learning style also plays an essential role besides mastery of the multiple integral concepts. This statement is in line with the research results of Kashefi, Ismail, & Yusof (2010), which concluded that the lack of instructions and the use of guiding questions adapted to student learning styles resulted in the less effective use of learning methods developed at Universiti Teknologi Malaysia (UTM) to overcome learning obstacles in multivariable calculus courses. Regarding learning styles Puspita (2020) concluded that differences in learning styles and achievements in differential calculus courses influence prospective mathematics teachers' accomplishments. From various research results, it indicates that the difficulties experienced by students can be related to the integral concept of multiple, prerequisite material, learning methods, and learning styles.

In Didactical Design Research (DDR), it is inseparable from the notion of didactic as its trademark. Didactics can be defined as the whole art of teaching all things to man (Comenius, 1657), the science of knowledge diffusion and acquisition in society (Chevallard, 2017), the epistemology of knowledge diffusion and acquisition in society (Suryadi, 2023). In simple terms, diffusion is defined as a learning process by lecturers, while acquisition is defined as a learning process by students. The process of diffusion and acquisition of knowledge will occur in a scientific community environment because there will be shifts from one institution to another. The knowledge produced by scientists will experience a change when it is presented in the curriculum as the knowledge that is taught, as well as will experience a shift when students learn the knowledge. Based on this description, the didactic design can be interpreted as a design regarding the diffusion of knowledge carried out by lecturers to facilitate knowledge acquisition by students.

There are three stages in DDR, namely: (1) analysis of the didactic situation before learning in the form of a hypothetical didactic design; (2) metapedadidactic analysis, and (3) retrospective analysis (Puspita, 2023). The focus of this study is the initial stage of DDR in the form of an analysis of the impact of didactic design on students' knowledge acquisition process, which is marked by the emergence of learning obstacles. The design of this study provides sufficient space to study student learning obstacles as a consideration in designing didactic designs aimed at overcoming learning obstacle findings. Brosseau (2002) defines obstacles as obstacles caused by external factors, while specifically, Suryadi (2019) states that the external factors in question are didactic designs.

Based on the source, Brousseau (2002) divides learning obstacles into three types, namely: (1) ontogenic learning obstacles, obstacles caused by students' mental unpreparedness and cognitive maturity in receiving knowledge; (2) didactical learning obstacles, learning obstacles caused by the didactic system in the form of sequences and stages in the curriculum as well as presentation in a learning process; and (3) epistemological learning obstacles, learning obstacles caused by limitations in students' mastery and understanding of something that is limitedly associated with a certain context adapted to the experiences they have experienced.

Suryadi (2019) divides ontogenic learning obstacles into three types: (1) psychological type, in the form of students' unpreparedness to learn as a result of cognitive maturity (related to the level of ability to think), and psychological aspects (weak interest or motivation towards the material); (2) the instrumental type, namely the inability of students to participate in learning caused by not mastering key technical matters of a problem being solved; (3) conceptual type, namely difficulties related to the conceptual level contained in the learning design.

About the type of learning obstacle, the meaning of didactic can be interpreted: (1) Didactic as an art means that the design developed is oriented towards the process of diffusion and acquisition of knowledge, which has a constructive and enjoyable impact, especially for students so that they avoid ontogenic learning obstacles; (2) Didactics as a science implies that the process of knowledge diffusion that is designed must be systemic in nature, namely clear in terms of sequence, stages, and coherence so that students avoid didactic learning obstacles; (3) Didactic as epistemology can be interpreted that the design developed must ensure that the process of diffusion and acquisition of knowledge is epistemic in producing knowledge, namely as a process of Justified true belief (Suryadi, 2023).

The results of various studies that have been carried out, especially on prospective mathematics teacher students, indicate that the concept of multiple integrals is still considered problematic. The difficulties experienced by students in understanding various concepts related to multiple integrals will result in delays in the knowledge acquisition process. Acquisition of knowledge is defined as the process of extracting, structuring, and organizing knowledge from one or more sources. Previous research has answered problems related to various difficulties experienced by students regardless of the source and how the knowledge acquisition process is constructed. Based on this, the research questions are: (1) What do prospective mathematics teacher students experience as learning obstacles in the concept of the double integral over non-rectangular areas, the double integral in polar coordinates, the triple integral in cylindrical or spherical coordinates, and the application of the double integral? (2) What is the process of acquiring knowledge of prospective mathematics teacher students on the concept of the double integral over non-rectangular areas, the double integral in polar coordinates, the triple integral in cylindrical or spherical coordinates, and the application of the double integral?

METHOD

This research is qualitative, specifically Didactical Design Research (DDR). DDR was developed starting in 2010 (Suryadi, 2019), seeks to explore the characteristics of learning design and its impact on students' knowledge construction (Suryadi, 2019; Fuadiah et al., 2019), and is a form of educational innovation (Sidik et al., 2021; Wahyuningrum et al., 2019). DDR is research on knowledge diffusion and acquisition (Suryadi, 2023).

DDR research is based on two paradigms, namely interpretive and critical. The interpretive paradigm underlies researchers in understanding the didactic design problems of the knowledge acquisition process of prospective mathematics teacher students on the concept of multiple integrals. Multiple integrals are one of the concepts in multivariable calculus lectures. Research data comes from test results, interview results, and documentation studies. The critical paradigm is the basis for designing alternative didactic designs to overcome learning obstacle findings. Based on the uniqueness of the method and objectives of this research, the researcher decided to use the DDR method.

The participants in this study were fourth-semester students in the Mathematics Education Study Program at a university in West Java, Indonesia. The study participants were aged between 18 to 22, totaling 46 people, 39 women and seven men. Of the 145 credits that must be taken to complete the program, they have completed approximately 80, two of which are multivariable calculus courses. With a weight of only two credits, this is certainly a problem, considering that the material in this course has a moderate to high level of difficulty. This statement aligns with Kashefi, Ismail, & Yusof (2012), who said that multivariable calculus is among the most challenging subjects for undergraduate students in many study programs.

The multivariable calculus course is a continuation of the differential calculus and integral calculus courses. It is also the basis for studies in vector calculus courses and group studies of other analysis courses. In multivariable calculus lectures, many variable functions are studied, which are extensions of one variable function; analyze the concept of limit, continuity, differential, and integral function of one variable to obtain a formulation into the form of a function of many variables; apply various limit, continuity, differential and integral theorems of multivariable functions. This study focused on the multiple integral concepts as a sub-subject in multivariable calculus lectures.

Students can solve problems related to multiple integrals based on their understanding of the concepts: multivariable functions, various integration techniques, the meaning of multiple integrals and their geometric meanings, and the concept of polar coordinates and their transformations. Students in integral calculus courses have obtained integral techniques for the function of one variable, while other concepts were obtained during multivariable calculus lectures.

Qualitative research has four data validity criteria: credibility, transferability, dependability, and confirmability (Sugiyono, 2016; Moleong, 2017; Denzin, Norman, and Lincoln, 2018). Researchers carried out the credibility of the research data through (a) being directly involved at the research location during the data collection process; (b) data collection and analysis carried out carefully and in detail; (c) using technique and source triangulation; (d) conduct peer reviews; (e) have sufficient references and maintain authentic evidence of research data. With these stages, it can guarantee that research findings have correct accuracy from the point of view of researchers, participants, and readers because complete and accountable data support them. Transferability is guaranteed by determining research participants, research settings, and data processing. The transferability stage guarantees that the researcher can provide the reader with sufficient information related to the problem under study. Readers can find out the degree of similarity between the research findings and the cases they face or the possibility of being transferred to other problems with characteristics like the research that has been done. Dependability is carried out by controlling the entire research process starting from problem identification, instrument preparation, data processing, and analysis. Confirmability is achieved by analyzing the objectivity, transparency of research findings and discussions.

Test instruments were developed to identify various learning obstacles and the continuity of the knowledge acquisition process for prospective teacher students on the integral concept of multiple. The process of acquiring the knowledge that one wants to capture through the test starts with the student's ability to use and connect the prerequisite material; determine the value of the double integral over a rectangular area, over any area, or in polar coordinates; determine the value of the triple integral both in Cartesian coordinates, cylindrical coordinates, and in spherical coordinates; ends with the ability to apply various integral multiple concepts in solving related problems.

There are four problems posed in the test. (1) Double integral over the non-rectangular area; in case a, the integral can be solved by the given order of integration, whereas in case b, the integral can be solved by changing the order of integration first. (2) Double integral in polar coordinates, aimed at seeing the process of acquiring students' knowledge, starting with recognizing the shape of the origin, variable transformation, and multiple integral solutions in polar coordinates. (3) Triple integral, aimed at acquiring knowledge related to the concept of triple integral in cylindrical or spherical coordinates. The acquisition process begins with recognizing the characteristics of cylindrical and spherical coordinates, variable transformation, the transformation of the form of the triple integral in cylindrical or spherical coordinates, and the solution of the triple integral with the appropriate coordinate system. (4) Multiple integral application, aimed at knowing the knowledge acquisition process related to students' ability to apply the multiple integral concepts in determining the volume of solid objects.

The test is given to students who are taking multivariable calculus lectures, aimed at knowing the process of acquiring knowledge on the concept of multiple integrals. Knowledge acquisition is a learning process carried out by students to produce knowledge. Acquisition in this study is limited to constructing integral multiple concept knowledge based on various learning obstacles experienced by prospective mathematics teachers. Knowledge acquisition in this research is defined as a learning process carried out by students. Technically, knowledge acquisition is the process of extracting, structuring, and organizing knowledge, whether obtained from reading results, discussion results, or information obtained from lecturers during lectures.

Semi-structured interviews were conducted with several students to corroborate the learning obstacle findings. Data on test results supported by interview results, review of lecture notes/source books, and plans for the next semester's lectures were analyzed simultaneously. Data analysis techniques include the following stages: identification, clarification, reduction, and verification, to be presented in a narrative descriptive manner.

In simple terms, the research process is presented in Figure 1.

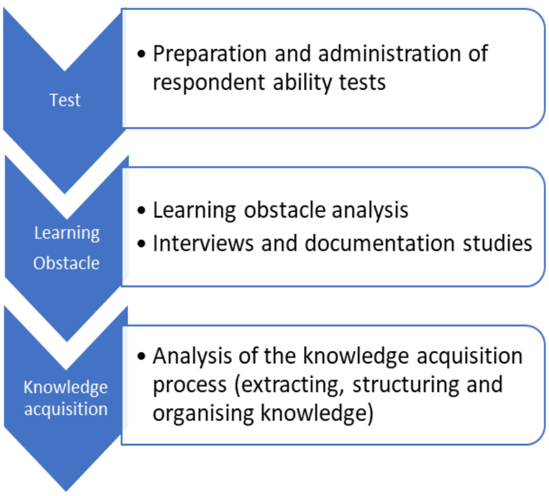


Figure 1. Research Stages

RESULTS AND DISCUSSION

Results

In the first problem, given two cases, (a) Calculate the value of $\int_1^3 \int_{-y}^{3y} xe^{y^3} dxdy$ and (b) Calculate the value of $\int_0^1 \int_y^1 e^{-x^2} dxdy$. This problem is intended to see students' knowledge acquisition process in solving double integrals over non-rectangular areas. The acquisition process begins with recognizing the characteristics of the shape of the function associated with a given integration sequence, making a sketch of the integration area if it is decided to change the order, determining the values for each variable, completing the integral according to the sequence that has been decided.

In case (a), generally, students can solve the questions in the order provided without changing the order of integration. A few students described the integration area although it was unnecessary. Several mistakes were made by students in terms of (i) performing algebraic operations, (ii) interpreting function values when using the Fundamental Theorem of Calculus on definite integrals, (iii) understanding the geometric meaning of double integrals as volumes and recognizing integral features that require change the order of integration, (iv) understand the visualization of the integration area sketch with the boundaries given to the integral. Sequentially these errors are presented in Figure 1.

Based on test results, interview results, and documentation studies, it can be concluded that students experience ontogenic learning obstacles of the type shown in Table 1 (see Appendix).

In case (b), students generally understand that this multiple integral problem cannot be solved in the order provided, but changes must be made first. The steps taken are correct by drawing the intended integration area and then repositioning the boundaries of the two variables is carried out. Originally the variable y had a constant upper and lower limit changed to a constant bounded x variable. However, there are problems where students can correctly draw the intended integration area and complete the double integral boundary. Still, they make mistakes in the substitution technique, as shown in Figure 2.

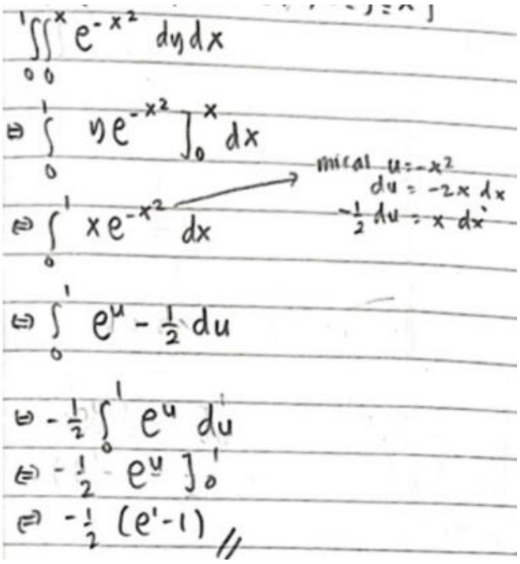
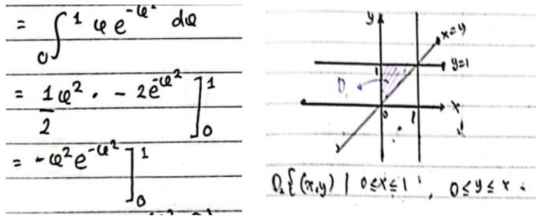


Figure 2. Errors in the substitution technique

Based on the results of the interviews, information was obtained that students did not understand the meaning of the integral bounds associated with the variables involved, so when the substitution technique was carried out, changing the variable was not followed by changing the bounds value of the integral. Based on the source, the mistakes made are categorized as ontogenic learning obstacles of the instrumental type. Students do not master technical matters related to substitution techniques in definite integrals which results in the problems presented being unable to be resolved properly. Besides that, the mistakes made can also be categorized as an ontogenic learning obstacle of the psychological type; a lack of prerequisite knowledge characterizes this. The prerequisite knowledge that is not mastered is the nature of integral linearity, as shown in Fig 3 section (i), and understanding of the graph of the origin area of the integral on the coordinate plane when the x and y variables have a certain bound, as shown in Figure 3 part (ii).



(i) (ii)

Figure 3. Error due to lack of prerequisite knowledge.

Based on the analysis that has been done, it can be concluded that the knowledge acquisition process related to the concept of a double integral over non-rectangular areas has not been running perfectly. In general, students can recognize the characteristics of the shape of the function with the order of the available integration variables and reposition the limit values for the integration variables by sketching the origin of the area first when it is decided to change the order of integration. The process is continued by determining the limit values for each variable. The acquisition process was constrained by the lack of prerequisite knowledge regarding algebraic operations, the linearity of integrals, technical matters related to function values and substitution techniques for definite integrals, and a weak understanding of applying double integrals as solid volumes.

In the second problem, students are asked to solve: Calculate $\iint_D e^{2x^2+2y^2} dA$, where D is the area in the second quadrant and bounded by $x^2+y^2=1$ and $x^2+y^2=5$. This problem is intended to see the process of acquiring student knowledge regarding the concept of a double integral in polar coordinates. In this case, students must recognize the general characteristics of problems that can be solved with a double integral in polar coordinates. These features are related to the shape area sketch for the integral in the plane $z = 0$, the variables involved in polar coordinates, the values for polar radius and angles, the transformation of variables from Cartesian coordinates to polar coordinates, the transformation of the integral form of Cartesian coordinates into polar coordinates.

Several mistakes are still made by students, including (1) the radius at the polar coordinates is negative; (2) they do not understand how the substitution technique works on definite integrals, which indicates students do not understand the meaning of the values for the integration variables; (3) the value of the integration variable is determined correctly but does not understand how the Fundamental Theorem of Calculus works on definite integrals; (4) do not understand the magnitude of the radius and angle values in polar coordinates associated with quadrants. These errors are shown sequentially in Figure 4.

Based on the results of the interviews, information was obtained that: (1) students think that the value of the radius depends on the quadrant where the integration area is located so that it can have a negative value; (2) the method of reading the radius value is not based on the center point so that the lower limit is greater than the upper limit. These answers indicate that students have a limited understanding of polar coordinates. They are familiar with the Cartesian coordinate system. Based on the source, it can be concluded that students experience ontogenic learning obstacles, as shown in Table 2 and epistemological learning obstacles, as explained next.

The learning obstacles mentioned in Table 2 can also be categorized as epistemological learning obstacles. Students have limited understanding regarding how to determine the value for the radius. In Cartesian coordinates, the determination of the value of a variable is read from left to right or from bottom to top. In contrast, the center point is used as a reference in determining the limit in polar coordinates. Epistemological learning obstacles are also encountered when the substitution technique is at definite integral. A change variable does not follow the change in the value variable of integration because, in an indeterminate integral, the substitution technique is only used to bring the integrant form into standard form. After obtaining the result, it is returned to the initial variable.

Based on the analysis that has been carried out, it can be concluded that the knowledge acquisition process related to the concept of double integral in polar coordinates has not been running perfectly. In general, students can recognize the shape of the region of origin and transform variables from Cartesian coordinates into polar coordinates. The final stage of the acquisition process, namely determining the multiple integral solution in polar coordinates, is constrained due to weak mastery regarding several things. The constraints are related to how definite integrals work, substitution techniques for definite integrals, and polar coordinate characteristics related to limits for polar radius and angles. The acquisition process is exacerbated by students' limited understanding of substitution techniques. They are used to working in indeterminate integrals, where substitution is carried out to bring up a standard form who's integral is known and not associated with bounds of integration. This error indicates that students do not understand the meaning of the bounds of integration as a variable value after the function is integrated.

In the third problem, students are asked to solve the following: Find the value of $\iiint_D y\,dv$, where D is a solid at octane one bounded by $z = 16 - x^2 - y^2$ and area z = 0. This problem is aimed at knowing the acquisition process of knowledge related to the concept of triple integral in cylindrical or spherical coordinates. The acquisition begins with recognizing cylindrical and spherical coordinates' characteristics, a variable transformation from Cartesian coordinates into cylindrical or spherical coordinates, and the triple integral solution with the appropriate coordinate system.

In general, mistakes made by students are caused by not understanding the characteristics of problems that require using cylindrical coordinates or spherical coordinates. The attributes in question are related to the sketch shape of the integration area on the plane $z = 0$, visualization of the graph sketch of the function $z = f(x, y)$, transformation of variables from Cartesian coordinates into cylindrical or spherical coordinates, and transformation of the triple integral form from Cartesian coordinates to cylindrical and spherical coordinates.

Based on test results, interview results, and documentation studies, information was obtained related to various mistakes made by students, including: (1) not knowing the characteristics of tube coordinates; (2) cannot draw the functions of two variables correctly, which results in an error in choosing the triple integral technique, the function graph sketch that should be a paraboloid using the tripel integral technique in tube coordinates is the right choice because it is drawn as a ball the student chooses the deep triple integral technique spherical coordinates; (3) does not recognize the variables used in cylindrical coordinates and spherical coordinates; (4) does not understand the transformation from Cartesian coordinates to cylindrical coordinates, both variable and integral transformations. The mistakes experienced by students sequentially are presented in Figure 5 (See Appendix).

Based on the source, the difficulties experienced by students can be categorized as ontogenic learning obstacles, as shown in Table 3 (See Appendix).

Based on the analysis that has been done, it can be concluded that the knowledge acquisition process related to the concept of triple integral in cylindrical coordinates and spherical coordinates has not run perfectly. Some students can sketch the graph $z = 16 - x^2 - y^2$ correctly but do not understand the characteristics of a cylinder and spherical coordinates, resulting in an error in choosing the type of coordinates for the multiple integral. In the problems given, students choose a spherical coordinate technique that should be a cylindrical coordinate. If one pays attention to understanding the triple integral in spherical coordinates, radius size, angle size, and the necessary transformations have been carried out correctly. Solving the triple integral in spherical coordinates is carried out correctly, starting from transforming the Carterius coordinates into spherical coordinates. However, this does not solve the problem because it does not answer what the problem requires. Some students can draw function graphs; define boundaries on Cartesian coordinates; and recognize the characteristics of cylinder coordinates by precisely defining limits for radius, angle, and variable z; however, an error occurred in the transformation of the triple integral from Cartesian coordinates into the triple integral in cylindrical coordinates. It can be concluded that acquiring the concept of triple integral in cylindrical and spherical coordinates is constrained due to errors in transforming variables and weak mastery of the characteristics of the graph of functions of two variables.

The fourth problem is related to the application of integral multiple, in which students are asked to solve the problem: Find the volume of a solid object bounded by $9z = x^2 + y^2$, cylinder $x^2 + y^2 = 6x$, and plane $z = 0$. This problem is aimed at knowing the knowledge acquisition process of prospective mathematics teacher students related to applying the multiple integral concepts. In this study, the application of multiple integrals is focused on the ability to determine the volume of solid objects.

In general, students understand the geometric meaning of $\iint_D f(x,y)dA$ as the volume of a solid body below the surface $f(x,y)$ Above D. However, various difficulties arise when the graphical sketch of area D in the plane $z = 0$ is in the form circle, either a complete circle or a part of a circle. This condition makes it difficult if the integral is solved using Cartesian coordinates, so it is recommended to use polar coordinates. The difficulty level increases when the given circle is not centered at $(0,0)$.

Various difficulties experienced were: (1) did not understand the general form of the equation of a circle; (2) being unable to sketch a circle on the R^3 coordinate system resulting in an error in determining the limits for angles on polar coordinates; (3) unable to change the quadratic equation into the general form of the circular equation as a result of not being able to master the properties of the quadratic form; (4) do not understand the characteristics of the radius and angle in polar coordinates, which results in being unable to determine the radius and angle limits if the circle formed is not centered at $(0,0)$; (5) do not understand the characteristics of the angles in R^3, even though students can change the quadratic equation into a general form of the circle equation and can describe it in the coordinate system in R^3; (6) do not understand the transformation of Cartesian coordinates into cylindrical coordinates; (7) do not understand the integral of trigonometric functions, related to the cosine of the double angle; (8) unable to sketch the graph of the function $x^2 + y^2 = 6x$ in R^3, so it cannot determine the required angle range in polar coordinates. The difficulties experienced by students are presented sequentially in Figure 6 (see Appendix).

From the results of the interviews, information was obtained that: (1) students could not change the given quadratic equation into a circular equation due to not knowing the general form of the circle equation, which has no center at $(0,0)$; (2) students can change the quadratic equation into the available form of a circle equation that is not centered at $(0,0)$, can draw a circle with a center, not $(0,0)$ on the R^2 coordinate system but have problems when it is marked on the R^3 coordinate system; (3) students do not know the limits of the angle values on R^3 because they do not understand the angle magnitude associated with the octane where the graphic sketch is located; (4) students cannot determine the range of values for the radius of a circle that is not centered at $(0,0)$, associated with polar coordinates.

Based on test answers, interview results, and document analysis, it can be concluded that students experience ontogenic learning obstacles, as shown in Table 4.

Apart from the ontogenic obstacle, the mistakes made can also be categorized as an epistemological learning obstacle. Students have a limited understanding of the coordinate system in R^3 and are used to determining the radius and angle values in R^2.

Based on the analysis that has been done, it can be concluded that the knowledge acquisition process related to the application of multiple integrals, primarily associated with the volume of solid objects, has not run perfectly. The initial acquisition process went well. Generally, students understood the geometric meaning of the double integral as a solid volume. Still, the subsequent acquisition process was constrained when the sketch of the integration area on the plane $z = 0$ was circular. The difficulty increased when the given circle was not centered at $(0,0)$. Prerequisite knowledge related to the general form of circular equations, inability to convert quadratic equations into the general format of circular equations, low understanding of graphs in the R^3 coordinate system, low understanding of the characteristics of angles and radii in polar coordinates when the circle is not centered at $(0,0)$, as well as a limited understanding of the coordinate system in R^3, students are used to determining the radius and angle values in R^3. Various things have been put forward are problems that affect the knowledge acquisition process related to applying double integrals.

Discussion

The findings of various difficulties experienced by students in the double integral concept in this study are in line with the research of Apriandi & Krisdiana (2016); Simorangkir & Sinaga (2022); Utari, Septy & Hutauruk (2021); Siti (2015); and Fahrudin (2018). Research by Apriandi, Krisdiana, and Fahrudin concluded that students still had difficulties performing calculations or operations on integrals. At the same time, the difficulty in choosing and using procedures in solving multiple integral problems is concluded from the research of Simorangkir, Utari, Siti, and Fahrudin. The difference with previous research in this study, apart from identifying student errors, also identified the source of difficulties so that the learning obstacles category could be determined. In this study, an analysis of the knowledge acquisition process of the twofold integral concept was also carried out on non-rectangular areas. What problems are experienced become obstacles for students in acquiring knowledge of the double integral concept?

Research findings related to the difficulties experienced by students in the concept of double integral in polar coordinates align with the research of Apriandi & Krisdiana (2016) and the study of Siti (2015). The two studies concluded that students still had difficulty writing integration forms in polar coordinates due to not mastering the form of variable transformation from Cartesian coordinates to polar coordinates. Even the basic concepts related to polar functions are poorly understood by students (Muchlis, 2017). In this study, in addition to identifying student errors in the concept of a double integral in polar coordinates, sources of difficulty were also identified to determine the learning obstacles category. Besides that, in this study, an analysis of the knowledge acquisition process of the twofold integral concept in polar coordinates was also carried out. What problems still arise and become obstacles for students in acquiring knowledge of double integral in polar coordinates?

No research has been found that examines the difficulties experienced by students in the concept of triple integral in cylindrical and spherical coordinates. In this study, in addition to identifying student errors in the triple integral concept in cylindrical and spherical coordinates, sources of difficulty were identified to determine the learning obstacle category. Besides that, in this study, an analysis of the knowledge acquisition process of the concept of triple integral in cylindrical coordinates was also carried out while still linking it to the knowledge acquisition process related to the idea of multiple integral in spherical coordinates. What problems are still experienced and become obstacles for students in acquiring knowledge of the triple integral concept in cylindrical and spherical coordinates?

Research findings related to the difficulties experienced by students in applying the twofold integral concept are in line with the research of Muchlis (2017), Nurmitasari (2017), and Takaendengan, Asriadi & Takaendengan (2022). In his research, Muchlish concluded that the percentage of students' difficulty levels in the integral concept increased from the smallest to the highest. The idea of the double integral application is a sub-material with the most serious difficulty percentage compared to other sub-materials. Nurmitasari concluded that judging from Bloom's Taxonomy, the level of learning difficulty in multivariable calculus courses with the concept of multiple integrals is at the level of application and analysis. In comparison, Takaendengan concluded that the low ability of the prerequisites caused the difficulties experienced by students. In this study, apart from identifying student errors in determining the volume of solid objects, sources of problems were also identified to choose the learning obstacles category. Learning obstacles placed in applying integral folds are ontogenic learning obstacles in their three types and epistemological learning obstacles. Besides that, this research also analyzes the knowledge acquisition process involving double integrals, in this case, the volume of solid objects. What problems still arise and become obstacles for students in the knowledge acquisition process related to integral applications?

Implication of Research

The various findings of learning obstacles and acquiring knowledge of multiple integrals' concepts have implications for didactic design. Didactic design should be designed in such a way that it can help students construct knowledge without significant obstacles. Didactic design can be started by presenting a problem as a stimulus for students to think about. The second stage is creating a situation that allows students to formulate concepts, the third stage is by creating a situation that provides opportunities for students to validate as justification for the responses that arise, and the last step is to provide space for students to be able to apply the concept to other problems in different contexts.

Limitation

In this research, the acquisition of knowledge related to fold integrals is only based on the findings of learning obstacles experienced by prospective mathematics teacher students. Various types of learning obstacles are based on test results, interviews, and documentation studies.

CONCLUSION

Ontogenic learning obstacles with psychological, instrumental, and conceptual types were experienced by prospective mathematics teacher students in the integral fold concept. Besides that, epistemological learning obstacles were also identified. The psychological type of ontogenic learning obstacle is characterized by a lack of prerequisite knowledge regarding algebraic operations, the linearity of integrals, the surface form of two-variable functions, the fundamental theorems of calculus on definite integrals, the general format of circular equations, and the transformation of variables from Cartesian coordinates into polar coordinates. The ontogenic learning obstacle of the instrumental type is characterized by a lack of mastery of technical matters related to the meaning of function values, substitution techniques for definite integrals, visualization of functions of two variables in R^3 coordinates, and graphic sketches of circle equations that are not centered at $(0,0)$. The conceptual type of ontogenic learning obstacle is characterized by not mastering the integral application concept, triple integral characteristics in tube coordinates, and the idea of radius size and angle size associated with the octane where the curve is located. Epistemological learning obstacles are encountered because students have a limited understanding of determining the boundary values for radii and substitution techniques for definite integrals. They are used to working on indefinite integrals. Epistemological learning obstacles are also encountered because students have a limited understanding of the coordinate system in R^3. They are used to determine the radius and angle values in R^2. Based on the analysis that has been done, it can be concluded that the knowledge acquisition process is related to the concept of the double integral over non-rectangular areas, the idea of the double integral in polar coordinates, the concept of the triple integral in cylindrical coordinates and spherical coordinates, as well as the application of the fold integral primarily related to the volume of solid objects has not run perfectly. The process of acquiring knowledge is constrained by the lack of prerequisite knowledge, mastery of technical matters, which are the key to specific problems, weak mastery of concepts, and limited understanding related to the habits that students do.

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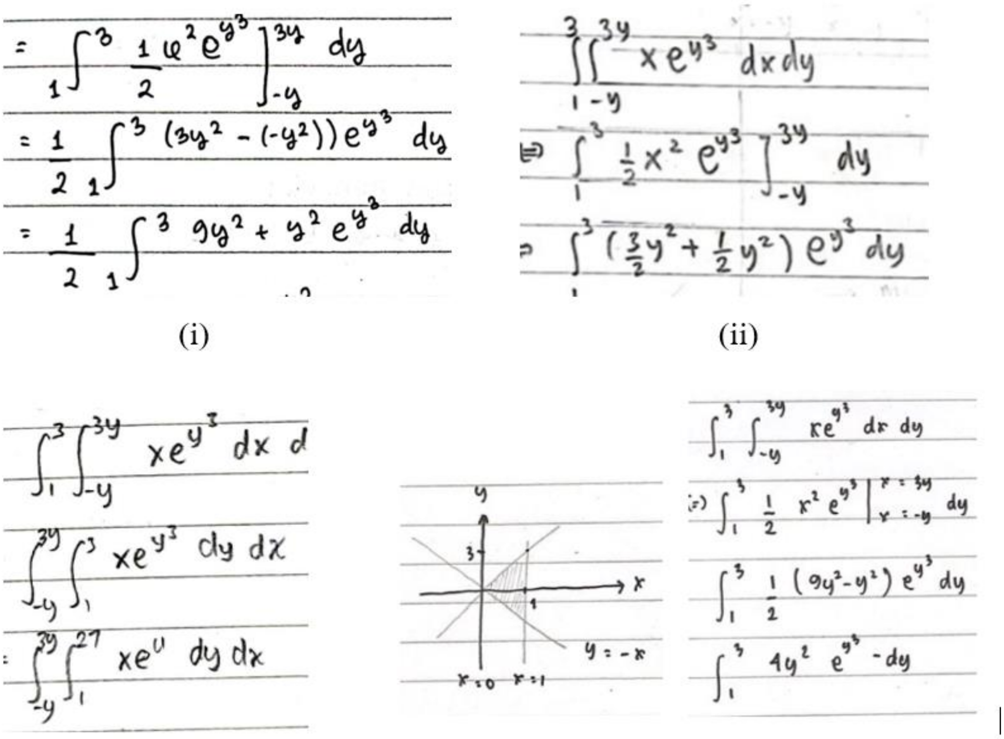
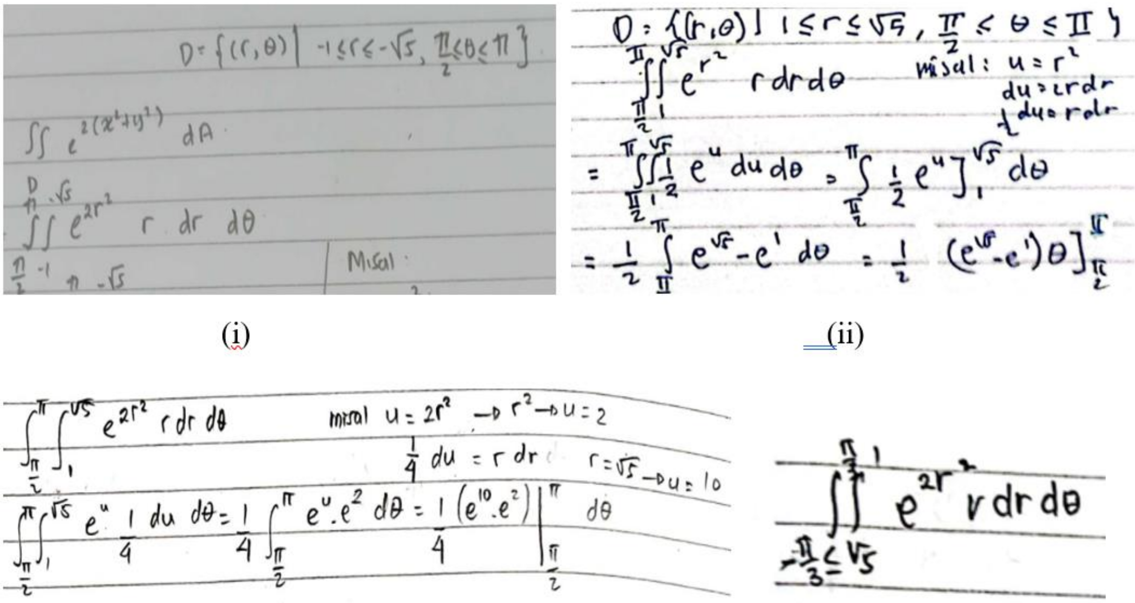
Appendix of Article entitled: Didactical Design Research: Knowledge Acquisition of Mathematics Teacher Prospective Students on Multiple Integral Concepts.



(i) (ii)

(iii) (iv)

Figure 1. Errors in the double integral problem over non-rectangular areas



(i) (ii)

(iii) (iv)

Figure 4. Errors related to double integrals in polar coordinates

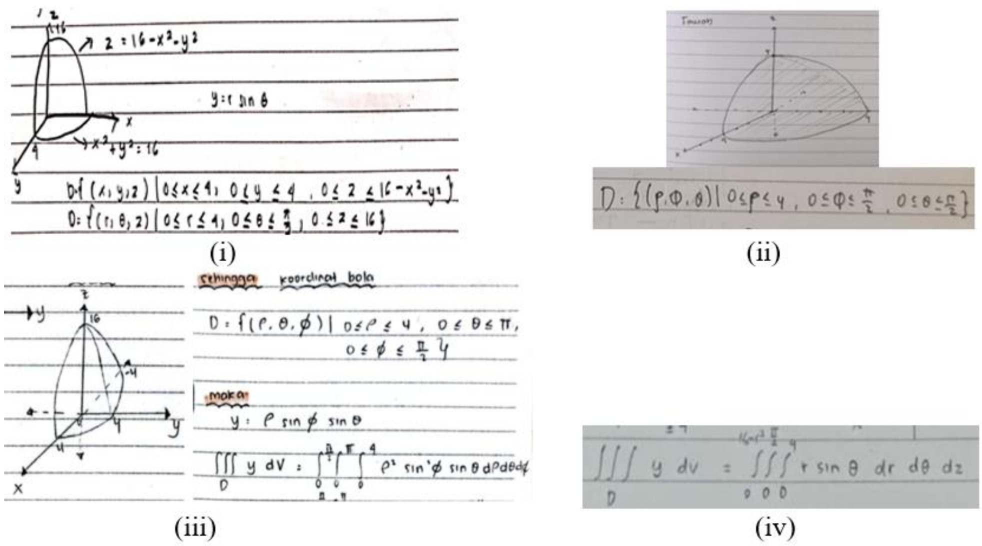


Figure 5. Errors related to triple integrals in cylindrical coordinates.

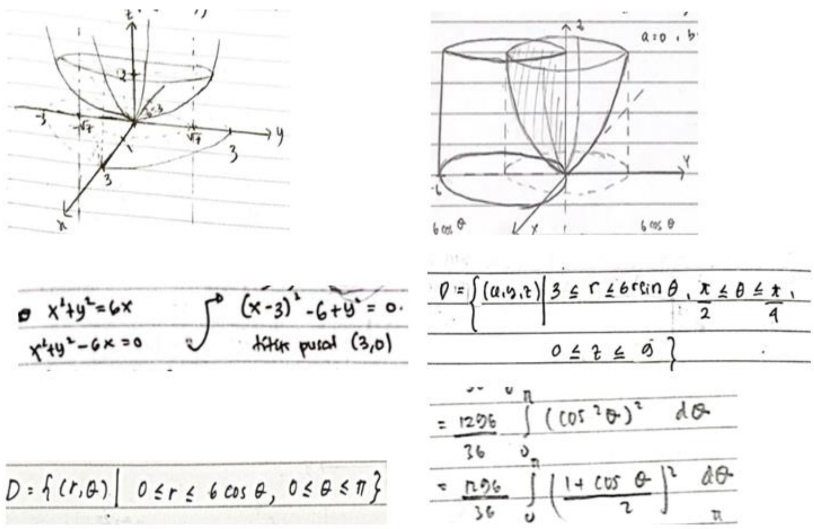


Figure 6. Errors related to multiple integral applications.

Table 1. Types of Ontogenic Learning Obstacle Double Integral Concept Cases (a)

No	Types of Learning Obstacles	Indicators
1	Psychological Type	characterized by weak knowledge of prerequisites related to algebraic operations
2	Instrumental Type	characterized by not mastering technical matters related to function values
3	Conceptual Type	marked by not mastering the concept related to the application of double integrals.

Table 2. Types of Learning Obstacles in the Double Integral Concept in Polar Coordinates

No	Types of Learning Obstacles	Indicators
1	Psychological Type	characterized by a lack of prerequisite knowledge regarding the fundamental theorem of calculus on definite integrals
2	Instrumental Type	characterized by not mastering technical matters related to substitution techniques in definite integrals
3	Conceptual Type	characterized by not mastering the concept related to the magnitude of the radius and angle values in polar coordinates

Table 3 Learning Obstacle Types of Triple Integral Concepts in Tube Coordinates

No	Types of Learning Obstacles	Indicators
1	Psychological Type	characterized by the lack of prerequisite knowledge regarding the transformation of Cartesian coordinates into polar coordinates in the double integral
2	Instrumental Type	characterized by not mastering technical matters related to how to visualize graphs of functions of two variables as surfaces in R3
3	Conceptual Type	characterized by not mastering the concept related to the characteristics of the triple integral in cylindrical coordinates

Table 4. Learning Obstacle Types of Double Integral Application Concepts

No	Types of Learning Obstacles	Indicators
1	Psychological Type	characterized by the lack of prerequisite knowledge regarding the general form of the circle equation
2	Instrumental Type	characterized by not mastering technical matters related to the form of a sketch of a circle equation graph in general form
3	Conceptual Type	characterized by not understanding the concept related to the radius and angle size associated with the octane where the curve is located.