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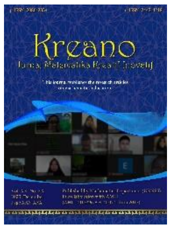

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Formal Mental Models of Junior High School Students in Understanding the Concept of Integers

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Abstract

Mental models are internal representations students form about a concept, which can be inferred through their external representations. This study aims to describe the characteristics of junior high school students' formal mental models in understanding the concept of integers. Conducted in a class of 44 seventh-grade students, this qualitative study employed a case study methodology. Subjects were selected using a purposive sampling, with two subjects chosen for in depth exploration and interviews. Data were analyzed using the Miles and Huberman model, encompassing data condensation, data presentation, and conclusion drawing. The results revealed that the students at the formal mental model level demonstrated the characteristics of being able to place positive integers, zero, and negative integers on a number line, compare positive and negative integers, compare two negative integers, sort integers, add and subtract integers, and sort them accordingly. By refining the characteristics of mental models of value and order of integers, and integrating Bofferding's ideas on the mental model of value and order of integers and mental model of directed magnitude, this study successfully expands the theoretical framework of students' mental models in understanding the concept of integers, extending it to include addition and subtraction operations. Practically, it provides educators with insights into the characteristics of formal mental models that seventh-grade students need to master, supporting the development of meaningful instructional strategies to strengthen students' conceptual understanding and promote the advancement of mental models to the formal level. In addition, these findings serve as a reference for future research on students' mental models in understanding integers and performing integer operations.

Keywords: Integers; Mental Models; Understanding Concept.

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Abstrak

Model mental diartikan sebagai representasi internal yang dimiliki siswa mengenai suatu konsep yang kemudian dapat diketahui melalui representasi eksternal. Penelitian ini bertujuan untuk mendeskripsikan karakteristik model mental formal siswa SMP dalam memahami konsep bilangan bulat. Penelitian ini dilakukan di kelas VII SMP yang terdiri dari 44 siswa. Metode penelitian yang digunakan adalah kualitatif dengan jenis penelitian studi kasus. Pemilihan subjek menggunakan teknik purposive sampling, yakni terdapat 2 subjek untuk dieksplorasi dan diwawancarai. Teknik analisis data menggunakan model Miles dan Huberman meliputi kondensasi data, penyajian data, dan penarikan kesimpulan. Hasil penelitian menunjukkan bahwa karakteristik siswa pada level model mental formal adalah dapat menempatkan bilangan bulat positif, nol, dan bilangan bulat negatif pada garis bilangan, dapat membandingkan bilangan bulat positif dan bilangan bulat negatif, dapat membandingkan dua bilangan bulat negatif, dapat mengurutkan bilangan bulat, dapat menjumlahkan dan mengurangkan bilangan bulat sekaligus mengurutkannya. Dengan mempertajam karakteristik model mental nilai dan urutan bilangan bulat, serta mengintegrasikan gagasan Bofferding tentang model mental nilai dan urutan bilangan bulat dan model mental besaran terarah, penelitian ini berhasil memperluas kerangka teori tentang model mental siswa dalam memahami konsep bilangan bulat, sehingga mencakup pula operasi penjumlahan dan pengurangan. Secara praktis, studi ini memberikan wawasan kepada pendidik mengenai karakteristik model mental formal yang perlu dikuasai oleh siswa kelas tujuh, mendukung pengembangan strategi pengajaran yang bermakna untuk memperkuat pemahaman konseptual siswa dan mendorong perkembangan model mental mereka ke tingkat formal. Selain itu, temuan ini dapat dijadikan acuan untuk penelitian lebih lanjut mengenai model mental siswa dalam memahami bilangan bulat dan melakukan operasi bilangan bulat.

INTRODUCTION

One of the things that attracts a lot of attention in the process of learning mathematics is how students can understand a concept and connect it with other concepts to build knowledge. Concept understanding is the main foundation for developing higher-level thinking skills, so students who understand concepts well will be better able to generalize and transfer their knowledge compared to students who only memorize definitions (Agustina et al., 2022). Jäder & Johansson (2025) suggested that concept understanding is the ability of students to relate between ideas in mathematics, use and transfer between representations (concrete and abstract), and show flexibility of thinking in explaining and applying concepts meaningfully. Furthermore, Schaathun (2022) argued that the essence of understanding mathematical concepts is not just the acquisition of knowledge but also an active and transformational process that connects new material with pre-existing knowledge. Schaathun also emphasized that a learner can be considered to have understood a concept only when they can recognize and apply it intuitively and consciously in a novel and real-world context (Schaathun, 2022). This underscores the potential for mathematics learning to reshape students' perspectives of the world when integrated with real-world applications (Gurmu et al., 2024).

Learning plays an important role in the process of constructing students' knowledge. Disch et al., (2023) emphasize that students' knowledge is actively constructed based on prior knowledge, as new concepts need to be integrated into the existing knowledge framework. This means that students' knowledge is not formed by teachers, but by the students themselves. Important information about the structure of students' understanding can be obtained through mental models. Generally, mental models are defined as internal representations that shape a person's understanding of the real world (Nagel et al., 2024). In education, mental models are defined as internal representations that can describe the structure of knowledge and the level of students' conceptual understanding (Agustina et al., 2022). This means that mental models serve as internal representations used to describe students' cognitive processes when understanding a particular concept. Sukiyanto et al., (2022, 2023) state that mental models refer to the internal representations that students have of a concept, which are then constructed into external representations in the form of images, writing, or verbal explanations. This aligns with Bofferding's (2014) view that mental models reflect how students understand, process, and visualize information about a concept based on their experiences, prior knowledge, and learning. Mental models are dynamic, meaning they can change and develop as students gain more knowledge, understanding, and experience (Rumite et al., 2023). Therefore, by identifying students' mental models, educators can obtain an understanding of students' thinking processes, recognize obstacles in the learning process, and create learning strategies that are more suited to their needs (Pedrera et al., 2025).

Studies on mental models in mathematics education have spanned various educational levels, from elementary (Bofferding, 2014; Utami et al., 2018), junior high school (Prayekti, et al., 2020; Sukiyanto et al., 2022, 2023), senior high school (Greefrath et al., 2021, 2023; Prayekti et al., 2019), to college (Rumite et al., 2023). Bofferdings' research identified five mental models of integer values and sequences including initial, transition I, synthetic, transition II, and formal mental. In addition, Bofferdings' research also produced a directed magnitude mental model (addition and subtraction of integers) including initial, synthetic, and formal mental models (Bofferding, 2014). Utami et al., (2018) continued Bofferdings' research on mental models of integer values and sequences and produced a pre-initial mental model, resulting in six levels of mental models including pre-initial, initial, transition I, synthesis, transition II, and formal mental models. Further research on mental models of integer values and sequences was also studied by Sukiyanto et al., (2022, 2023) at the junior high school level. Sukiyanto et al., (2022, 2023) refined these levels further by dividing the Transition I mental model level into "identifying" and "exploring" subcategories. Thus, so far there are seven levels of integer mental models for values and sequences that previous researchers have developed.

Integers are fundamental concepts to build students' understanding of more complex mathematical concepts (Zuhriawan et al., 2024). However, the fact is that many students still have difficulties in understanding the concept of integers (Acosta & Soliba, 2024; Felipe, 2024; Harun et al., 2023; Lamb et al., 2023; Nurnberger-Haag et al., 2022; Pineda, 2024; Rosa et al., 2023; Rozikin & Ronji, 2024; Sercenia et al., 2023; Sukiyanto et al., 2023, 2022; Suzen et al., 2024; Vallejo-Vargas & Reid, 2024; Zuhriawan et al., 2024). Harun et al., (2023) said that students have understood the concept of positive integers, but still experience obstacles in understanding the concept of negative integers. This can be seen when students compare and sort integers (Sukiyanto et al., 2022, 2023). Furthermore, Pineda (2024) and Vallejo-Vargas & Reid (2024) emphasized that some students also still have difficulties in operating integers, especially subtraction of negative integers. Therefore, this research seeks to examine students' understanding of the topic of integers from the angle of students' mental models.

The researcher conducted a preliminary study on integers in a seventh-grade junior high school class. Two unique phenomena were observed: 1) Some students could not place negative integers on the number line, although they recognized negative integers. The characteristics found in these students differed from those theorized by Utami et al., (2018) and (Sukiyanto et al., 2022, 2023). 2) Some students exhibited the characteristics of the formal model of integer value and order (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018) but need help to perform addition and subtraction with integers. Meanwhile, research by Utami et al., (2018) and (Sukiyanto et al., 2022, 2023) should have addressed integer addition and subtraction. Furthermore, Bofferding (2014) work on the mental models of the integer value and order and on directed magnitude (addition and subtraction of integers) analyzed these concepts separately.

To address this research gap, the researcher proposes the development of new ideas to refine the understanding of these phenomena. Specifically, this involves refining the characteristics of the mental model of the integer value and order and integrating Bofferding's 2014) ideas regarding the mental model of the integer value and order and the mental model of the directed magnitude (addition and subtraction of integers). This approach aligns with Peled (1991) assertion that students' first level of knowledge of integers begins with understanding their order and progresses to the ability to add and subtract them. The proposed contributions of this research are as follows: Refine the Characteristics of Mental Models: enhance the pre-initial and initial mental model characteristics for the integer value and order, as described by (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018). Merge Transition I Mental Models: reunite the Transition I-identifying and Transition I-exploring mental models, as proposed by (Sukiyanto et al., 2022, 2023), into the Transition I mental model, in accordance with (Bofferding, 2014; Utami et al., 2018), due to the proximity of the material. Divide Transition II Mental Models: divide the Transition II mental model on the integer value and order (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018) into two subcategories: Transition II-inconsistent and Transition II-consistent, so the mental model of Transition II-consistent shifts the position of the formal mental model. Then, include Bofferding's 2014) directed magnitude mental model, which addresses integer addition, subtraction, and sorting, into the formal mental model. This integration contributes to a more comprehensive and complex theoretical framework for students' mental models in understanding integers.

The formal mental model represents the highest level of mental model, indicating that students at this level possess a more comprehensive understanding of the concept of integers. Such students have successfully progressed through all preceding mental model levels. This research focuses on analyzing the cases of students at the formal mental model level to explain the characteristics of the formal mental model level and the previous mental model levels. Thus, the study aims to describe the characteristics of junior high school students' formal mental models in understanding the concept of integers.

METHOD

This research used a qualitative approach with a case study design. According to Creswell (2018), the qualitative approach aims to understand human problems in a social context by creating a comprehensive and complex picture, reporting detailed views from sources of information, and conducting the research in a natural setting without intervention. The cases observed in this study involve students at the formal mental model level. This research was conducted in a seventh-grade junior high school of 44 students. The instrument used in the study was a set of test questions on integers, consisting of five items: 1) placing integers on the number line, 2) comparing positive and negative integers, 3) comparing two negative integers, 4) ordering integers, and 5) performing addition and subtraction of integers while sorting them. The characteristics of students' mental models in understanding integer concepts, as modified by the researcher based on the Bofferding (2014); Sukiyanto et al., (2022, 2023); Utami et al., (2018) study, are presented in Table 1 below.

Table 1. The Characteristics of Students' Mental Models

Mental Models	Characteristics that Emerge
Pre-Initial	• The child can place positive numbers on the number line.
Initial	• The child can place zero and positive integers on the number line, or • The child places zero, positive integers, and negative numbers on the number line. However, the order of negative integers is treated like positive integers, or • The child places positive integers and negative integers. However, 0 and -1 are not written on the number line.
Transition I	• The child places zero, positive, and negative integers on the number line. • The child can compare a positive integer and a negative integer.
Synthetic	• The child can compare two negative integers.
Transition II-Inconsistent	• The child can sort positive integers but is inconsistent in sorting negative integers.
Transition II-Consistent	• The child can sort positive and negative integers correctly.
Formal	• The child can add and subtract integers as well as sort them.

Based on Table 1, students at the pre-initial mental model level exhibit the characteristics of the pre-initial mental model level. Students at the initial mental model level meet the characteristics of the initial mental model level and have surpassed the pre-initial mental model level and so on, up to the formal mental model level. In other words, students at the formal mental model level meet the characteristics of this level and have surpassed all previous levels: pre-initial, initial, transition I, synthetic, transition II-inconsistent, and transition II-consistent.

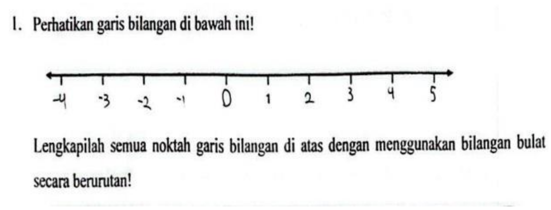
The selection of subjects in this study used a purposive sampling technique, meaning that the selection was based on research objectives. The research subjects were students at the formal mental model level. From the responses of 44 students, seven were identified as being at this level. These seven students were divided into two groups, one group with 5 students and the other with 2 students based on the similarity of the answers and the reasons for the answers given. One student from each group was selected as a subject based on the communication fluency, as confirmed by the class teacher. Thus, there were two subjects in this study. Data analysis was conducted using the Miles and Huberman model, involving the following steps: data condensation, data display, and drawing and verifying conclusions (Miles et al., 2014).

RESULT AND DISCUSSION

Results

This research is guided by the characteristics of mental models developed by researchers in Table 1. Based on the results, the distribution of students' mental model levels in understanding the concept of integers is presented in Table 2 below.

From Table 2, two of the seven students at the formal mental model level, referred to as S1 and S2, were described and analyzed based on their answers to test questions that illustrate their understanding of integers. In the first problem, completing integers on the number line, S1 and S2 correctly placed the integers on the number line. Their answers to question 1 can be seen in Figure 1 dan 2 below.

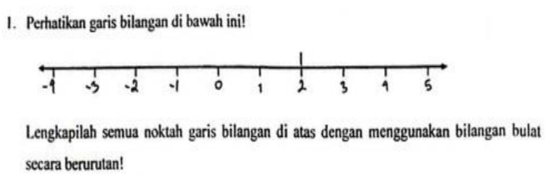


Translated:
1. Look at the number line below!

-4 -3 -2 -1 0 1 2 3 4 5

Complete all the dots on the number line above using integers in order!

Figure 1. S1's Answers to Question Number 1



Translated:
1. Look at the number line below!

-4 -3 -2 -1 0 1 2 3 4 5

Complete all the dots on the number line above using integers in order!

Figure 2. S2's Answers to Question Number 1

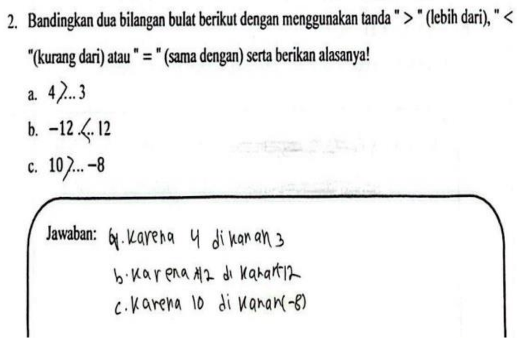
Based on their answers, S1 and S2 constructed the representations of integers in their minds into external representations on the number line. They accurately positioned positive integers, zero, and negative integers in the correct locations and order. S1 and S2 also provided appropriate reasoning for their decisions, as demonstrated in the following interview with S1.

P : Why did you place 2 to the right of 1?
S1 : Because 2 is greater than 1.
P : Why did you place 0 to the left of 1?
S1 : Because 1 is greater than 0.
P : Why did you place -1 to the left of 0?
S1 : Because -1is less than 0.

The interview with S2 was as follows.

P : Why did you place 0 to the right of 1?
S2 : Because before 1 is 0.
P : Why did you put 2 to the right of 1?
S2 : Because after 1 is 2.
P : Then why did you place this number (pointing to -1) to the left of 0?
S2 : Because to the left of 0 is -1, Ma'am.

In the second problem, comparing the values of two integers, S1 and S2 provided correct answers supported by appropriate reasons. S1's answer to question 2 is shown in Figure 3 dan 4 below.



2. Bandingkan dua bilangan bulat berikut dengan menggunakan tanda " > " (lebih dari), " < "(kurang dari) atau " = " (sama dengan) serta berikan alasanya!

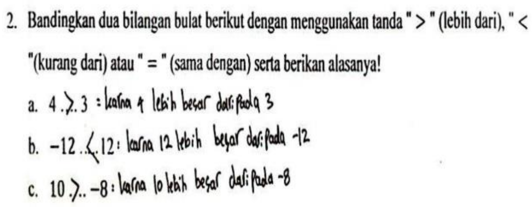
a. 4 >... 3
b. −12 .<.. 12
c. 10 >... −8

Jawaban: a. karena 4 di kanan 3
b. karena 12 di kanan −12
c. karena 10 di kanan (−8)

Translated:

2. Compare the following two integers using the signs" > " (more than)," < " (less than) or" = " (equal to) and give a reason!
   a. 4 > 3, because 4 is located to the right of 3
   b. −12 < 12, because 12 is located to the right of -12
   c. 10 > −8, because 10 is located to the right of -8

Figure 3. S1's Answers to Question Number 2



2. Bandingkan dua bilangan bulat berikut dengan menggunakan tanda " > " (lebih dari), " < "(kurang dari) atau " = " (sama dengan) serta berikan alasanya!

a. 4 .>.. 3 : karna 4 lebih besar daripada 3
b. −12 ..<.12 : karna 12 lebih besar daripada -12
c. 10 .>.. −8 : karna 10 lebih besar daripada -8

Translated:

2. Compare the following two integers using the signs" > " (more than)," < " (less than) or" = " (equal to) and give a reason!
   a. 4 > 3, because the value of 4 is greater than that of 3
   b. −12 < 12, because the value of 12 is greater than that of -12
   c. 10 > −8, because the value of 10 is greater than that of -8

Figure 4. S2's Answers to Question Number 2

Based on Figure 3 and 4, S1 and S2 correctly compared the values of two positive integers (problem 2a), the values of positive and negative integers with the same magnitude symbol (problem 2b), and the values of positive and negative integers with different magnitude symbols, recognizing that positive integers magnitude symbols are greater than negative integer magnitude symbols (problem 2c). S1 and S2 and also justified with valid reasoning. S1 constructed his internal representation into an external representation by explaining that on a number line, numbers further to the right have greater values, while numbers further to the left have smaller values. This is evident in the following interview.

P : Why is 4 greater than 3?
S1 : Because 4 is to the right of 3 on the number line, so 4 is greater than 3.
P : Why is -12 less than 12?
S1 : Because 12 is to the right of -12 on the number line, so 12 is greater than -12.
P : Why is 10 greater than -8?
S1 : Because 10 is to the right of -8, Ma'am.

S2 constructed his internal representation into an external representation by offering two complementary explanations. The first opinion is that numbers further to the right on the number line have greater values, while numbers further to the left have smaller values. The second opinion is that any number with a negative sign must have a smaller value than a positive number, whether the symbols are the same or different. This reasoning is shown in the following interview.

P : Okay, why is 4 greater than 3?
S2 : Because after 3 is 4, so 4 is greater.
P : How do you know 4 is greater than 3?

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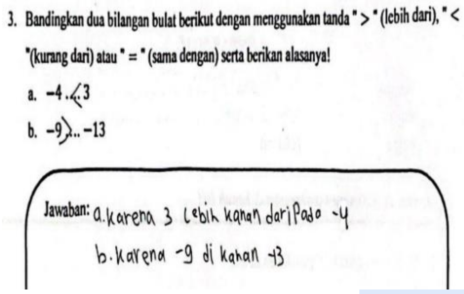
S2 : From the number line, Ma'am.
P : What does that mean?
S2 : The farther to the right, the greater the value; the farther to the left, the smaller the value.
P : Okay, why is -12 less than 12?
S2 : Because negative is smaller and positive is greater.

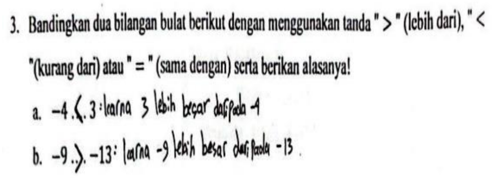
In the third, problem comparing the values of two integers, S1 and S2 gave correct answers accompanied by appropriate reasoning. The answers provided by S1 and S2 to question 3 can be seen in Figure 5 and 6 below.


3. Bandingkan dua bilangan bulat berikut dengan menggunakan tanda " > " (lebih dari), " < "(kurang dari) atau " = " (sama dengan) serta berikan alasanya!
a. -4 .<. 3
b. -9 .>.. -13

Jawaban: a. karena 3 lebih kanan daripada -4
b. karena -9 di kanan -13

Translated:
3. Compare the following two integers using the signs" > " (more than)," < " (less than) or" = " (equal to) and give a reason!
   a. $-4 < 3$, because 3 is located to the right of -4
   b. $-9 > -13$, because -9 is located to the right of -13

Figure 5. S1's Answers to Question Number 3


3. Bandingkan dua bilangan bulat berikut dengan menggunakan tanda " > " (lebih dari), " < "(kurang dari) atau " = " (sama dengan) serta berikan alasanya!
a. -4 .<. 3 : karna 3 lebih besar daripada -4
b. -9 .>. -13 : karna -9 lebih besar daripada -13 .

Translated:
3. Compare the following two integers using the signs" > " (more than)," < " (less than) or" = " (equal to) and give a reason!
   a. $-4 < 3$, because the value of 3 is greater than that of -4
   b. $-9 > -13$, because the value of -9 is greater than that of -13

Figure 6. S2's Answers to Question Number 3

From Figure 5 and 6, S1 and S2 correctly compared the values of positive and negative integers with different magnitude symbols, noting that positive numbers have smaller magnitude symbols than negative integer magnitude symbols (question 3a). They also compared the values of two negative integers (question 3b). S1 explained that -4 is less than 3 because, on the number line, 3 is to the right of -4, and -9 is greater than -13 be-cause -9 is to the right of -13. Although S1 knew that 10 is greater than -8 (questions 2c), this understanding did not affect his ability to correctly compare two negative integers. The following interview transcript illustrates this.

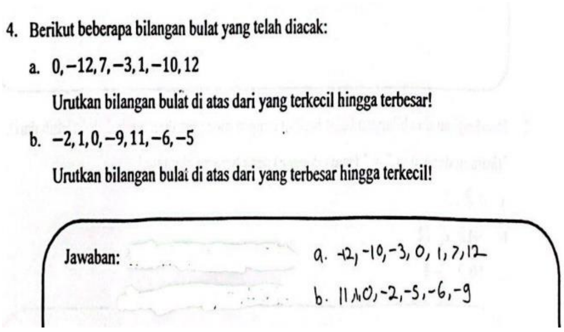
P : Why is -4 less than 3?
S1 : Because on the number line, 3 is to the right of -4, so 3 is greater than -4.
P : Why is -9 greater than -13.
S1 : Because -9 is to the right of -13, Ma'am?

In contrast to S1, S2 explained that any number with a negative sign has a smaller value than any positive number, regardless of the magnitude symbols. When comparing two negative numbers, S2 relied on the number line to demonstrate their reasoning, as shown in the following interview transcript.

P : Okay. Why is -4 less than 3?
S2 : Because -4 is a negative integer, while 3 is a positive integer.
P : Okay, why is -9 greater than -13?
S2 : It is greater because -9 is to the right of

-13 on the number line.

In the fourth problem, ordering integers, S1 and S2 provided correct answers along with appropriate reasoning. Their answers to question 4 can be seen in Figure 7 and 8 below.



4. Berikut beberapa bilangan bulat yang telah diacak:
   a. $0, -12, 7, -3, 1, -10, 12$
   Urutkan bilangan bulat di atas dari yang terkecil hingga terbesar!
   b. $-2, 1, 0, -9, 11, -6, -5$
   Urutkan bilangan bulat di atas dari yang terbesar hingga terkecil!

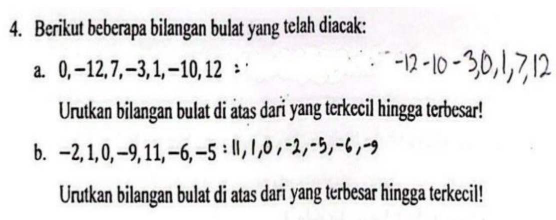
Jawaban: a. -12, -10, -3, 0, 1, 7, 12
b. 11, 1, 0, -2, -5, -6, -9

Translated:

4. Here are some integers that have been randomized:
   a. $-12, 7, -3, 1, -10, 12$
   Sort the integers above from smallest to largest!
   b. $-2, 1, -9, 11, -6, -5, 0$
   Sort the integers above from largest to smallest!

Answer:
a. $-12, -10, -3, 0, 1, 7, 12$
b. $11, 1, 0, -2, -5, -6, -9$

Figure 7. S1's Answers to Question Number 4



4. Berikut beberapa bilangan bulat yang telah diacak:
   a. $0, -12, 7, -3, 1, -10, 12$: -12 -10 -3, 0, 1, 7, 12
   Urutkan bilangan bulat di atas dari yang terkecil hingga terbesar!
   b. $-2, 1, 0, -9, 11, -6, -5$: 11, 1, 0, -2, -5, -6, -9
   Urutkan bilangan bulat di atas dari yang terbesar hingga terkecil!

Translated:

5. Here are some integers that have been randomized:
   c. $-12, 7, -3, 1, -10, 12$
   Sort the integers above from smallest to largest!
   d. $-2, 1, -9, 11, -6, -5, 0$
   Sort the integers above from largest to smallest!

Answer:
c. $-12, -10, -3, 0, 1, 7, 12$
d. $11, 1, 0, -2, -5, -6, -9$

Figure 8. S2's Answers to Question Number 4

From Figure 7 and 8, S1 and S2 successfully sorted the randomized integers. For question 4(a), which involved sorting integer values from smallest to largest, S1 explained that -12 is the leftmost integer on the number line, and therefore, has the smallest value, followed by -10, -3, 0, 1, 7, and finally 12, which is the largest. For question 4(b), which involved sorting integers from largest to smallest, S1 noted that 11 is the rightmost integer on the number line, and therefore, has the largest value, followed by 1, 0, -2, -5, -6, and -9, which is the smallest. This reasoning is reflected in the following interview transcript.

P : Why did you answer like this? (pointing to the answer sheet)
S1 : Because on the number line, -12 is the leftmost, then I sorted to the right.

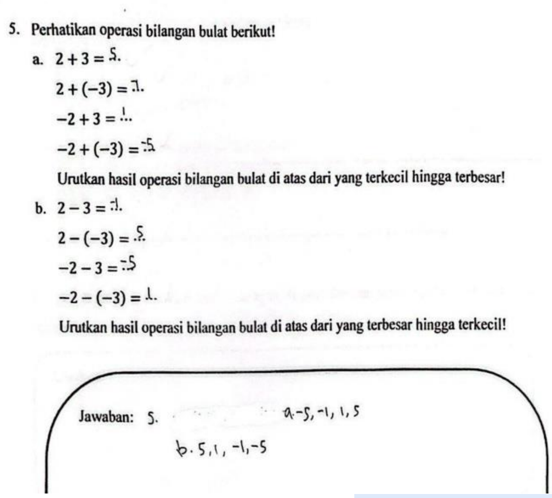
Similarly, S2 provided a similar reason, as shown below.

P : Okay, why did you sort it like this? (pointing to the answer sheet).
S2 : Because -12 is the leftmost (smallest), and 12 is the rightmost (largest).

Based on the interview above, S1 and S2

consistently used the number line to represent their understanding of integers.

In the fifth problem, involving addition and subtraction of integers while sorting them, S1 and S2 answered correctly but provided different reasons. S1's answer to Question 5 are seen in Figure 9 below.


5. Perhatikan operasi bilangan bulat berikut!
a. $2+3=5$
$2+(-3)=-1$
$-2+3=1$
$-2+(-3)=-5$
Urutkan hasil operasi bilangan bulat di atas dari yang terkecil hingga terbesar!
b. $2-3=-1$
$2-(-3)=5$
$-2-3=-5$
$-2-(-3)=1$
Urutkan hasil operasi bilangan bulat di atas dari yang terbesar hingga terkecil!

Jawaban: 5. a. -5, -1, 1, 5
b. 5, 1, -1, -5

Translated:

5. Consider the following addition of integers!
   a. $2+3=5$
      $2+(-3)=-1$
      $-2+3=1$
      $-2+(-3)=-5$
   Sort the sum of the above from smallest to largest!
   b. $2-3=-1$
      $2-(-3)=5$
      $-2-3=-5$
      $-2-(-3)=1$
   Sort the subtraction results of the integers above from largest to smallest!

Figure 9. S1's Answers to Question Number 5

From Figure 9, S1 wrote $2+(-3)=-1$. The result is obtained from $2-3=-1$. S1 explained that adding a positive integer to a negative integer is equivalent to subtracting a positive integer from another positive integer. The addition sign changes to subtraction. S1 applied similar reasoning when answering $-2+3=1$. The result was obtained from $3-2=1$. This can be shown in the following interview excerpts.

P : Why $2+(-3)=-1$*?*
S1 : Because $2-3=-1$*.*

In the following question, S1 wrote $-2+(-3)=-5$. S1 explained that the negative value will be greater if a negative integer is added to a negative integer. This can be shown in the following interview transcript.

P : Why $-2+(-3)=-5$*?*
S1 : Because both are negative. So just add them up, Ma'am.

In the next question, S1 wrote that $2-(-3)=5$. The result is obtained from $2+3=5$. When a positive integer is subtracted from a negative integer, it is equivalent to removing the negative value in the operation. In this case, to eliminate -3, 3 must be added. Thus, the result is 5. This can be shown in the following interview transcript.

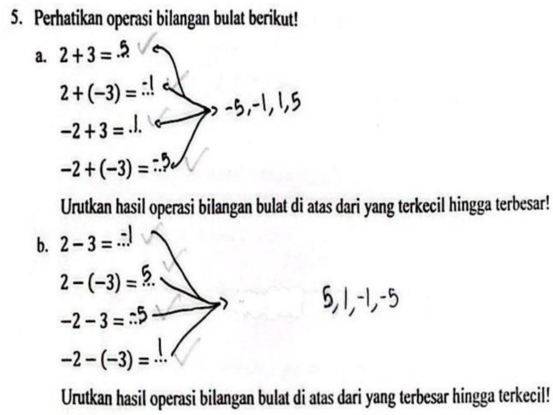
P : Why $2-(-3)=5$*?*
S1 : Because $2+3=5$*, Ma'am.*

In the next question, S1 wrote that $-2-(-3)=1$. The result is obtained from $-2+3=1$, and the result $-2+3=1$ is obtained from $3-2=1$. S1 has applied his understanding of addition and subtraction of integers consistently. This can be shown in the following interview transcript.

P : Why $-2-(-3)=1$*?*
S1 : Because $-2+3=1$*.*
P : Why $-2+3=1$*?*
S1 : Because $3-2=1$*, Ma'am.*

In the fifth problem, S1 did not use the number line rule to represent his understanding. Nevertheless, S1 demonstrated the ability to add and subtract integers and to sort them appropriately.

Furthermore, S2's answer can be seen in Figure 10 below.


5. Perhatikan operasi bilangan bulat berikut!
a. 2 + 3 = 5
2 + (−3) = −1
−2 + 3 = 1
−2 + (−3) = −5
−5, −1, 1, 5
Urutkan hasil operasi bilangan bulat di atas dari yang terkecil hingga terbesar!
b. 2 − 3 = −1
2 − (−3) = 5
−2 − 3 = −5
−2 − (−3) = 1
5, 1, −1, −5
Urutkan hasil operasi bilangan bulat di atas dari yang terbesar hingga terkecil!

Translated:

5. Consider the following addition of integers!
   a. $2 + 3 = 5$
   $2 + (-3) = -1$
   $-2 + 3 = 1$
   $-2 + (-3) = -5$
   Sort the sum of the above from smallest to largest!
   b. $2 - 3 = -1$
   $2 - (-3) = 5$
   $-2 - 3 = -5$
   $-2 - (-3) = 1$
   Sort the subtraction results of the integers above from largest to smallest!

Figure 10. S2's Answers to Question Number 5

Based on Figure 10, S2 wrote $2 + (-3) = -1$. The result is obtained from $2 - 3 = -1$. S2 explained that if a positive number is added to a negative number, it is equivalent to subtracting a positive number from another a positive number. Since the subtracting number is larger, the result is negative. Similar reasoning was given by S2 when writing $-2 + 3 = 1$. The result is obtained from $3 - 2 = 1$. Since the subtracting number is smaller, the result is positive. This can be shown in the following interview transcript.

P : Why $2 + (-3) = -1$?
S2 : Because of the difference, Ma'am.
P : What do you mean?
S2 : $2 + (-3)$ is equal to $2 - 3$, Ma'am.
P : Okay, why $2 - 3 = -1$?
S2 : Because the subtracting number is greater, Ma'am. That is why the result is negative.
P : Okay, why $-2 + 3 = 1$?
S2: Because the positive number is greater, Ma'am. That's why the result is 1.

S2 also explained other reasons for obtaining these results, such as using a number line. This can be shown in the following interview transcript.

P : Are there any other ways to add and subtract integers that you use?
S2 : Yes, Ma'am. Use the number line.
P : How do you do it?
S2 : For example, $-2 + 3 = 1$, Ma'am. We start by moving backward from 0 to -2. Then, because we add a positive number, we move forward 3 steps. The result is 1.
P : Then, what about $2 + (-3) = -1$?
S2 : We start from to 2, Ma'am. Because we add a negative number, we move back 3 steps. The result is -1.

In the next question, S1 wrote that $-2 + (-3) = 5$. S1 explained that the negative integer increases if a negative integer is added to another negative integer. This can be shown in the following interview transcript.

P : Why $-2 + (-3) = -5$?
S2 : Because both are negative, Ma'am. So, the result is negative and just add them.

S2 also explained other reasons for using the number line. This can be shown in the following interview transcript.

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P : Then what about $-2+(-3)=-5$*?*
S2 : Because it is a negative integer plus a negative integer, it moves to the left, Ma'am. From 0, it moves back to the left, -2, and then back to the left for 3 steps. Now the result is -5.

In the next question, S2 wrote 2-(-3)=5. The result is obtained from2+3=5. S2 explained that when subtraction meets a negative integer, the subtraction is reversed into addition, and the negative integer is reversed into a positive integer. This can be shown in the following interview transcript.

P : Okay, why $2-(-3)=5$*?*
S2 : Because the subtraction formula reverses into addition when it meets a negative integer, and the negative integer turns into a positive one.
P : Okay, that's why the result is 5?
S2: Yes, Ma'am.

S2 also explained other reasons for using the number line. This can be shown in the following interview transcript.

P : What about $2-(-3)=5$*?*
S2 : We start from 0 and move forward to 2, Ma'am. Because the subtraction meets a negative sign, it turns into an addition sign, and -3 turns into 3. It becomes $2+3=5$*.*

In the next question, S1 wrote that $-2-3=-5$. S2 explained that $-2-3=-5$ is the same as $-2+(-3)=5$.This can be shown in the following interview transcript.

P : Then why $-2-3=-5$*?*
S2 : Because it is the same as $-2+(-3)=5$ *, Ma'am.*

In the next question, S2 wrote that $-2-(-3)=1$. S2 explained that when a subtraction sign meets a negative integer, the subtraction sign is reversed into an addition sign, and the negative integer becomes positive. Thus, $-2+3$ or $3-2$ is obtained. Furthermore, because the subtracting number is smaller, the result is positive, that is 1. This can be shown in the following interview transcript.

P : Then, why $-2-(-3)=1$*?*
S2 : Because the subtraction sign turns into addition, then -3 turns into 3. So, $-2+3=1$*.*

S2 also explained other reasons for using the number line. This can be shown in the following interview transcript.

P : What about $-2-(-3)=1$*?*
S2 : We start moving backwards from 0 to -2, Ma'am. Since the subtraction sign meets negative, it turns into an addition sign and -3 turns positive, so we move to the right again for 3 steps. The result is 1.

In the fifth problem, S2 effectively used the number line rule to represent his understanding. S2 demonstrated the ability to add and subtract integers and sort them.

Discussion

This study aims to describe the characteristics of formal mental models of junior high school students in understanding the concept of integers. The characteristics of formal mental models identified in subjects S1 and S2 are as follows: In Question 1, S1 and S2 answered correctly. Both subjects were able to place positive integers, 0, and negative integers on the number line. S1 and S2 demonstrated the understanding of the integers in their minds as an external representation on the number line. This indicated that S1 and S2 had surpassed the pre-initial and initial mental models and have met the characteristics of the first Transition I mental model: the ability to place positive integers, zero, and negative integers on a number line.

In Question 2, S1 and S2 answered correctly, providing appropriate reasoning. Both subjects successfully compared positive and negative integers. S1 explained that integers positioned farther to the right on the number line have greater values, while those farther to the left have smaller values. Therefore, positive integers are larger than negative integers. S2 explained that all negative integers are smaller than positive integers, regardless of the relative magnitude of their symbols. This explanation differs from findings by (Sukiyanto et al., 2022, 2023) which suggest that students generally compare positive and negative integers only when the magnitude symbols are identical (Sukiyanto et al., 2023) and compare positive and negative integer values for the same and different magnitude symbols (Sukiyanto et al., 2022). Thus, S1 and S2 have fulfilled the characteristics of the second Transition I mental model: the ability to compare positive and negative integers (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018).

In Question 3(b), S1 and S2 answered correctly with appropriate reasoning. Both subjects compared two negative integers. S1 and S2 constructed their internal representations into external representations. They clarified that a number farther to the right on the number line has a greater value, while a number farther to the left has a smaller value. Conversely, if it is located further to the left then the value of the number will be smaller. Thus, S1 and S2 have met the characteristics of synthetic mental (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018).

In Question 4 (a&b), S1 and S2 sorted the integers correctly. They explained in detail how they could sort the randomized integers with the help of a number line. S1 and S2 stated that the farther to the right an integer is, the greater its value, while the farther to the left an integer is, the smaller its value. They consistently used the number line to represent their understanding of the integer value and order. Previous studies Bofferding, (2014); Sukiyanto et al., (2022, 2023); Utami et al., (2018) have shown that students capable of consistently sequencing integers exhibit characteristics of formal mental models. However, in this study, the characteristics of formal mental models extend to the addition and subtraction of more complex integers, as per the theory proposed by Peled (1991). Thus, S1 and S2 have demonstrated the characteristics of Mental Model Transition II-Consistent and have passed Mental Model Transition II-Inconsistent. Bofferding (2014) noted that students who can consistently sequence whole numbers understand negative numbers as part of an integrated number system and can comprehensively understand the concepts of value and number order.

In Question 5, S1 and S2 answered correctly and provided appropriate reasoning. However, in this last question, their explanation revealed a significant difference. S1 demonstrated the ability to add and subtract integers accurately. For example, $2-(-3)$, the result is obtained from $2+3=5$. S1 explained that when the subtraction sign meets a negative sign, the subtraction sign changes to addition, and the negative sign becomes positive. While this reasoning is acceptable, a more precise explanation is that when a positive number is subtracted from a negative number, it is the same as eliminating the negative value in the operation. In this case, to eliminate -3, 3 must be added, so the result is 5. This aligns with Bofferding's assertion that interpreting addition as 'getting more' and subtraction as 'getting less' equates negative addition to positive subtraction and negative subtraction to positive addition (Bofferding, 2014). Although S1 did not use a number line, S1 answered all Question 5 and sorted the addition and subtraction correctly. Thus, S1 has met the characteristics of the formal mental model.

S2, on the other hand, provided two alternative explanations in the process of obtaining the answer. The first mirrored S1's approach, while the second alternative used a number line. For instance, when adding $-2+3=1$, S2 illustrated that the arrow moves backward from 0 to -2, then because there is an addition sign, the arrow moves forward 3 steps, resulting in 1. S2 also consistently applied the number line throughout the subsequent sub-questions in Question 5. In the sub-question $-2-(-3)=1$, S2 explained that the arrow moves backward from 0 to -2, then because the subtraction sign meets a negative, it changes to addition. -3 becomes 3, so the arrow moves to the right, resulting in 1. This indicates S2's understanding of integer magnitudes and their directions on the number line whether positive or negative. S2 demonstrated knowledge of when to move the arrow to the right or left or move forward or backward. However, S2 is imperfect in representing their understanding when the subtraction sign meets a negative number on the number line. In the sub-question $-2-(-3)=1$, the correct explanation would involve the arrow moving backward from 0 to -2 facing right. Because the subtraction sign meets a negative, the arrow turns left and moves back 3 steps, resulting in 1. Basically, the number line can effectively be used to represent the concept of addition and subtraction of integers. This aligns with the opinion of Bennett et al., (2012) stating that the number line is the most accurate medium for teaching students the concept of addition and subtraction of integers. Like S1, S2 also correctly ordered the addition and subtraction results, meeting the characteristics of a formal mental model.

In general, students at the formal mental model level have a more comprehensive understanding of the concept of integers. The characteristics include the ability to place positive integers, zero, and negative integers on a number line, compare positive integers and negative integers, compare two negative integers, order integers, and add and subtract integers as well as order them. These characteristics differ from the characteristics of the formal mental model theorized (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018) which are limited to the mental model of the value and order of integers and discuss the mental model of directed magnitude (addition and subtraction) separately.

Implication of Research

Students' mental models in understanding the concept of integers have been studied in several previous research (Bofferding, 2014; Sukiyanto et al., 2022, 2023; Utami et al., 2018). This study expands the scope of students' mental models in understanding the concept of integers by combining Bofferding's (2014) ideas on mental models of integer value and order and mental models of directed magnitude (addition and subtraction of integers) with Peled's theory (Peled, 1991). The study contributes theoretically by offering a comprehensive framework of students' mental models in understanding the concept of integers. Practically, it provides educators with insights into the characteristics of formal mental models that seventh-grade students need to master, supporting the development of meaningful instructional strategies to strengthen students' conceptual understanding and promote the advancement of mental models to the formal level. In addition, these findings serve as a reference for future research on students' mental models in understanding integers and performing integer operations.

Limitation

This research has several limitations: It focuses solely on students who are at the formal mental model level. This focus was chosen because the formal mental model represents the highest level of the mental model, meaning that students at the formal mental model level have progressed through all previous mental model levels. This approach allows for a broader scope in describing the characteristics of students' mental models in understanding the concept of integers. The data source for this research is limited to one seventh-grade junior high school.

CONCLUSION

Based on the research results, it is known that students who are at the formal mental model level have fulfilled all the characteristics in each level of the previous mental model. In other words, students at the formal mental model level have a more comprehensive understanding of the concept of integers. The students at the formal mental model level demonstrated the characteristics of being able to place positive integers, zero, and negative integers on a number line, compare positive and negative integers, compare two negative integers, sort integers, add and subtract integers, and sort them accordingly. By refining the characteristics of the mental model of value and order of integers and integrating Bofferding's ideas on the mental model of value and order and the mental model of directed magnitude, this study successfully expands the theoretical framework of students' mental models in understanding the concept of integers, extending it to include addition and subtraction operations.

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