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Design of Proving Worksheet Based on APOS Theory on Trigonometric Functions of Sum and Difference of Two Angles

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Abstract

Trigonometry material is considered difficult by students because there are many formulas and concepts that must be memorized, so students have difficulty in analyzing and describing the problems given. One way for students to understand the formula is by learning proof. In proof learning related to APOS theory to construct students' ability to understand the formula. In this study, a learning media will be made in the form of Worksheet based on APOS theory. The subjects in this study were 11th grade students at SMA Negeri 2 South Kikim. The purpose of this research is to produce a valid, practical, and impactful worksheet theory-based proof worksheet for learning. This study is research and development of a product development type. The development process will be carried out using the ADDIE method (Analysis, Design, Development, Implementation, and Evaluation). The data collection techniques used by researchers are interviews, questionnaires, and test questions. The development starts from the initial design of the APOS-based proof worksheet, then validated by three validators. The results of the validation carried out to three experts received a score of 91.85%, so that the worksheet was categorized as very valid and feasible to use. Furthermore, small group trials were carried out, the results obtained a practicality score of 80%, thus the worksheet was categorized as practical. Furthermore, a field test was conducted, to determine the impact of worksheet in learning. The results show that the worksheet has an impact with an effective category with a score of 76.19%.

Keywords: *Proof Learning; APOS; Sum and Difference of Two Angles; Worksheet*

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Abstrak

Materi Trigonometri dianggap sulit oleh siswa karena banyak rumus dan konsep yang harus dihafalkan, sehingga siswa kesulitan dalam menganalisi dan menguraikan permasalahan yang diberikan. Salah satu cara agar siswa dapat memahami rumus yaitu dengan pembelajaran pembuktian. Dalam pembelajaran pembuktian berkaiatan dengan teori APOS untuk mengkonstruk kemampuan siswa dalam memahami rumus tersebut. Pada penelitian ini, akan dibuat suatu media pembelajaran berbentuk LKPD pembuktian berbasis teori APOS. Adapun subjek dalam penelitian ini adalah siswa kelas XI di SMA Negeri 2 Kikim Selatan. Tujuan dari penelitian ini adalah untuk menghasilkan LKPD pembuktian berbasis teori APOS yang valid, praktis, serta memiliki dampak terhadap pembelajaran. Jenis penelitian ini adalah penelitian pengembangan tipe pengembangan produk (development study). Proses pengembangan akan dilakukan dengan menggunakan metode ADDIE (Analysis, Design, Development, Implementation, and Evaluation). Teknik pengumpulan data yang digunakan peneliti yaitu wawancara, angket, dan soal tes. Pengembangan dimulai dari mendesain awal LKPD pembuktian berbasis APOS, kemudian divalidasi oleh tiga orang validator. Hasil dari validasi yang dilakukan kepada tiga orang ahli mendapatkan skor sebesar 91,85%, sehingga LKPD dikategorikan sangat valid dan layak digunakan. Selanjutnya dilakukan uji coba kelompok kecil hasilnya mendapatkan skor kepraktisan sebesar 80% Dengan demikian LKPD dikategorikan praktis. Selanjutnya dilakukan field tes, untuk mengetahui dampak LKPD dalam pembelajaran. Hasilnya menunjukan bahwa LKPD memiliki dampak dengan kategori efektif dengan skor 76,19%.

Kata Kunci: *Pembelajaran Pembuktian; APOS; LKPD; Jumlah dan Selisih Dua Sudut*

INTRODUCTION

Trigonometric function material is one of the materials that must be learned by students in Senior High School (SMA). The material of the sum and difference of two angles is mandatory material about trigonometric functions which has many formulas (Novita, Isnainah, & Joni, 2022). Armiati, Budi AS (2021) In the field, trigonometry is one of the meters that is difficult for students because the material is abstract. Trigonometry is a subject matter that often uses a formula, where the flow will always develop, and is not memorized material so that if you do not understand the formula there is concern about learning difficulties for the next (Novianti & Rianjanto, 2021). This is in line with research Nurmeidina & Djamilah (2020) that trigonometry is considered difficult by students because there are many formulas and concepts that must be memorized, so students have difficulty in analyzing and describing the problems given.

Based on the above problems, to help students understand the formula, a learning model is needed, one of which is learning with proof (Herizal, 2020). This is also in line with Saftari, Darmawijoyo & Hartono (2020) saying that one of the effective ways of learning in mathematics is evidence-based learning, so learning related to proof must be developed. Evidence acts as the soul in mathematics (Mañosa, 2021), therefore evidence can be one of the means in learning mathematics (Laamena, Christina, Nusantara, Irawan , & Muksar, 2018). In analyzing evidence, the ability to understand and relate evidence to one another through argumentation is required (Ahmadpour & Reid, 2019). Proof is a unique series of logical arguments that make a statement true (Hanna & Reid, 2019). Students will get a better understanding of a mathematical concept with mathematical proof (Herizal, 2020).

The use of evidence is a tool to improve mathematics learning which includes the influence on students' ability to solve problems, reasoning, and understanding of mathematical concepts (Herizal, 2020). Evidence-based learning is learning mathematics through proof to strengthen students' concepts (Shinariko, Hartono, & Darmawijoyo, 2022). In China, there is also development research on teaching materials about proof (Fan, Mailzar, Alafaleq, & Wang, 2018). It is also found in Zhang & Chuanxia (2019) for

similar research on textbook development. Furthermore Rocha (2019) gave advice to conduct research that teaches simple proofs that are widely ignored by math teachers. Students also often have difficulty understanding a mathematical material that is explained, due to learning that is only teacher-centered so that students tend to be passive when learning (Jufri, 2021). Therefore, evidence-based Worksheet based learning media is also needed in the learning process in the classroom. is one of the guides for independent learning for students and can play a role in improving concept understanding (Ummaeroh, Gusmania, & Hasibuan , 2019) saying LKPD

APOS theory (Action, Procecss, Object, Schema) is a theory that focuses on the mental attitude of students during learning in constructing mathematical concepts (Arnon, et al., 2014). The following research on APOS, namely (Saftari, Darmawijoyo, & Hartono, 2020) conducted research on the development of student activities using APOS theory to understand the concept of Riemann sum. Furthermore Syamsuri & Marenthi (2018) discusses the analysis of students' cognitive processes in proof activities. Research Syamsuri, Purwanto, Sbanji, & Irawati (2017) explains why students have not been able to construct the proof using APOS theory in the analysis stage. Students' mental structure in learning proof can be described using APOS theory (WIjayanti, Waluya, Kartono, & Isnarto, 2019). APOS theory is not only used to mentally construct students in learning, but it is also used to communicate learning ideas and is a tool for students to learn basic mathematical concepts (Dubinsky & McDonald, 2001). APOS theory can be used to analyze students' ability and understanding of proof (Chamberlain & Vidakovic, 2021). Furthermore, APOS theory can improve students' mathematical concept understanding ability (Putri, Mukhaiyar, & Ananda, 2022).

Some previous research has developed a model of how students understand about a proof (Ahmadpour & Reid, 2019). With the results of the study that is obtained a model of how students understand about proof and suggest the use of proof models for further research. Then research conducted by (Rocha, 2019) with experimental research methods, said that simple proofs are rarely discussed in learning, and suggested that applying simple proofs in learning is important. Furthermore, (Fan, Mailzar, Alafaleq, & Wang, 2018) in his research said that teaching materials in Indonesia contained little about proof, then suggested improving teaching materials about proof. (Saftari, Darmawijoyo, & Hartono, 2020) in his research that focuses on the development of APOS learning activities, which says that learning activities influence mathematical understanding skills. In his research also suggested further development on the use of activities that have been developed. Research conducted by (Shinariko, Hartono, & Darmawijoyo, 2022) which focuses on learning evidence that affects mathematical abilities with qualitative descriptive research methods. In his research it was found that students' mathematical representation skills were still low and suggested evidence-based learning for students to get used to recognizing proof correctly. Furthermore (Anggraeni, Somakim, & Hapizah, 2022) in his research that focuses on developing proof-based worksheet that produce worksheet.

Based on the background that has been described above, the researcher wants to conduct a study entitled "Design of Proving Worksheet Based on APOS

Theory on the material of the trigonometric function of the sum and difference of two angles".

METHOD

The method used in this research is development research (Research and Development) using the ADDIE method (Analysis, Design, Development, Implementation, Evaluation). This study aims to produce valid, practical, and effective APOS theory-based proof Worksheet in learning. The subjects in this study were grade XI students at Senior hight School Number 2 Kikim Selatan in the 2023/2024.

The steps of the ADDIE model according to Aldoobie (2015) are (1) Analysis at this stage is to analyze the needs, curriculum, and analysis of learning media; (2) Design at this stage designs the initial product to be developed in the form of an APOS theory-based proof Worksheet design called Storyboard, preparation of materials to be used in Worksheet and making research instruments; (3) Development at the development stage, researchers will make worksheet based on the Storyboard that has been made, then the worksheet that has been made is called Prototype1. Furthermore, researchers validated the initial product that had been made to experts (validators). At this stage, a new product that has been declared valid by the validator is called Prototype 2; (4) Implementation at the application stage, products that have been declared valid by the validator will then be tested on students in small groups. After testing the product, researchers will distribute questionnaires to determine the practicality value of the developed worksheet; (5) Evaluation (evaluation) the evaluation stage is used to determine whether the LKPD that has been developed is effective to use or not.

The data collection techniques in this research are walktrough, questionnaire and test questions. Walktrough was used for the product validation process. Questionnaires were used after the small group trial to determine the practicality of the product. The test questions were given after doing the learning by using worksheet, to determine the effectiveness of worksheet in learning. The questionnaire used is a closed questionnaire with a Likert scale. The analysis techniques carried out are: (1) validity analysis technique; practicality analysis technique; (3) effectiveness analysis technique. All three used quantitative descriptive analysis. The data obtained was calculated the average value which was then converted according to the Table 1.

Table 1. Criteria for the validity

Average Interval	Category
$84 < R \leq 100$	Very Valid
$68 < R \leq 84$	Valid
$52 < R \leq 68$	Fairly Valid
$36 < R \leq 52$	less Valid
$20 < R \leq 36$	Not Valid

(Source: Syaharuddin dan Mandailina, 2017)

The product validation stage ends if the average results of the quantitative assessment at this stage obtain at least the Quite Valid category. Furthermore, for the practicality category, the average results are converted according to the following table:

Table 2. Category of Practicality

Score	Category
$0 - 20$	Not Very Practical
$21 - 40$	Not Practical
$41 - 60$	Fairly Practical
$61 - 80$	Practical
$81 - 100$	Very Practical

Source: Riduwan (Purnamasari & Rochmawati, 2015)

The practicality test stage ends when the criteria for interpreting the average score of the questionnaire reaches ≥

41% (Puji, Gulo, & Ibrahim, 2014). To determine the effectiveness of the developed worksheet, the data from the percentage of student completeness is then converted according to the following table:

Table 3. Effectiveness Interval

Effectiveness Interval	Category
$P > 80\ \%$	Very Effective
$60\ \% < P \leq 80\ \%$	Effective
$40\ \% < P \leq 60\ \%$	Fairly Effective
$20\ \% < P \leq 40\ \%$	Less Effective
$P \leq 20\ \%$	Not Effective

Source: (Ariskasari & Pratiwi, 2019)

It is said to be effective if the percentage of student learning test results after using the Proof worksheet based on APOS theory reaches classical learning completeness of at least 60% according to the KKM criteria that apply at school, namely 70 (Rahmadi, 2015).

RESULTS AND DISCUSSION

Results

Analysis Phase

At this stage, researchers analyzed the existing problems. Based on the problems in the background that the material of the sum and difference of two angles is material that has many formulas and students cannot solve the problem if they forget the formula. Therefore, proof learning is needed to strengthen students in understanding the formula and remembering the formula by constructing the way students learn through APOS theory. Furthermore, APOS theory-based proof learning will be presented in worksheet because worksheet can make student-centered learning, it is hoped that with worksheet students will be more active in learning. Furthermore, researchers analyzed the curriculum used in learning. Senior Hight School Number 2 South Kikim uses the 2013 Curriculum. The results of the curriculum analysis obtained are (1) Basic Competency (KD) *(see Table 4)* and (2) Learning objectives. The laerning objectives are students can understand the concept of sum and difference of two angles; Students can understand the relationship between sine and cosine expressed in the sum and difference of two angles formula; Students can solve problems related to the formula and the sum of difference of two angles; and Students can determine the trigonometric equation that is identical to the known equation using the sum and difference of two angles.

Table 4. KD dan IPK

Basic Competency	**Competency Achievement Indicators**
3.2 distinguish the use of the sum and difference of two angles	• Understand the concepts of sin, cos, and tan. • Understand the relationship between sine and cosine functions expressed in sine and cosine sum and difference formulas.
4.2 solve problems related to the sum and difference of two angles formula	• Solve problems related to the sum and difference of two angles formula. • Determine the trigonometric equation that is identical to the known equation using the sum and difference of two angles formula.

Design Phase

At this stage, the researcher collects references that will be used to make worksheet. The researcher designs the worksheet storyboard which contains: (1) Title of worksheet; (2) basic competencies; (3) learning objectives; (3) time allocation for completion; (4) instructions for use; (5) work steps (according to APOS theory); (6) tasks to be done. Researchers also

designed how to use APOS theory (Action, Process, Object, Schema) on worksheet.

At this stage, researchers collect reference sources that will be used for material on worksheet. as for the reference sources, namely the basic trigonometry book by Ali Syahbana in 2017, class XI mathematics book Curriculum 2013, and Internet sources and YouTube. After all the designs are ready and the material has been collected, then proceed to the development stage.

Development Phase

- *Creation Of Worksheet*

After the preparation stage is complete, then making the worksheet in accordance with the storyboard that has been made. Making worksheet is made using Ms. Word and for the cover using the Canva application. The following are the results of the worksheet that have been made.

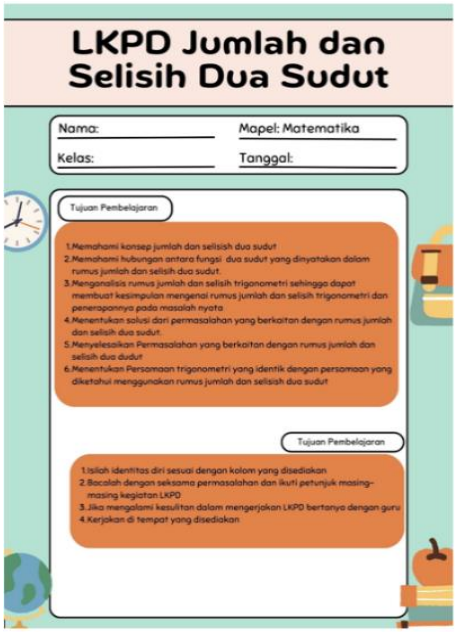


Figure 1. Home Page

On the front page there is the title of the worksheet, the identity of the student who will work on it, learning objectives, and instructions for use. For the front page, researchers use the Canva application to make it more attractive.

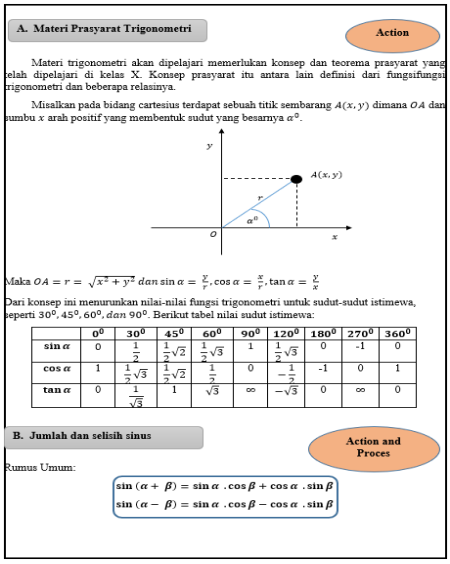


Figure 2. Step action and proces

Next is the student activity steps page. Student activities are made to prove the formula for the sum and difference of two angles. In proving student activities are designed in accordance with APOS theory which consists of Action, Process, object, and schema. On the page, there are action and proces steps. for object and schema steps can be seen as in the following image.

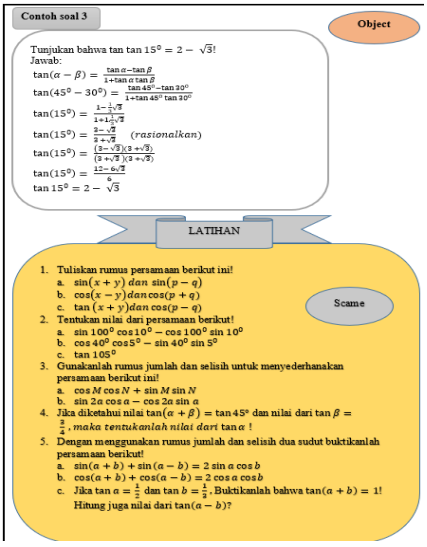


Figure 3. Step object and schema.

- *Validation Worksheet*

Worksheet that has been made, named Prototype 1. The next stage is to validate the product to validators consisting of 3 lecturers. The following are suggestions and comments from validators on the L worksheet made.

Table 5. Comments and Suggestions

Code of Validator	Comments and Suggestions
Validator 1	1. You can use angle relations to prove the value of special angles. 2. Add more varied problems. 3. Add proof steps to make it easier for students. 4. In the object step do not answer, let students answer themselves. 5. For student reasoning on actions, just make the table tidier.
Validator 2	1. There is typo words 2. On the cover, it should be instructions for use not learning objectives anymore. 3. The proof of cosine angle should also use triangles as well 4.
Validator 3	1. In the Schema section, the questions used are too many and there are questions that do not match what the students have done. 2. Correct the words there are still typos

After conducting product validation, revise the product according to the comments and suggestions of the validator. The following are the results of worksheet revisions based on comments and suggestions from validators *(See Table 6 at Appendix for before-after validation).*

After improving the worksheet, the researcher returned to the validator to conduct validation again. After that the researcher asked the validator to fill out a questionnaire to provide an assessment of the revised worksheet. The following are the results of the validator's assessment of the revised worksheet.

Table 7. Results of Validation

Aspects	Score	Category
Content	90,6	Very Valid
Construct	90,4	Very valid
Language	94,4	Very valid

Based on the Table 7, the results of the worksheet assessment given by the validator through a questionnaire with a Likert scale. The results show that the content aspect of the three validators scored 90.6. So that based on the table of content validity guidelines on worksheet is categorized as very valid. Furthermore, the construct aspect obtained a score of 90.4 so that it was categorized as very valid. The language aspect is also categorized as very valid with a score of 94.4. Overall, the APOS theory-based proof worksheet gets a score of 91.8, thus the APOS theory-based proof worksheet is categorized as very valid and feasible to be tested.

Implementation Phase

In the application stage, worksheet that have been valid and declared feasible to be tested will be tested on small groups of six students. Students who are subjects for this application stage trial are not schools for field tests. Six students who became small group testers were students of SMK Bina Cipta Palembang. The researcher gave the worksheet to students where the students worked on the worksheet in groups. Of course, the researcher accompanied them in working on the worksheet. After completing the worksheet, the researcher gave a questionnaire to the six students. The questionnaire given is a practicality questionnaire with a Likert scale. The questionnaire contains practicality indicators from the APOS theory-based proof worksheet *(See Table 8 for the result).*

Table 8. Small Group Score

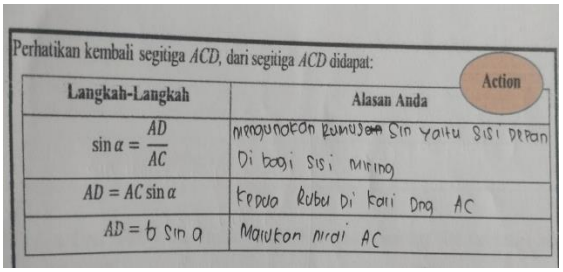
Responden	Score	Category
DM	72	Practical
NS	78	Practical
RY	76	Practical
RDR	82	Very Practical
PH	96	Very Practical
SV	76	Practical
Average	**80**	
Category	**Practical**	

Based on the results of the practicality questionnaire, out of six students, there were four students who gave a questionnaire assessment in the practical category. While the other two students gave a questionnaire assessment in the very practical category. This can be seen during the trial, two students, namely RDR and PH, were very active when working on the LKPD. besides that, the two students often asked the researcher if they did not understand. RDR and PH students also often teach their friends in working on the worksheet.

After the LKPD was declared Practical, the researchers then conducted further trials, namely the field test. The field test was conducted at Senior Hight School Number 2 South Kikim. In this trial, researchers tested the worksheet in classroom learning activities. The trial was conducted for 3 meetings from September 11-13, 2023. In the first and second meetings, researchers carried out learning activities in the classroom using worksheet media based on APOS theory. Learning activities also use a group division system. Students work on the APOS theory-based proof worksheet with a system of cooperation between friends in one group. At the third meeting, the researcher gave a learning outcome test question to find out the effectiveness of the APOS theory-based proof worksheet in classroom learning.

Evaluation Phase

At this stage, an evaluation is carried out after conducting a field test. After conducting the field test, the researcher at the third meeting gave a learning outcome test question. The test question aims to determine the level of effectiveness of the use of proof worksheet based on APOS theory on the material of the trigonometric function of the sum and difference of two angles. After making corrections from the results of the learning outcomes test questions, it can be seen that out of 21 students there were five students who did not complete (completeness criteria 70). If it is percented, only 76.19% of students are complete in working on the learning outcomes test questions with the APOS theory-based proof worksheet. Thus, it can be concluded that the APOS theory-based proof worksheet is effective in learning with a percentage of 76.19%. The following is one of the answers of student groups in working on proving worksheet based on APOS theory.


Perhatikan kembali segitiga *ACD*, dari segitiga *ACD* didapat:

Action

Langkah-Langkah	Alasan Anda
$\sin\alpha = \frac{AD}{AC}$	mengunakan rumus Sin yaitu sisi depan di bagi sisi miring
$AD = AC \sin\alpha$	kedua ruas di kali dng AC
$AD = b \sin\alpha$	Maukan nilai AC

Figure 4. Student answer Action Step

In the picture above, students pay attention to the ACD triangle on the worksheet, then students are directed to find the value of the AD side. The AD side is one of the requirements to get the formula for the sum of the sine angles. There students can also be seen giving their arguments about this step. This means that students understand every step that is done. The step in the picture above is an

action activity where students collect initial information that will be used for the next step, namely proces. the following is a picture of student answers at the proces step.

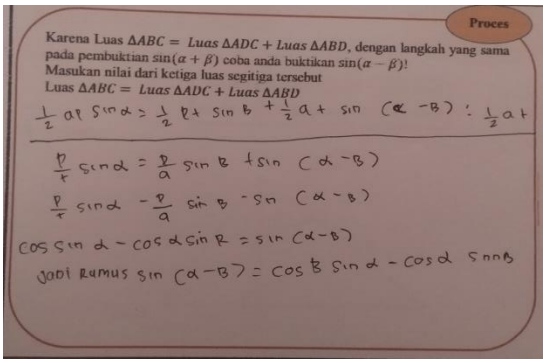


Figure 5. Student answer Proces Step

Based on the picture above, at the process stage students with the help of commands from the worksheet can find the formula for the sum of the sine angles. It can be seen from the student's answer, students have entered what is obtained from the action activity which is then mathematical manipulation so that the formula for the sum and difference of two angles is obtained. So, student activities in this APOS theory-based proof worksheet up to the material to find the sum and difference of tangent angles.

Discussion

Based on the results of three experts as validators, the APOS theory-based proof worksheet is declared very valid. This is in accordance with the objectives of the researcher, namely, to produce a valid APOS theory-based proof worksheet. Although in the validation activities there were revisions made by the researcher in accordance with the suggestions and comments from the validator.

Based on the small group trial conducted on six students at Vocational High School Bina Cipta Palembang, the APOS theory-based proof worksheet was declared Practical with an average score of 80. Of the six students, there were two students who gave an assessment in the very practical category, while the other four students gave an assessment in the practical category. This is in line with the researcher's goal of producing a practical APOS theory-based proof worksheet.

Based on the results of the field test conducted at SMA N 2 South Kikim, it is obtained that the proof LKPD based on APOS theory is effective if used in learning based on the results of test questions given after learning activities using the proof LKPD. This is also in line with the researcher's goal of producing an effective APOS theory-based proof worksheet in learning. So, it can be concluded that the proof worksheet based on APOS theory can be used in classroom learning.

This can be seen during the trial in class, students were initially confused about the APOS theory-based proof worksheets that were distributed. but after being given an explanation and reading the filling instructions, students began to work on the worksheet. students work in groups, in the process students are getting used to exploring proof through working on the worksheets provided. with student learning outcomes after learning is also good with a percentage level of completeness of 76.19% so that APOS theory-based proof worksheets are categorized as effective and can be used in learning.

This research also agrees with research conducted by (Saftari, Darmawijoyo, & Hartono, 2020) who developed student activity sheets based on APOS theory which were declared valid, practical and had potential effects in learning. Furthermore (Anggraeni, Somakim, & Hapizah, 2022) developed a proof-based worksheet on logarithm material with the results of LKPD research declared valid,

practical and has a potential effect on student perceptions. (Lestari, 2018) also conducted research on the development of worksheet based on APOS theory on flat-sided space building material, with the results of worksheet research declared valid, practical and has a potential effect so that worksheet can be used in learning.

Implication of Research

The impact of this research is related to learning in the classroom. The teacher's habit of teaching is only teacher-centered now with worksheet can be student-centered. In addition, the habit of memorizing formulas, especially in the material of the sum and difference of two angles, will be abandoned because with this worksheet students are required to learn to prove where the formula comes from and understand it. With the help of APOS theory steps that focus on the mental attitude of students during learning in constructing mathematical concepts.

Limitation

This study discusses the material of trigonometric functions with the subchapter of the sum and difference of two angles. The media produced in this study are printed learning media in the form of proof worksheet based on APOS theory.

CONCLUSION

Based on the results of the research that has been done, it can be concluded that the APOS theory-based proof worksheet that has been validated by three validators gets a score of 91.85%, so that the APOS theory-based proof worksheet is categorized as very valid and feasible to be tested. Furthermore, a small group trial consisting of six students resulted in a practicality score of 80%. Thus, the proof worksheet based on APOS theory is categorized as practical. Furthermore, a field test was conducted, to determine the impact of worksheet in learning. The results show that the APOS theory-based proof worksheet has an impact with an effective category with a score of 76.19%. This means that the APOS theory-based proof worksheet can be used in classroom learning.

REFERENCE

Ahmadpour, F., & Reid, D. (2019). Students' ways of understanding a proof. *Mathematical Thinking and Learning, 21*(1), 85-104.

Aldoobie, N. (2015). ADDIE model. *American international journal of contemporary research, 5*(6), 68-72.

Anggraeni, M., Somakim, & Hapizah. (2022). Pengembangan LKPD Berbasis Pembuktian Pada Materi Logaritma di Kelas X SMA. *Lentera Sriwijaya: Jurnal Pendidikan Matematika, 4*(1), 42-48.

Ariskasari, D., & Pratiwi, D. D. (2019). Pengembangan Modul Matematika Berbasis Problem Solving Pada Materi Vektor. *Desimal: Jurnal Matematika, 2*(3), 249-258.

Armiati, A., & Budi, A. S. (2021). Identifikasi Efektifitas Pembelajaran Trigonometri Kelas X Masa Pandemi COVID 19 Melalui Whatsapp Group. *Jurnal Gantang, 6*(1), 11-17.

Arnon, I., Cottril, J., Dubensky, E., Oktac , A., Feuntes, S. R., & Trigueros. (2014). *APOS Theory A Framework for Research and Curriculum Development in Mathematics Education.* Springer New York Heidelberg Dordrecht London.

Chamberlain, D., & Vidakovic, D. (2021). Cognitive Trajectory of Proof by Contradiction for Transition-to-proof Students. *Journal of Mathematical Behavior, 62*, 1-15.

Dubinsky, E., & McDonald, M. A. (2001). *APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research*. Dordrecht: Springer Netherlands., 275-282.

Fan, L., Mailzar, Alafaleq, M., & Wang, Y. (2018). A Comparative Study on the Presentation of Geometric Proof in Secondary Mathematics Textbooks in China, Indonesia, and Saudi Arabia. *Research on Mathematics Textbooks and Teachers' Resources,* 53-65.

Hanna, G., & Reid, D. (2019). Mathematics Education in the Digital Era Proof Technology in *Mathematics Research and Teaching. Mathematics Education in the Digital Era (MEDE, volume 14)*. Springer.

Herizal, H. (2020). Faktor yang Mempengaruhi Kemampuan Pembuktian Matematis Siswa. *Vygotsky: Jurnal Pendidikan Matematika dan Matematika, 2*(1), 33-42.

Jufri, M. (2021). Penerapan Model Problem Based Learning Untuk Meningkatkan Pemahaman Siswa Pada Materi Trigonometri. *Sigma, 2*(1), 49-62.

Laamena, Christina, M., Nusantara, T., Irawan, E. B., & Muksar, M. (2018). How Do the Undergraduate Students Use an Example in Mathematical Proof Construction: A Study Based on Argumentation and Proving Activity. *International Electronic Journal of Mathematics Education, 13*(3), 185-198.

Lestari, D. (2018). Pengembangan LKS Berbasis Teori APOS Pada Materi Bangun Ruang Sisi Datar di SMP dengan Konteks Rumah Adat Musi Banyuasin. *Kreano: Jurnal MatematikaKreatif-Inovatif, 9*(1), 1-9.

Mañosa, V. (2021). The Invisible Heartbeat: The Beauty and Soul of Mathematics. *Psychological Perspectiv,* 118-134.

Novianti, V., & Rianjanto, M. L. (2021). Analisis Kesulitan Siswa SMK Dalam Menyelesaikan Soal Materi Trigonometri. *JPMI: Jurnal Pembelajarn Matematika Inovatif, 4*(1), 161-168.

Novita, N., Isnainah, I., & Joni, D. (2022). Analisis kesulitan siswa kelas XI MIPA pada materi Trigonometri di SMA N 1 Kecamatan Gunung Omeh. *Koloni: Jurnal Multidisiplin Ilmu, 1*(4), 381-392.

Nurmeidina, R., & Djamilah, S. (2020). Pelatihan Tips dan Trik Trigonometri Mudah Untuk Siswa SMA. *Jurnal Pendidikan dan Pengabdian Masyarakat, 2*(3), 362-365.

Puji, K. M., Gulo, F., & Ibrahim, A. R. (2014). Pengembangan Multimedia Interaktif untuk Pembelajaran Bentuk Molekul di SMA. *Jurnal Penelitian Pendidikan Kimia: Kajian Hasil Penelitian Pendidikan Kimia*, *1*(1), 59-65.

Purnamasari, A., & Rochmawati. (2015). Pengembangan Alat Evaluasi Pembelajaran Berbasis Teknologi Informasi Komunikasi dengan Wondershare Quiz Creator Materi Sistem Penilaian Persediaan. *Jurnal Pendidikan Akuntansi (JPAK)*, *3*(2), 1-9.

Putri, R., Mukhaiyar, & Ananda, A. (2022). Penerapan Model Pembelajaran M-APOS untuk Meningkatkan Kemampuan Pemahaman Konsep Matematis. *Jurnal Ilmiah Pendidikan Matematika Al Qalasadi, 2*(6), 167-177.

Rahmadi, F. (2015). Pengembangan Perangkat Pembelajaran Berbasis Pemecahan Masalah Berorientasi Pada Kemampuan Penalaran daan Komunikasi Matematika. *Pythagoras*: *Jurnal Pendidikan Matematika*, *10*(2), 137-145.

Rocha, H. (2019). Mathematical proof: from mathematics to school mathematics. *Philosophical Transactions of the Royal Society A*, *377*(2140), 20180045.

Saftari, M., Darmawijoyo, & Hartono, Y. (2020). Development Of Student Activities Sheet Based on Apos Theory to Understand The Concept Of Riemann Sum. *Math Didactic: Jurnal Pendidikan Matematika, 6*(1), 110-123.

Shinariko, L. J., Hartono, Y., & Darmawijoyo. (2022). Developing Proof-Based Learning Using Apos Theory Approach In Exponential For Enhancing Students'reasoning Ability. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika, 11*(4), 2903-2914.

Syamsuri, S., & Marethi, I. (2018). APOS analysis on cognitive process in mathematical proving activities. *International Journal on Teaching and Learning Mathematics., 1*(1), 1-12.

Syamsuri, S., Purwanto, P., Subanji, S., & Irawati, S. (2017). Using APOS theory framework: Why did students unable to construct a formal proof. *International Journal on Emerging Mathematics Education, 1*(2), 135-146.

Ummaeroh, R., Gusmania, Y., & Hasibuan, N. H. (2019). Pengaruh Penggunaan Lks Berbasis Inkuiri Terhadap Pemahaman Konsep Matematis Siswa Kelas XI IPA SMA. *PYTHAGORAS: Jurnal Program Studi Pendidikan Matematika., 8*(2), 93-98.

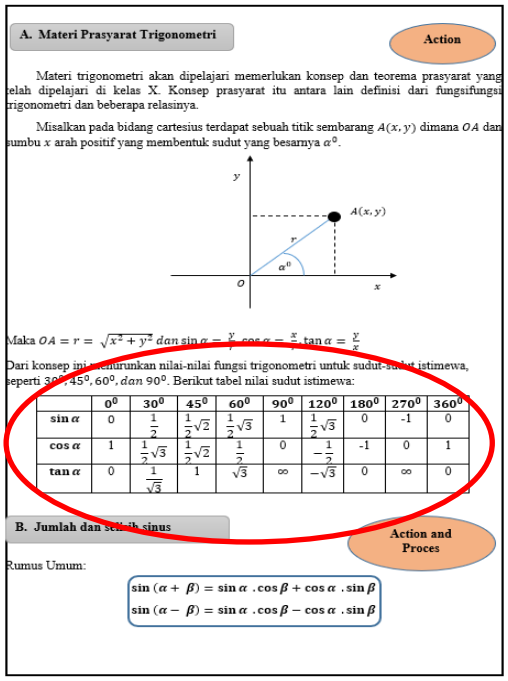
Wljayanti, K., Waluya, S. B., Kartono, & Isnarto. (2019). Mental structure construction of field independent students based on initial proof ability in APOS-based learning. *Journal of Physics: Conference Series, 1321*(3).

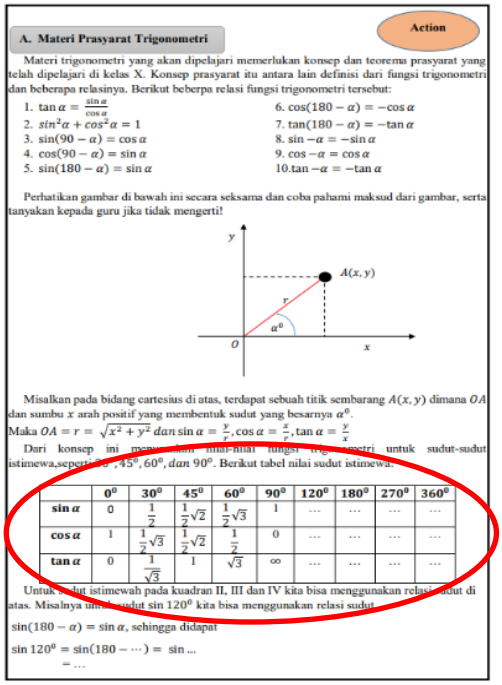
Zhang, D., & Chuanxia, Q. (2019). Reasoning and proof in eighth-grade mathematics textbooks in China. *International Journal of Educational Research,* 98, 77-90.

Appendix of article entitled Design of Proving Worksheet Based on APOS Theory on Trigonometric Functions of Sum and Difference of Two Angles

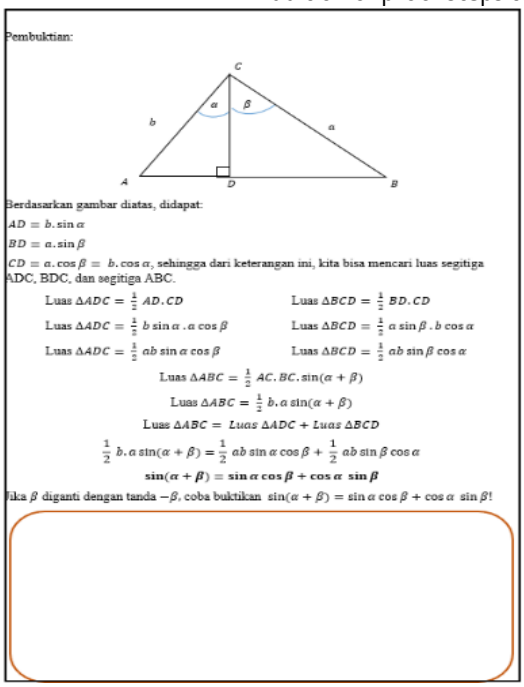
Table 6. Improvement Table

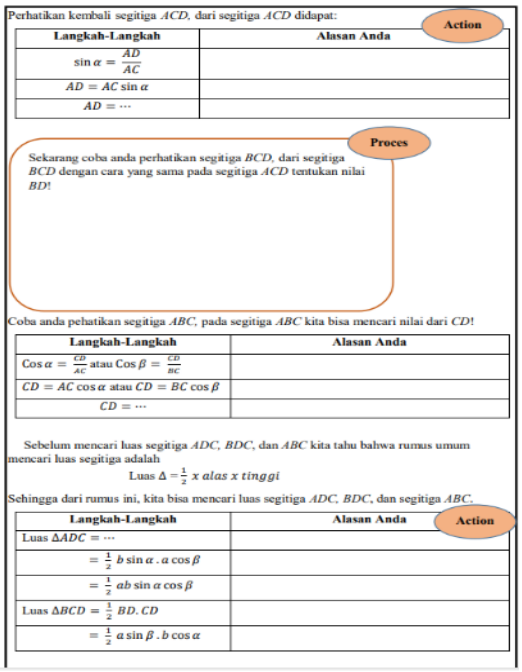
Before Revision	After Revision
Improvement of special angles should be done by the students themselves using angle relations.	





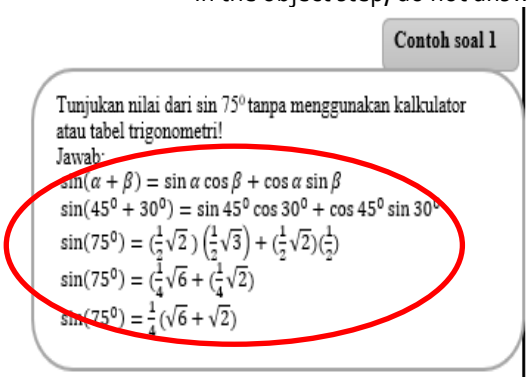
Before Revision	After Revision
Addition of proof steps to make it easier for students.	

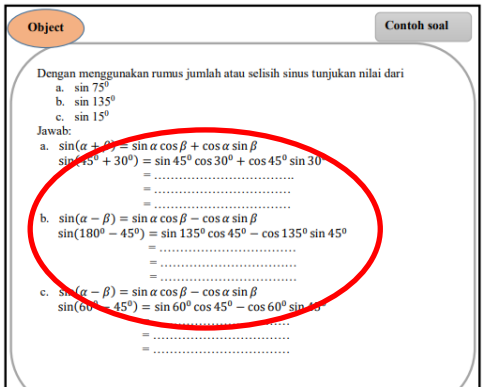




Before Revision	After Revision

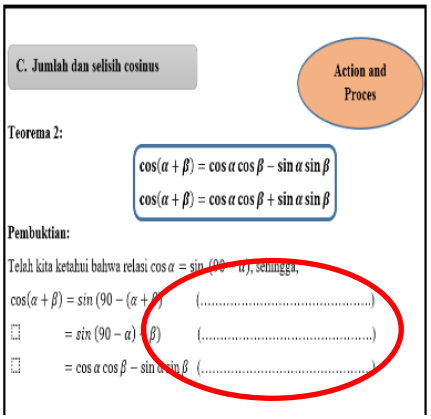
In the object step, do not answer, let students answer themselves.

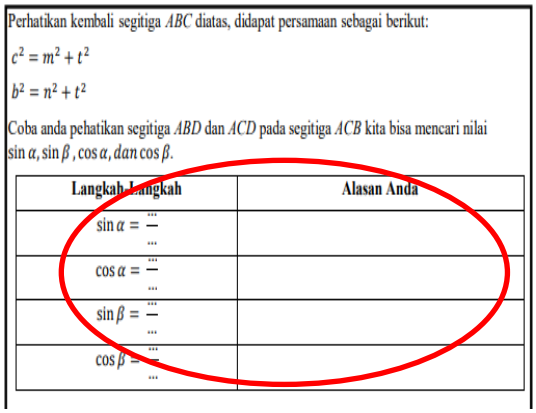




Before Revision	After Revision

For student reasoning on actions, just make the table tidier





Before Revision	After Revision

In the Schema section, the questions used are too many and there are questions that do not match what the students have done.