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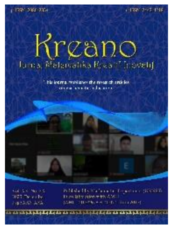

Designing Learning Instructional Theory Topics Special Lines Triangle with Augmented Reality: A Design Research

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Abstract

Geometry is still most commonly taught using two-dimensional media such as a blackboard. What is presented on the whiteboard can have varied points of view depending on the circumstances. The digitalization of learning is progressing in tandem with the most recent technical breakthroughs, such as Augmented Reality (AR). The purpose of this research is to create a learning design for specific triangle line material in the context of the Musi 6 Bridge utilizing AR. The research was conducted on junior high school students. This design research has three stages: experiment preparation, design experiment, and retrospective analysis. Observation, interview, documentation, and test were used to obtain data. This study developed a Local Instruction Theory (LIT) comprising five sequenced activities using the Musi 6 Palembang Bridge context and augmented reality (AR) technology. It consists of 5 activities: observing AR of Musi 6 bridge, defining height line, defining bisector line, defining median of a triangle, defining axis lines in a triangle. The study's findings revealed that, students in cycle-2 did not do better on the preliminary test than students in cycle-1, their grasp of special lines of triangles performed better than students in cycle-1. This difference shows how the activity sheet in cycle-2 provides more convenience for students in understanding the material with additional activities at the beginning of cycle-2 to recall the prerequisite material contained in the TKA. Revisions to the activity sheet made on the activity sheet include: easier AR access, AR is made more real in front of students without background, Geogebra applet is made easier to access and operate by students. This study shows that integrating AR-GeoGebra into geometry instruction (1) improves conceptual understanding through interactive 3D visualization and (2) bridges hypothetical-actual learning trajectories via real-time conjecture adjustments, while emphasizing the need for infrastructure support and teacher training. With contextual modifications, the results suggest an adaptive pedagogical model that can be used in all STEM fields.

Keywords: Special Lines; Triangles; Augmented Reality; Design Research.

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Abstrak

Geometri masih paling sering diajarkan menggunakan media dua dimensi seperti papan tulis. Apa yang disajikan di papan tulis dapat memiliki sudut pandang yang berbeda-beda tergantung pada keadaannya. Digitalisasi pembelajaran mengalami kemajuan seiring dengan terobosan teknis terkini, seperti Augmented Reality (AR). Tujuan penelitian ini adalah membuat desain pembelajaran materi garis segitiga tertentu pada konteks Jembatan Musi 6 dengan memanfaatkan AR. Penelitian ini dilakukan pada siswa sekolah menengah pertama. Penelitian desain ini memiliki tiga tahap: persiapan eksperimen, desain eksperimen, dan analisis retrospektif. Observasi, wawancara, dokumentasi, dan tes digunakan untuk memperoleh data. Penelitian ini mengembangkan Teori Pembelajaran Lokal (LIT) yang terdiri dari lima kegiatan berurutan menggunakan konteks Jembatan Musi 6 Palembang dan teknologi augmented reality (AR). LIT terdiri dari 5 kegiatan yaitu mengamati AR jembatan Musi 6, menentukan garis ketinggian, menentukan garis bagi, menentukan garis berat, menentukan garis sumbu pada suatu segitiga. Temuan penelitian mengungkapkan bahwa, siswa pada siklus-2 tidak mengerjakan tes pendahuluan lebih baik dibandingkan siswa pada siklus-1, pemahaman mereka terhadap garis-garis khusus segitiga lebih baik daripada siswa pada siklus-1. Perbedaan ini menunjukkan bahwa lembar kegiatan pada siklus-2 memberikan kemudahan bagi siswa dalam memahami materi dengan kegiatan tambahan di awal siklus-2 untuk mengingat kembali materi prasyarat yang ada pada TKA. Revisi lembar kegiatan yang dilakukan pada lembar kegiatan antara lain: akses AR lebih mudah, AR dibuat lebih nyata di depan siswa tanpa latar belakang, applet Geogebra dibuat lebih mudah diakses dan dioperasikan oleh siswa. Studi ini menunjukkan bahwa mengintegrasikan AR-GeoGebra ke dalam pembelajaran geometri (1) meningkatkan pemahaman konseptual melalui visualisasi 3D interaktif dan (2) menjembatani lintasan pembelajaran hipotetis-aktual melalui penyesuaian dugaan waktu nyata, sekaligus menekankan perlunya dukungan infrastruktur dan pelatihan guru. Dengan modifikasi kontekstual, hasilnya menunjukkan model pedagogis adaptif yang dapat digunakan di semua bidang STEM.

INTRODUCTION

Geometric studies contain significant conversations about dimensions, sizes, and distances. Due to the complexity of geometric concepts (e.g., angles, distances, and transformations) and the need to think and visualize in different dimensions, geometry is often regarded by learners as one of the most challenging areas of mathematics (Hwang & Hu, 2023; Khalil et al., 2022). PISA 2022 results show that Indonesian students' mathematical performance, particularly in the geometry, continues to lag significantly below the global average (OECD, 2023). The data reveal that Indonesian students can only answer issues with direct formulas and cannot integrate multiple geometric concepts to settle on a solution to a problem (OECD, 2023; Ulya et al., 2024). This aligns with findings that geometry instruction in Indonesia remains procedural rather than conceptual, limiting spatial reasoning development (Prahmana et al., 2023)

The geometry learning process at school is one of the variables that enhances pupils' achievement of understanding. Geometry is still most commonly taught using two-dimensional media such as a blackboard. In practice, teaching geometry with a blackboard is not simple. Teachers and students must continue to describe the same objects to reinforce several geometric ideas. What is presented on the whiteboard often leads to multiple interpretations due to perceptual limitations (Haqq & Putra, 2023). Students frequently maintain incorrect understandings of geometric measurements like area and volume due to difficulties in mentally visualizing and manipulating multi-dimensional shapes (Battista, 2023). Misconceptions about spatial aspects are possible when using 2D media, including whiteboard diagrams, to teach 3D geometric concepts (Battista, 2023; Wijaya et al., 2022). Two-dimensional pictures on whiteboards or other 2D learning medium have the ability to give birth to interpretations that give rise to mistakes in three-dimensional geometry learning that discusses elements (points, distances, and planes). As a result, additional space in supplying geometry learning material is required for innovation in geometry learning. These findings underscore the critical need for innovative spatial learning tools that transcend the constraints of planar representations (Prahmana et al., 2023).

In order to help students overcome these difficulties, a geometry topic learning design is needed that includes students' learning trajectories in understanding concepts and solving problems related to geometry. In research in the last decade, design research in learning has become a topic of discussion that has attracted much attention from researchers. A systematic literature review of 163 research publications between 2013 and 2020 provides an overview of how design research has grown in order to complement the need to improve the quality of learning in the classroom (Tinoca et al, 2022). Learning designs that include appropriate activities and stimulation for students help to enhance students' mathematical understanding (Kusuma et al., 2023; Nguyen & Tran, 2024).

Learning design should contain a series of meaningful learning activities for students in sequence towards comprehensive understanding. The assumption that contains the pattern and direction of students' mental activities in building an understanding of mathematical concepts is known as the Hypothetical Learning Trajectory or HLT (M. Simon, 2020; M. A. Simon & Tzur, 2004). HLT is arranged in the framework of mathematical idea construction activities and training thinking skills, especially the preparation of evidence (Agustiani & Nursalim, 2020; Cobb & Gravemeijer, 2006).

The National Council of Teachers of Mathematics (NCTM) emphasizes that technology integration enhances instructional quality, improves learning effectiveness, and expands pedagogical possibilities in mathematics education (Nguyen et al., 2023). This paradigm shift necessitates schools to adopt technological innovations in developing learning media. As today's students are digital natives, the digital transformation of learning environments has become inevitable—a hallmark of the Fourth Industrial Revolution (4IR) in education (Prahmana et al., 2024). These digital learning environments encompass all forms of learner-environment interactions, whether intentional or incidental (Kusumawardani et al., 2023).

AR dynamically superimposes virtual objects onto real-world contexts to facilitate conceptual understanding (Alghadari et al., 2024). Empirical studies confirm AR's efficacy as a learning medium, with Wardhani et al. (2023) demonstrating its superiority in visualizing 2D/3D mathematical objects in real time, thereby increasing student engagement and comprehension.

The National Council of Teachers of Mathematics (NCTM) concept is based on the idea that technology has the ability to improve quality, support effectiveness, and expands pedagogical possibilities in mathematics education (Nguyen et al., 2023). This paradigm shift necessitates schools to adopt technological innovations in developing learning media. This implies that schools are expected to use technology breakthroughs in the creation of learning media. Students in today's generation are the digital generation, hence digitalization of learning is unavoidable. Digitalization of learning is a feature of the digital learning environment, which is a feature of the fourth industrial revolution. The environment is described here as any sort of interaction between pupils and their surroundings, whether consciously or unconsciously, directly or indirectly (Prahmana et al., 2024). These digital learning environments encompass all forms of learner-environment interactions, whether intentional or incidental (Kusumawardani et al., 2023).

Among emerging technologies, Augmented Reality (AR) has gained significant traction in mathematics instruction. Augmented Reality (AR) is a new technology that is commonly employed in mathematics study right now. AR is a technology that incorporates virtual content into real-world settings to aid in teaching and learning (Saundarajan et al, 2020). AR dynamically superimposes virtual objects onto real-world contexts to facilitate conceptual understanding (Alghadari et al., 2024). According to Wardhani et al. (2023), augmented reality (AR) technology is extremely effective in visualizing 2D/3D mathematical objects in real time, thereby increasing student engagement and comprehension Whether in the form of 2D or 3D objects. AR shows objects that are appealing to students in real time to promote understanding and drive learning.

Aside from the use of technology, it is critical to construct mathematical learning designs that are meaningful for students in order to assist the accomplishment of 21st century capabilities. According to Freudenthal, mathematics learning is basically a human activity (Sembiring, 2014; Hadi, 2017), which can be read as pupils being given the opportunity to learn through mathematization activities in mathematics learning. As a result, tangible efforts are required to investigate alternative circumstances or contexts that are appropriate for students' cognitive ability. Not only is it important as a backdrop for learning mathematics, but cultural context also plays a role in improving students' character, both cultural character and religious and patriotic character (Richardo, 2020).

Freudenthal's ideas are highly relevant to the needs of modern education. However, the challenge is how to create a learning environment that allows students to actually "do" mathematics, not just "receive" mathematics. This is where Augmented Reality (AR) emerges as an innovative and transformative teaching tool. AR has tremendous potential to bridge the gap between abstract mathematical concepts and the real world, thus aligning perfectly with Freudenthal's philosophy. AR has the potential to create immersive and engaging learning experiences and emphasizes the importance of meaningful mathematical design that is rooted in real-world and cultural contexts (Arifin & Efriani, 2025; Voulgari et al, 2024).

Based on the background description, the researcher concluded that developing a learning design in the form of a hypothetical learning trajectory (HLT) of geometry material using a cultural context in collaboration with AR technology is necessary to support the effectiveness of mathematics learning. It is hoped that this learning can help and make it easier for students to understand geometric concepts.

METHOD

This study employs Design Research. This study employs an acyclic (repetitive) process from a learning experiment in the form of a diagram that illustrates the experiment's concept. The research was conducted at class VII of SMPN 52 Palembang and SMP 44 Palembang. The procedure carried out in this study went through three stages: preparing for the experiment, designing the experiment, and retrospective analysis (Cobb & Gravemeijer, 2006).

Preparation for the Experiment

A literature review is conducted at this stage by reviewing literature such as journals, papers, and books relating to the triangle idea material. During this literature review, the researcher also constructed a research instrument. The Palembang Musi 6 Bridge building then served as the beginning of the Hypothetical Learning Trajectory (HLT), which will be designed by expert review. A hypothetical learning trajectory is a theoretical paradigm for developing mathematical training that consists of three parts: a learning goal, a collection of learning problems, and a supposed learning process (M. Simon, 2020; M. A. Simon & Tzur, 2004). The HLT, Activity Sheets, initial ability test questions and final test questions to be reviewed by experts where we involved 2 mathematics education lecturers, 1 information systems lecturer, and 1 IT practitioner who developed digital learning media in schools. The expert review activity did not end in the valid category but served to provide input and discussion related to the learning design complete with the instruments to be used. Engaging multidisciplinary experts guarantees that the instructional design is both grounded in theory and applicable in real-world settings (Plomp & Nieveen, 2013). Furthermore, collaborating with IT specialists and educators during instrument development facilitates the effective incorporation of digital tools in math education (Schoenfeld, 2014). Unlike traditional validation processes, expert reviews in design research adopt a developmental approach, where specialists provide feedback to progressively enhance prototypes through multiple iterations (Akker, 2006; McKenney & Reeves, 2019).

Design Experiment

A teaching experiment was carried out during the second step. The teaching experiment, according to (Akker et al., 2006), intends to test and refine the Hypothetical Learning Trajectory (HLT) conjecture generated in the early stages, as well as to gain a knowledge of how it operates. The goal of this teaching research is to collect data in order to answer the research questions. So, at this point, the actions initiated in the early phases are carried out in three cycles. The first cycle was carried out as a pilot experiment, with the goal of fine-tuning and improving both the content and sequence of activities generated in order to produce a better HLT design for the subsequent cycle. Furthermore, the following 2-cycles, namely teaching experiments, is carried out as an actual teaching process in the classroom, where the sequence of activities is carried out. At this level, it prioritizes data collection related to HLT implementation and data from student answers in each sequence of activities carried out as material for developing local instructional theory.

Retrospective Analysis

Retrospective analysis is the examination of the full data set in order to contribute to the development of a local instructional theory and (improvement of) the interpretative framework (Cobb & Gravemeijer, 2006; McKenney et al., 2006). At this step, HLT is used as a guideline and reference point in analysing the data obtained during the experiment design stage of instructional experiments. In the classroom, this HLT is compared to actual theory and student learning processes. The investigation was explained in terms of both instances that supported and rejected the hypothesis. The results of this analysis served as answers to the research questions. The primary outcome is not a working design, but rather the rationales for how it works (Bustang et al., 2013; Cobb & Gravemeijer, 2006). This conclusion and analysis are utilized to respond to the framing of the research problem.

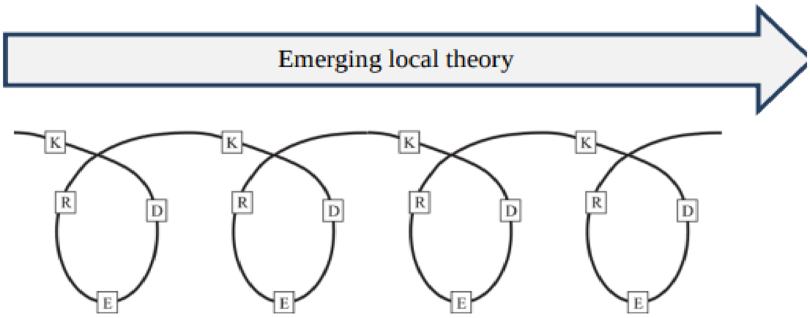


Figure 1. Cyclic process of knowledge, design, experiment, reflection and (new) knowledge. (Akker et al., 2006; Cobb & Gravemeijer, 2006; Van Eerde, 2013)

In order to strengthen the data analysis, the discussion of the results of this study also goes through the validity and reliability of the data. Design research demands special focus on data accuracy issues like validity and reliability (Cobb and Gravemeijer, 2006). This dual focus on methodological quality serves to establish both the credibility of findings and their reproducibility in educational contexts (McKenney & Reeves, 2019).

In this case, validity involves ensuring precision of all data elements captured during different processes of the design implementation to analysis interpretation. The role of HLT in construct validity as well as conclusion traceability is critical (Bakker, 2018). Moreover, the HLT facilitates researchers to check whether students achieve the learning goals often yielding confirmation beyond what was expected enhancing the study's interpretive validity (Drijvers et al., 2021).

Validity in this study emphasizes the accuracy of the data collected as a whole from the design process to the analysis process. Validity in this learning design study emphasizes: HLT as a means to support validity and Trackability of the conclusion.

Data reliability means the extent to which the data can be trusted. The indicator of reliable measurement data is the existence of relatively similar results in several measurement activities on the same subject, as long as the aspects measured in the subject have not changed. Reliability refers to the trustworthiness of research data, proven by consistent outcomes after multiple samplings under similar conditions (Plomp & Nieveen, 2013). In a qualitative design research paradigm, reliability is achieved through two primary procedures: (1) data triangulation which is corroborating data through different types of evidence such as evaluative observational notes, interviews, and material artifacts (Creswell & Poth, 2018); and (2) cross-interpretation where several researchers look at the same data set individually so as to reduce bias stemming from single researcher influence and improve appraisal objectivity (Schoenfeld, 2014). Reliability in this study was carried out qualitatively. Qualitative reliability can be done in two ways, namely: Data Triangulation and Cross Interpretation. Utilizing multiple sources of data and analytical triangulation bolsters validity and reliability at the same time (Schoenfeld, 2014; Cobb et al., 2017).

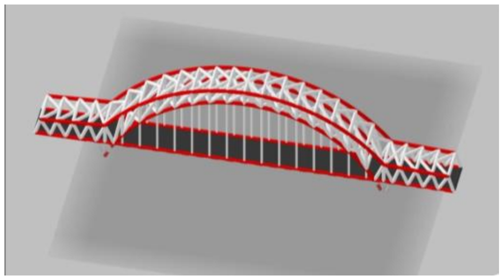
RESULT AND DISCUSSION

Results

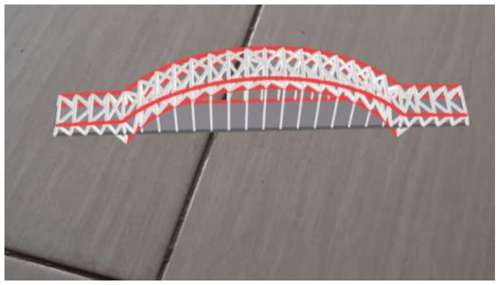
The research produced a Hypothetical Learning Trajectory (HLT), Activity Sheet with Augmented Reality, and Initial Ability Test (Pretest) to be employed in teaching experiment activities in this design research study. The teaching experiment activity is divided into three stages: the pilot experiment, the teaching experiment cycle 1, and the teaching experiment cycle 2. Following completion of each stage of the teaching experiment, a retrospective analysis is performed to reflect on and evaluate the learning design, particularly HLT, before proceeding to the teaching experiment stage. furthermore. The outcome of this design research effort is augmented reality learning material on special triangular lines based on Local Instruction Theory (LIT). The activities performed in the teaching experiment activity are described below.

Activity 1

The first activity, students use augmented reality (AR) technology to investigate the concept of triangle special lines in the context of the Musi 6 Palembang Bridge. Students scan the barcode on the student activity sheet provided. Once pupils have access to it, an illustration of the Musi 6 bridge with a 3-dimensional display and other activities will surface. Students can view images of the Musi 6 bridge from various angles using this 3-dimensional display. The appearance of the bridge in teaching experiment cycle 2 was altered as a consequence of discussions and advice from IT specialists. The AR display in question is depicted in the image below.



Cycle 1



Cycle 2

Figure 2. The appearance of the bridge

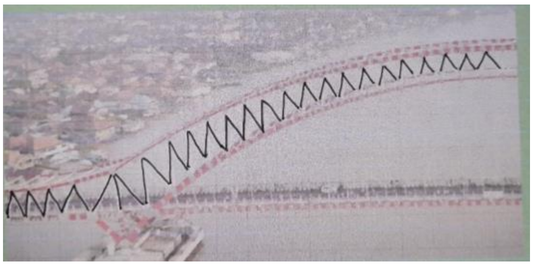
Students observe by moving or manipulating the 3-dimensional media of the Musi 6 bridge to discover where the triangle shape is on the iron frame of the Musi 6 bridge. In this problem, the teacher directs the group and inquires as to what is impeding the pupils' progress. Students in both cycle 1 and cycle 2 seemed very interested in the AR display of the bridge, the following is a transcript of the teacher and student conversation.

Student: Should I just stare at it or do something else?
Teacher: This media can be shifted and moved in the same way.
(showing pupils an example that may be shifted and moved to be seen from multiple points of view)
Student: So it's like a game, bro, you can move it yourself
Teacher: Why do you refer to it as a game?
Student: Because it can be moved, it is similar to playing a cellphone game.
Teacher: Well, that's more interesting, right? Now move it yourself, pay

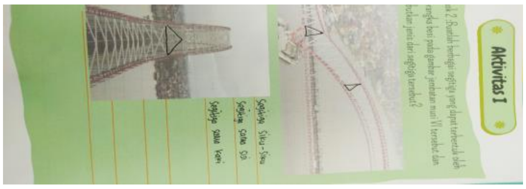
attention to the commands

Student moves the Musi 6 bridge media by sliding it from right to left then from right to bottom, paying attention to how it looks like a triangle.

From the results of the activities in cycle 1, there were groups that matched the previous conjecture which depicted the triangle shape in the framework well, but there were groups whose drawing results were outside the conjecture. Meanwhile, in cycle 2, all groups were able to describe triangles as expected.



Cycle 1



Cycle 2

Figure 3. Triangle of the bridge

Then the student activities continued using the Geogebra application, students were asked to determine the midpoint of a line and determine the line that divides equal angles. The student's answer below shows that the student has understood the concept of the midpoint. Students also understand the concept of dividing angles using equal lines. This is in accordance with the HLT that has been designed. See Figure 4.

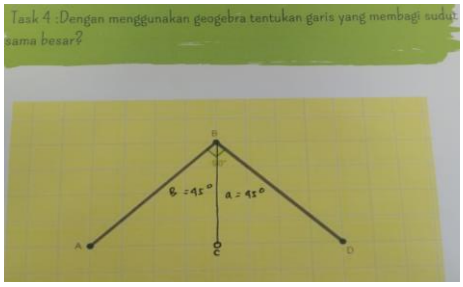
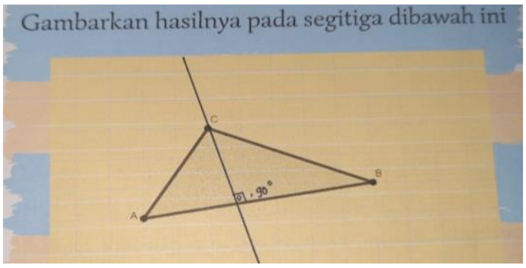


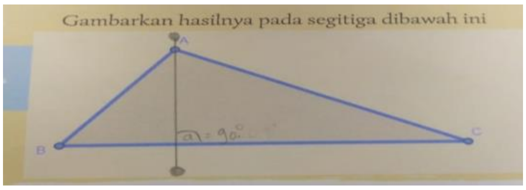
Figure 4. Student's answer about the concept of the mid-point

Activity 2

The second activity, Students use the Geogebra application to draw a height line of triangle to the triangle as the triangle's height line and see changes in the height line based on the shape of the triangle. Before defining the height line, students are given media in Geogebra to determine which pole can indicate the height of the bridge. In this activity, students are able to show that the upright frame pillars are the height of the Musi 6 bridge. Then students are given an example of the steps in drawing the height line in a triangle. Students are then instructed to use Geogebra to draw a height line in accordance with the instructions, then re-describe the outcomes on the activity-sheet.



Cycle 1



Cycle 2

Figure 5. Student's answer about the height line

Activity 3

The third activity, Students define the bisector by creating a line with Geogebra that may split the angles of an equal triangle. After that, students are given a triangle with lines already drawn on it, and they adjust the shape of the triangle to better comprehend the bisectors of each type of triangle. Because drawing bisector lines is different from drawing basic height lines, pupils sometimes struggle while using the augmented reality in barcodes. A group that initially struggled to draw the triangle's bisector with accuracy was eventually able to do so after receiving instructions. The dividing line can be effectively drawn by each group.

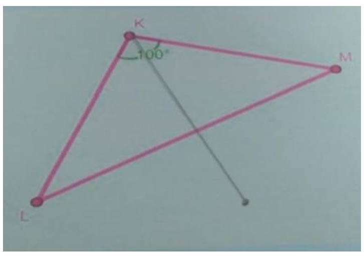


Figure 6. Student's answer in drawing bisector line

Students practice a variety of triangles during the assignment while paying close attention to the triangles' bisectors. In this project, students use Geogebra, which has been created to allow the triangle's axis lines to adapt to changes in the triangle if one of its sides is moved. Students who successfully complete this task will know that a triangle's bisector still divides its angles evenly.

Activity 4

The fourth activity, students use Geogebra to create a median of a triangle by identifying the midpoint of one side of a triangle and connecting the point to the corner facing that side, and then they experiment with different sorts of triangles. Students can work on the median of a triangle by carefully examining and focusing on the provided examples, along with some guidance from the researcher. It can be well described by students. The researcher's hypothesis that students will find it challenging to determine the middle led to the creation of this heavy line. With the provided instructions and examples, however, students are able to adequately characterize the median of a triangle.

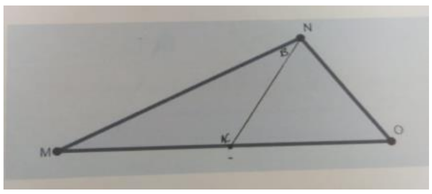
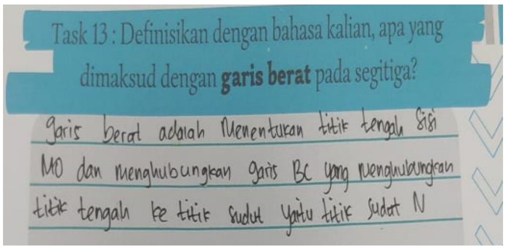


Figure 7. Student's answer in drawing median of a triangle

Nevertheless, some groups of students continue to define the line weight indefinitely. If you look closely at the thick lines they drew, you can see how they have defined it in their own language. According to what they observe, Group 2 has defined it, but it is less precise than the definition of the median of a triangle.



Task 13: Define in your own words, what is meant by a median in a triangle?

A median is determining the midpoint of side MO and connecting line BC which connects the midpoint to the vertex, which is vertex N

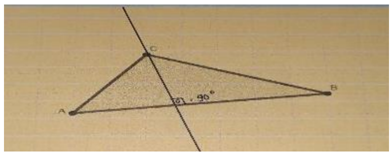
Figure 8. Student's answer in defining median of a triangle

Activity 5

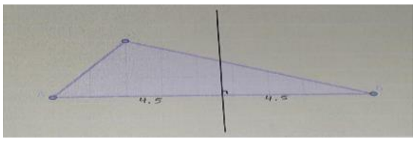
The fifth activity, students draw the axis of a triangle using Geogebra by identifying the midpoint of one of the triangle's sides and then drawing a line perpendicular to that side at the midpoint. Following that, students conclude the significance of axis lines in triangles and attempt to adjust the type of triangle that already includes axis lines so that students better comprehend the axis lines. Students are given a line that is perpendicular to the Musi 6 bridge, and they must determine that this line is perpendicular to the Musi 6 bridge's center point in order to know which pole is perpendicular and exactly in the middle of the Musi 6 bridge before drawing the axis lines in the triangle.

Then, using multiple corner points to alter the axis lines, students in the activity pay close attention to the axis lines of various forms of triangles. Many students found the interpretation of the instructions offered in this task to be unclear. The presenter demonstrates how to use this medium so that you may reposition and modify the triangle's angles to create different kinds of triangles. Students carry out this while paying close attention to the triangle's axis lines. Students can declare that the axis remains perpendicular to the triangle's midpoint during this assignment.

When a group that has finished the activity on the axis line is directly interacted with by the teacher. They offered the same response—"a perpendicular line"—when they glanced at their response and noticed the response on the high line. The teacher then asks the class to compare the triangle's axis line and height line. They maintain that the fact that the line is straight like a pole remains unchanged. One of the students responded that the height line passes through point C whereas the axis line is perpendicular to line AB in response to the question of what distinguishes the components.



Height Line



Median of a triangle

Figure 9. Student's answer about the height line and median of a triangle

Discussion

Discussion of learning activities in design research involves more than just assessing whether the activities were completed as intended. In this research design, students' thinking conjectures emerge and develop. In this study, it is also examined how, when participating in design experiment activities, students' purported thinking conjectures are contained in the hypothetical learning trajectory (HLT) and their anticipations appear in the actual learning trajectory (ALT). The learning activities during or after a session must be changed, and the hypotheses must be adjusted to the new circumstances if the observed learning/hypothetical learning trajectory (HLT) differs from the intended learning/actual learning trajectory (ALT) (Van Eerde, 2013). Based on the retrospective analysis that was performed, all thinking conjectures that turn into hypotheses occur in actual learning in the pilot and teaching experiment stages, students' thinking conjectures in HLT and ALT are in agreement. The application of augmented reality and Geogebra supports the appropriateness of students' thinking hypotheses between HLT and ALT. The following is a comparison table of HLT and ALT in activity 3 cycle 2.

Table 1. HLT and ALT Compatibility Activity 3 Cycle 2

Activity 3	HLT	ALT
Students draw the bisector according to the steps given.	Students draw bisectors on triangles	Students are able to draw bisectors well.
Students define the bisector in their own language. Students answer the question	Students answer the line that divides angle C equally.	Students answer the question about line bisects angle equally.
Students explore the bisector with various types of triangles.	Students answer that the bisector will still divide angle C equally.	Students answer well that the line still divides angle C equally.

The use of augmented reality from the start of the activity provides added value to learning activities. Students learn through technology that is new to them so that it attracts students' attention. A good emotional relationship between the student and the teacher is precisely what is created by the employment of augmented reality tools in geometry lectures (Rashevska et al, 2020). Nguyen et al. (2024), who documented a 40% reduction in teacher interventions in AR-based geometry tasks. Observing real and interactive objects in augmented reality makes it easier for students to identify the concept of special lines of triangles. The AR allows for situated learning, which contends that learning takes place best when it is embedded in a physical environment and takes place in a particular context (Rossano et al, 2020).

As can be observed from the activity described above, students utilize ICT to complete tasks that define the idea of special lines and examine the patterns they observe when doing experiments utilizing augmented reality. Since each person learns and retains information differently, using technology to enhance learning is aimed at them (Gargrish & Kaur, 2020). Students then have the chance to search for relationships between special lines and patterns of special line tendencies based on the type of triangle or based on changes in the origin of special lines of triangles by using ICT, particularly augmented reality and the Geogebra application. Students frequently improve their abilities in the use and application of mathematics when they utilize ICT, especially augmented reality and Geogebra, as a tool to assist them in finding information, resolving issues, or comprehending what is happening (Das, 2019; Sudihartinih & Purniati, 2019).

Although using augmented reality and Geogebra takes time, doing so might make learning more active for the student (Suratno & Walijanti, 2023). While

Suratno & Walijanti's (2023) findings are in line with the difficulty of time-consuming AR/GeoGebra activities, structured task allocation offers a fresh solution. This method is based on the Distributed Cognition Theory (Zhang et al., 2024), which views time management as a cognitive burden that both students and teachers must share. Prahmana (2024) 4IR framework emphasizes the need for explicit time scaffolding in digital tools; this study operationalizes this recommendation through timed activity targets. As a solution to the long duration of time needed for learning activities, it is recommended for time management or resource allocation in the classroom using AR and GeoGebra. Students can be given a target time for completing each activity at the beginning of learning. In addition, a clear division of tasks for group members that teachers can convey through instructions for using activity sheets is believed to help students manage their time to complete tasks. Clear role division in AR groups decreased task completion time by 35%, according to (Wardhani et al., 2023).

Implication of Research

According to the findings, incorporating AR and GeoGebra into geometry instruction offers three significant improvements: (1) it allows for real-time adjustments to students' emergent conjectures, bridging the gap between hypothetical and actual learning trajectories; (2) it validates situated learning theory by demonstrating how interactive 3D visualization improves conceptual understanding of abstract geometric properties; and (3) it offers a replicable model for technology-enhanced design research that places an emphasis on adaptive pedagogy rather than static lesson plans. In order to fully realize AR's potential, schools should invest in infrastructure and teacher training, as the study also shows that these benefits require intentional scaffolding, such as structured time management and equitable access to devices. These revelations go beyond geometry and provide a foundation for implementing immersive technologies across STEM disciplines while cautioning against one-size-fits-all adoption without contextual adjustments.

Limitation

Although this study shows how AR and GeoGebra can improve geometry learning, there are a few things to be aware of. First, the study's limited sample size and particular educational setting may have limited its applicability to larger or more varied student populations. Second, because the study was dependent on specific technological instruments and gadgets, it might not be as applicable in environments where access to these resources is limited. Furthermore, the short-term nature of the study raises concerns regarding the approach's scalability across various mathematical topics and the long-term retention of learning gains. Lastly, even though structured task management helps to some extent, the time-intensive nature of AR-based activities may present real-world difficulties for educators with limited curriculum time. These limitations suggest the need for further research to validate and expand upon the current findings.

CONCLUSION

Based on the research findings and discussions about the research process, a Local Instruction Theory (LIT) was developed using the context of the Musi 6 Palembang Bridge and augmented reality technology. The LIT is made up of five activities. The first activity, students use augmented reality technology to investigate the concept of triangle special lines in the context of the Musi 6 Palembang Bridge. The second activity, Students use the Geogebra application to draw a perpendicular line to the triangle as the triangle's height line and see changes in the height line based on the shape of the triangle. The third activity, Students define the bisector by creating a line with Geogebra that may split the angles of an equal triangle. After that, students are given a triangle with lines already drawn on it, and they adjust the shape of the triangle to better comprehend the bisectors of each type of triangle. The fourth activity, students use Geogebra to create a median of a triangle by identifying the midpoint of one side of a triangle and connecting the point to the corner facing that side, and then they experiment with different sorts of triangles. The fifth activity, students draw the axis of a triangle using Geogebra by identifying the midpoint of one of the triangle's sides and then drawing a line perpendicular to that side at the midpoint. Following that, students conclude the significance of axis lines in triangles and attempt to adjust the type of triangle that already includes axis lines so that students better comprehend the axis lines.

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