



Visualization of Multivariable Calculus: Learning Needs Analysis for GeoGebra-Assisted Teaching Material Development

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Abstract

Students often struggle to visualize multivariable functions, interpret partial derivatives, and apply extrema, yet systematic needs analyses for designing meaningful Multivariable Calculus learning environments remain limited. This study aims to identify students' learning needs as a foundation for developing GeoGebra-assisted materials. Using a qualitative descriptive approach within the Gravemeijer and Cobb design research framework, data were collected through a needs-analysis questionnaire administered to eight purposively selected university students who had completed the course. Results show that six of the eight students experienced moderate to high levels of difficulty, especially in visualizing multivariable functions and understanding partial derivatives. Students also reported that videos and lecturer explanations were helpful, and all expressed readiness to use GeoGebra-based materials. The findings indicate the importance of designing structured learning environments that integrate dynamic visualizations. This study provides the groundwork for subsequent teaching experiment and retrospective analysis cycles in developing valid and generalizable instructional designs.

Keywords: Multivariable Calculus; Geogebra; Learning Needs; Learning Design; Design Research.

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Abstrak

Mahasiswa sering mengalami kesulitan dalam memvisualisasikan fungsi multivariabel, menafsirkan turunan parsial, dan menerapkan konsep nilai ekstrem, namun analisis kebutuhan yang sistematis untuk merancang lingkungan belajar Kalkulus Peubah Banyak masih terbatas. Penelitian ini bertujuan mengidentifikasi kebutuhan belajar mahasiswa sebagai dasar pengembangan bahan ajar berbantuan GeoGebra. Dengan pendekatan deskriptif kualitatif dalam kerangka penelitian desain Gravemeijer dan Cobb, data dikumpulkan melalui angket analisis kebutuhan yang diberikan kepada delapan mahasiswa terpilih yang telah menyelesaikan mata kuliah tersebut. Hasil menunjukkan bahwa enam dari delapan mahasiswa mengalami tingkat kesulitan sedang hingga tinggi, terutama dalam visualisasi fungsi multivariabel dan pemahaman turunan parsial. Mahasiswa juga menyatakan bahwa video dan penjelasan dosen membantu pemahaman, dan seluruhnya siap menggunakan bahan ajar berbasis GeoGebra. Temuan ini menjadi dasar bagi pelaksanaan teaching experiment dan analisis retrospektif untuk menghasilkan desain pembelajaran yang valid dan tergeneralisasi.

INTRODUCTION

In ideal conditions, learning Multivariable Calculus should foster not only procedural fluency but also deep conceptual understanding supported by coherent symbolic, graphical, numerical, and verbal representations (Al Dehaybes et al., 2025; Altindis et al., 2024; Andiani, 2016; Andiani et al., 2020, 2025; Bedada & Machaba, 2022; Hahn & Klein, 2025; Milenković & Vučićević, 2024). Students are expected to connect key multivariable concepts such as functions of two variables, gradients, partial derivatives, and multiple integrals through strong representational and spatial reasoning, particularly supported by 3D visualization tools (Cheong et al., 2023, 2024; Herrera et al., 2024; Jones et al., 2022). Ideal learning also emphasizes active engagement, where students rediscover concepts through exploration and modeling while lecturers act as facilitators who guide reflection rather than transmit procedures (Andiani et al., 2025; Biehler et al., 2024; Johnson et al., 2025; Ribeiro-Silva et al., 2022; Song & Cai, 2024; Weinhandl et al., 2025). Assessment, accordingly, should measure conceptual, representational, and application skills rather than calculations alone (Andiani et al., 2020, 2024; Vizek et al., 2024).

However, gaps persist between these ideals and classroom realities. Research reports that many students

struggle to visualize 3D surfaces and interpret symbolic changes geometrically, leading to weak conceptual understanding (Borji et al., 2022; Herrera et al., 2024). Although virtual reality, GeoGebra 3D, and other dynamic visualization tools have proven effective, they remain underused in practice, leaving instruction dominated by symbolic procedures (Cheong et al., 2023; Herrera et al., 2024; Schoenherr et al., 2024). Furthermore, visual tools are often implemented without adequate scaffolding, limiting their pedagogical impact (Abraham & Prediger, 2025; Huang et al., 2024; Malone et al., 2023; Nickl et al., 2024a, 2024b; Wang et al., 2025). Assessments in many calculus courses also still emphasize procedural skills, widening the gap between intended and actual learning outcomes (Ballon et al., 2024; Castillo et al., 2025; Spencer-Tyree et al., 2024). Consistent with previous studies, preliminary questionnaire results in this study confirm that students experience difficulties in visualizing multivariable concepts and express readiness to engage with GeoGebra-based materials (Abuhassna & Alnawajha, 2023; Andiani et al., 2025; Leung et al., 2024).

To reduce these disparities, the integration of technological visualization particularly GeoGebra with well designed instructional strategies becomes essential. GeoGebra enables dynamic

exploration of multivariable functions and has been shown to improve conceptual understanding and representation skills (Bedada & Machaba, 2022; Hevardani et al., 2025). Combining this technology with active learning approaches further enhances students' spatial reasoning and engagement (Cheong et al., 2023; Herrera et al., 2024). Additionally, the design research approach by Gravemeijer and Cobb (2006) offers a systematic framework for developing instructional materials through iterative cycles of design, teaching experiments, and retrospective analysis, ensuring that innovations align with students' real learning needs (Komatsu et al., 2024; Mackey et al., 2023; Prediger et al., 2015). Although prior studies have examined visualization tools or design research separately, research that integrates both in the context of Multivariable Calculus especially in Indonesian higher education remains limited.

Based on the ideal conditions, empirical gaps, and students' expressed needs, the main problems identified in Multivariable Calculus learning include limited visualization skills, minimal experience with interactive technologies such as GeoGebra, and the dominance of procedural instruction that undermines conceptual understanding. Therefore, this study aims to develop a theoretically grounded and technology-assisted instructional design that addresses these issues. Specifically, the objectives are: (1) to identify students' difficulties and learning needs related to representation and visualization; (2) to analyze students' perceptions and readiness toward GeoGebra-assisted materials; and (3) to provide an empirical and conceptual foundation for designing a GeoGebra-based Multivariable Calculus learning model using the Gravemeijer and Cobb approach.

This study contributes empirically by documenting students' learning needs; practically by offering a conceptual basis for technology-supported instructional design relevant to Indonesian higher education; and theoretically by integrating visualization tools with the Gravemeijer and Cobb design research framework, thereby advancing the literature and laying groundwork for future validation studies. Taken together, these contributions highlight the urgency of developing an instructional design that is both empirically grounded and theoretically informed. The persistent gap between ideal conceptual understanding and current learning practices in Multivariable Calculus, combined with students' demonstrated need for visualization-based support, underscores the necessity of a systematic and integrated approach. By merging the strengths of technological visualization and design research, this study aims to provide a sustainable foundation for improving learning outcomes and informing future instructional development.

METHOD

This study employed a qualitative descriptive approach to identify students' learning needs as the basis for developing GeoGebra-assisted teaching materials for Multivariable Calculus. Eight fourth-semester mathematics education students from a university in West Java were selected through purposive sampling, as their prior completion of the Multivariable Calculus course provided relevant and context-rich insights for a needs analysis study. Data were collected using an online questionnaire developed in Google Forms, consisting of sections on participant identity, learning experiences and needs, availability of GeoGebra-

based resources, and expectations for teaching material design. The instrument included closed ended rating scale items and open ended prompts and was validated by two experts one in mathematics and one in information technology who reviewed the items for clarity, relevance, and representativeness; minor revisions were made accordingly, and the final version was pilot tested with two students outside the main sample to ensure clarity and technical usability.

Data collection was conducted asynchronously, allowing students to respond reflectively and ensuring complete and manageable digital records. The analysis used qualitative descriptive techniques in which numerical ratings were converted into percentages to illustrate general trends, while open ended responses were coded thematically to identify recurring issues such as difficulties in visualizing multivariable functions, limited understanding of partial derivatives and extrema, and the need for interactive visual tools to support conceptual comprehension. Trustworthiness was strengthened through iterative comparison of codes, repeated category checking, and alignment with prior literature on technology-supported calculus learning. The researcher acted as the primary instrument of the study, designing the questionnaire, coordinating expert validation, managing data collection, and conducting the thematic coding and interpretation, while taking measures to minimize bias through iterative reflection and expert consultation.

The methodological procedure was also positioned within the Gravemeijer and Cobb design research framework by aligning the needs analysis with the three interconnected stages of preliminary design, teaching experiment, and

retrospective analysis. In this first year cycle, preliminary design involved identifying learning difficulties from existing studies and constructing the questionnaire to investigate initial conjectures; the teaching experiment was adapted into a systematic needs assessment activity in which students' responses were used to refine those conjectures; and retrospective analysis consisted of examining and interpreting the questionnaire data to generate thematic findings and form early design principles for developing GeoGebra-assisted instructional materials. Altogether, this methodological design provides a systematic and empirically grounded foundation for understanding students' learning needs and informing the initial stage of a broader instructional design developed through the design research paradigm.

In this qualitative descriptive study, the researcher served as the primary instrument responsible for designing the questionnaire, validating its content with experts, administering the data collection process, and conducting the thematic analysis. The researcher interpreted students' responses through iterative coding, continuously comparing emerging categories with existing literature to minimize subjective bias. To enhance trustworthiness, the researcher engaged in repeated checks of the coding scheme, consulted experts during validation, and ensured that interpretations remained aligned with the empirical focus of the needs-analysis phase within the design research framework.

RESULT AND DISCUSSION

Results

The analysis of the questionnaire responses provides a detailed overview of

students' learning challenges in Multivariable Calculus and clarifies the specific conceptual and procedural aspects that require strengthened instructional support. A majority of students (75%) categorized the course as moderately to highly difficult across core topics involving limits of two variables, visualization of surfaces, partial derivatives, and extreme values. The area reported as most challenging was partial derivatives, with 50% of students stating that they experienced persistent difficulties not only in carrying out the symbolic computations but also in interpreting what the derivatives represent in a multivariable context. Visualization of multivariable functions emerged as another major source of difficulty for 37.5% of respondents, who indicated that they struggled to interpret three-dimensional surfaces, understand the meaning of contour curves, and recognize how directional changes relate to gradients. Students' written comments further illuminate the depth of these difficulties. For example, one student noted, *"I can compute the partial derivatives, but I still cannot picture the shape of the surface,"* while another remarked, *"Contour lines are confusing because I cannot see how they represent the height or slope of the graph."* These statements describe the specific difficulties students encountered when working with geometric representations.

Additionally, 25% of students reported difficulties with limits and continuity of functions of two variables, often citing uncertainty about how a function behaves when approaching a point from infinitely many directions. Meanwhile, 37.5% struggled with extreme values, particularly in understanding the relationship between critical points, gradients, and the geometric structure of the surface. The proportions of these reported difficulties indicate that partial derivatives had the highest percentage of students experiencing challenges compared with the other topics. Table 1 presents a detailed overview of the proportion of students who reported experiencing difficulties across four key conceptual areas in Multivariable Calculus. Partial derivatives emerged as the most challenging topic, with 50% of students indicating that they struggle to understand both the geometric interpretations and the practical applications of these concepts. Visualization of multivariable functions was also a notable difficulty, reported by 37.5% of respondents, who specifically mentioned problems in accurately representing three dimensional surfaces, interpreting contour curves, and understanding gradients. Difficulties with limits and continuity in two variables were reported by 25% of students. In addition, 37.5% of students reported difficulties in

Table 1. Percentage of Student Difficulties Based on Learning Aspects

Learning Aspects	Percentage of Students Experiencing Difficulties	Information
Partial derivatives	50%	The main obstacle is the difficulty in understanding geometric and applicative concepts.
Visualization of multivariable function	37.5%	Weak in drawing 3D surfaces, contour curves, and gradients
Limit & continuity	25%	Difficulty generalizing the concept of limit from 1 variable to many variables
Extreme values	37.5%	Understand the procedure, but have difficulty applying the concept

applying extreme value concepts.

In terms of learning resources used by students in studying Multivariable Calculus, 87.5% of students identified lecturer explanations as their primary source of support. Students reported that explanations delivered during lectures were the resource they relied on most frequently when attempting to understand course concepts and solve related problems. Additionally, 75% of students indicated that learning videos were helpful in supporting their study of Multivariable Calculus. Students noted that they used these videos to review examples, revisit explanations, and follow procedural steps demonstrated in the materials. A similar proportion, 75%, reported that visual tools such as GeoGebra were beneficial for their learning. These students mentioned that the tools assisted them when working with graphical representations, examining surfaces, or checking the structure of multivariable functions. However, several students stated that they used these tools only occasionally and primarily when such tools were introduced or demonstrated during classroom sessions.

Based on the questionnaire responses, Table 2 summarizes the percentage of students who identified each type

of other supplementary resources. The distribution shows the frequency with which each medium was selected by students as supportive for their understanding of the material. The table also indicates whether students used these resources independently, during lessons, or both. The results further show the range of resources students accessed: some relied solely on lecturer explanations, some combined videos with visualization tools, while others used multiple resources including textbooks and online materials. The data also record variations in usage patterns, such as students who reported regular use of visual tools compared to those who used them only when assigned. Additionally, a portion of students indicated that they supplemented primary resources with external materials such as YouTube tutorials or online mathematics forums. These patterns are further detailed in Table 2, which presents a summary of the learning media used by students and the corresponding proportions of students who reported each medium as helpful.

Students report limited prior experience with GeoGebra. The data show that 62.5% of respondents have never used GeoGebra in any mathematics course, while 37.5% have used the software only

Table 2. Percentage of Students Based on Learning Media

Instructional Media	Percentage of Students	Information
Lecturer's explanation	87.5%	Still, the primary source of learning
Learning videos	75%	Supports repetition and independent understanding
GeoGebra	75%	Interactive media, but the student experience is limited
Previous GeoGebra experience	62.5% never, 37.5% limited	Demonstrates the gap between technological potential and real-world experience.
Readiness for using GeoGebra-based teaching materials	100%	Strong signal of acceptance of technological innovation

of learning medium as helpful. The table includes data on lecturer explanations, learning videos, visual tools such as GeoGebra, textbooks, online tutorials, and

to a limited extent. Several participants provided comments regarding their experience. One student states, *"I know GeoGebra exists, but I have never used it in*

class." Another student reports, *"I tried using GeoGebra before, but I did not understand how to manipulate the 3D view."* In terms of perceived usefulness, 75% of respondents indicate that GeoGebra could help them learn material related to Multivariable Calculus. Furthermore, all respondents (100%) report that they are willing to use GeoGebra-based teaching materials in future learning activities. The responses were recorded consistently across all participants, and prior studies by Schoenherr et al. (2024) are cited to provide contextual reference for patterns of technology acceptance (Schoenherr et al., 2024).

Students' expectations for learning materials varied, but most emphasized the need for interactive visualizations. Respondents suggested 3D graphics, GeoGebra-based mini-projects, and exploration activities that allow for active learning. Some respondents requested more dynamic representations, noting, for instance, *"It would help if I could rotate the surface and see the contour lines at the same time."* In addition to visualization, students desire a connection between learning materials and real world contexts. Several respondents stated that they preferred GeoGebra visualizations to be connected to everyday life applications or case studies.

Based on the findings from the questionnaire, Table 3 summarizes students' expressed needs and expectations regarding Multivariable Calculus teaching materials. The table lists specific features

and components identified by students as relevant to their learning. These features include the use of interactive visualizations to represent three-dimensional concepts, the inclusion of diverse types of learning media, the presence of problems and examples connected to practical or real world contexts, and the availability of exploratory and inquiry-based activities that require students' active participation. The responses recorded in Table 3 indicate the number and proportion of students who mentioned each type of resource or activity. For instance, the table shows how many students requested interactive visualizations, the frequency with which students mentioned the need for multiple learning media, and the number of students highlighting the importance of problem examples from real-world contexts. The table also presents the proportion of students who indicated a preference for exploratory or inquiry-based activities as part of their learning process.

Students reported expectations regarding the availability of diverse learning media for studying Multivariable Calculus. The responses show that students considered multiple types of media as relevant to their learning. Specifically, students mentioned traditional textbooks, instructional videos, and technology based resources such as interactive simulations or software tools. The questionnaire recorded how many students referred to each type of media, including cases where

Table 3. Questionnaire Findings

Aspect	Questionnaire Findings	Implications for Teaching Materials
Visualization	37.5% difficulty with 3D visualization	Module with interactive 3D graphics
Interactive media	75% found GeoGebra useful	GeoGebra-based mini project
Learning activities	Students want active classes	Exploration activities, guided reinvention
Real context	Students want everyday life applications	Case studies related to KPB
Lecturer support	87.5% rely on lecturers	GeoGebra structured instructions & guides

students preferred a combination of several resources. Several participants provided comments about lecturer explanations. Students reported expectations regarding the availability of diverse learning media for studying Multivariable Calculus. The responses show that students mentioned multiple types of media, including traditional textbooks, instructional videos, and technology-based resources such as interactive simulations or software tools. The questionnaire recorded how many students referred to each type of media, including cases where students preferred a combination of several resources.

Several students provided comments about lecturer explanations. Some students noted that detailed and inclusive explanations were important, particularly in relation to differences in prior educational experiences. Students also reported variations in their foundational understanding of Multivariable Calculus, referring to varied types of prior secondary education. In addition, students identified a need for more active learning activities. The responses included references to hands-on classroom exercises, exploratory problem-solving activities, and collaborative tasks. The questionnaire data recorded the proportion of students mentioning each type of activity and whether they expressed a preference for multiple types of active learning approaches simultaneously.

Regarding students' experiences and needs in studying Multivariable Calculus, the questionnaire collected information on demographic and educational backgrounds. The respondents included both male and female students, all of whom had previously taken a Multivariable Calculus course. Students reported varying levels of difficulty in understanding the material, with some describing it as moderately difficult and others as very

challenging. In terms of specific learning challenges, several students mentioned difficulties in visualizing multivariable functions, understanding limits and continuity, performing partial derivatives, and applying extreme values.

Students described the types of learning resources they found most helpful. Many referred to lecturer explanations and learning videos, while others mentioned textbooks, interactive applications such as GeoGebra, and group discussions. Regarding prior experience with GeoGebra, some students indicated they had never used it, while others had only limited exposure. One student commented, *"I know GeoGebra exists, but I have never used it in class,"* and another added, *"I tried using GeoGebra before, but I did not understand how to manipulate the 3D view."* Most students agreed that instructional materials should include interactive GeoGebra visualizations, and all respondents expressed willingness to use GeoGebra-based teaching materials developed with the Gravemeijer & Cobb approach. Students also reported preferences for learning activities and features in instructional materials. These included interactive 3D visualizations, mini-projects using GeoGebra, videos for repeated individual study, contextual exploration with real-life case studies, and detailed lecturer explanations, highlighting the variety of approaches students considered supportive of their learning.

Student responses were collected through a questionnaire to document learning difficulties, prior experience, and preferences related to Multivariable Calculus. The responses identified specific challenges in visualizing multivariable functions, understanding limits and continuity, performing partial derivatives, and applying extreme values. Students reported preferences for learning media, including lecturer explanations,

instructional videos, textbooks, GeoGebra, and group discussions. Prior experience with GeoGebra varied, with 62.5% of students reporting that they had never used it and 37.5% indicating limited experience. All students expressed willingness to use GeoGebra-based teaching materials. Students also mentioned desired features for instructional materials, including interactive 3D visualizations, mini-projects using GeoGebra, videos for repeated individual study, contextual exploration with real-life case studies, and detailed lecturer explanations. Additionally, hands-on classroom exercises, exploratory problem-solving activities, and collaborative tasks were cited. These responses provide the empirical basis for the preliminary design phase in which instructional materials are developed.

The questionnaire collected information on students' experiences, difficulties, and preferences in studying Multivariable Calculus. Several students reported difficulties in visualizing multivariable functions, understanding limits and continuity, performing partial derivatives, and applying extreme values. One student noted, *"I can compute the partial derivatives, but I still cannot picture the shape of the surface,"* while another commented, *"Contour lines are confusing because I cannot see how they represent the height or slope of the graph."* Regarding learning activities, a few students mentioned a preference for hands-on classroom exercises, exploratory problem-solving activities, and collaborative tasks. One respondent stated, *"I understand better when I can try things myself rather than only listening,"* highlighting the importance of active engagement. Some students suggested mini-projects using GeoGebra, and several expressed interest

in interactive 3D visualizations, with comments such as, *"Seeing the graph in 3D helps me connect the formulas to the shape of the surface."*

In terms of learning media, most students identified lecturer explanations and learning videos as helpful, while others mentioned textbooks, GeoGebra, or group discussions. Regarding prior experience with GeoGebra, some students had never used it, while others reported limited experience. One student shared, *"I know GeoGebra exists, but I have never used it in class,"* while another added, *"I tried using GeoGebra before, but I did not understand how to manipulate the 3D view."* When asked about the inclusion of GeoGebra visualizations in teaching materials, the majority of students agreed that such visualizations would be useful. All respondents indicated willingness to use GeoGebra-based teaching materials developed using the Gravemeijer & Cobb approach.

Students' expectations for interactive 3D visualizations, mini projects, and multimedia materials align with Gravemeijer's principle of *guided reinvention*. Through designed exploratory activities, students receive information and can rediscover multivariable concepts through interaction with visual media and mathematical discussions. Thus, this study's theoretical contribution lies in strengthening the argument that the Gravemeijer & Cobb framework can be adapted for advanced calculus learning, which has been relatively understudied. Applying this framework in the context of CDE not only enriches the international literature but also broadens the scope of *design research applications* to the realm of Multivariable Calculus.

From a research perspective, the questionnaire data serve as the basis for the preliminary design phase in the

second year. These results will be used to develop a prototype GeoGebra-based teaching material that targets explicitly students' key challenges, namely partial derivatives, extreme values, and visualization of multivariable functions. Thus, the research goes beyond describing needs and continues to design innovative solutions. The initial implementation phase (teaching experiment) will utilize these empirical findings as a reference. For example, the gap between procedural and conceptual skills will be tested through GeoGebra-based exploration activities. At the same time, student preferences for lecturer explanations will be considered when designing a combination of instructional strategies to ensure that the developed design remains contextual and aligned with the students' learning culture.

Next, the results of the initial implementation will be analyzed retrospectively to evaluate the appropriateness of the teaching materials. New findings from this phase will inform the next development cycle. Using an iterative approach, this research aims to produce increasingly mature teaching material designs over the years until they are ready for widespread implementation in the regular curriculum. Ultimately, this study provides empirical contributions through detailed data on student learning needs and offers both practical and theoretical value through the application of a design-based research approach. Building on these empirical results, this study also delineates the long term research agenda within the Gravemeijer and Cobb (2006) design research framework. The needs analysis presented here constitutes the starting point for a multi-year trajectory aimed at developing a comprehensive set of Multivariable Calculus learning materials. The planned outputs include a printed

instructional module, an interactive digital module supported by GeoGebra-based 3D visualizations, and a sequence of guided reinvention activities aligned with departmental curriculum standards. In the second year, a preliminary prototype of these materials will be constructed and refined through iterative teaching experiments, followed by successive cycles of revision and classroom implementation in subsequent years. The long term goal of this research agenda is to produce a validated, scalable, and curriculum integrated learning resource that systematically addresses the conceptual difficulties identified in this study.

Discussion

The findings of this study confirm that students experience persistent difficulties in Multivariable Calculus, particularly regarding partial derivatives, extreme values, and the visualization of multivariable functions. These results are consistent with Borji *et al.* (2022), who reported that students often fail to connect procedural manipulation with the conceptual meaning of mathematical symbols. While Borji's work emphasizes symbolic-conceptual gaps, the present study extends this understanding by showing that such difficulties are intertwined with weaknesses in three-dimensional visualization. This indicates that conceptual errors identified in symbolic tasks cannot be separated from students' limited capacity to mentally represent surfaces, contours, and directional changes.

An important implication emerging from these findings concerns the role of needs-based instruction. The trend highlighted by Herrera *et al.* (2024) shows that mathematics learning aligned with students' actual needs tends to increase

engagement and conceptual depth (Herrera et al., 2024). In this study, needs analysis plays a central role in interpreting student difficulties, especially in the context of technology based learning. Rather than serving as a preliminary administrative step, needs analysis becomes a strategic foundation for determining which aspects of the Multivariable Calculus curriculum require reinforcement, particularly in visual conceptual domains.

The difficulty in three-dimensional visualization experienced by 37.5% of students also aligns with Cheong et al. (2023), who found that weak spatial reasoning hinders understanding of gradients and the behavior of multivariable functions (Cheong et al., 2023). The present study, however, contributes a more specific insight: students' inability to visualize surfaces directly influences their conceptual misunderstanding of two-variable limits and partial derivatives. For example, several students misinterpreted the behavior of functions approaching a point from multiple directions—an error tied closely to how gradients and directional derivatives are conceptualized. This suggests that visualization difficulties form a chain of interconnected conceptual weaknesses that collectively obstruct students' understanding of multivariable ideas.

Students' preference for lecturer explanations (87.5%) demonstrates the continued dominance of traditional teaching methods in KPB learning. Nevertheless, their strong interest in adopting instructional videos and visual applications (75%) indicates readiness for blended learning. These findings reinforce Hevardani et al. (2025), who emphasized that the combination of traditional and digital strategies is more effective than either approach alone (Hevardani et al., 2025). The present study adds nuance by revealing *why* students prefer blended

learning: their conceptual challenges especially those involving spatial visualization demand dynamic technological tools that cannot be addressed through conventional lectures.

Students' limited experience with GeoGebra (62.5% had never used it) contrasts with their positive perceptions of its potential benefits. Consistent with Schoenherr (2024), technology adoption in mathematics learning appears driven more by perceived usefulness than prior exposure (Schoenherr et al., 2024). This finding underscores the importance of designing learning materials that directly respond to student needs, particularly interactive modules such as 3D visualizations, context-based mini projects, and multimedia explanations. These preferences echo the conclusions of Medina Herrera et al. (2024), who reported that interactive technology enhances cognitive engagement and motivation (Herrera et al., 2024). The present study extends these insights by identifying specific components that students find most essential particularly 3D visualization and contextual tasks elements that directly support key concepts in Multivariable Calculus.

Integrating needs analysis into the Gravemeijer & Cobb design research framework further strengthens the contribution of this study. Previous research tends to focus either on student difficulties or on the benefits of learning media, without connecting empirical student needs to the *preliminary design* stage of instructional development. This study provides such a bridge by using students' expressed needs to inform the early phases of design, thereby reinforcing the theoretical coherence of the framework and ensuring that pedagogical choices are empirically grounded. Consequently, needs analysis should be regarded not as a secondary

diagnostic step but as a core component of designing effective teaching materials for advanced mathematics.

Finally, the findings of this study open opportunities for developing more targeted GeoGebra-based learning materials for Multivariable Calculus. Unlike previous research which examined student difficulties (Borji et al., 2022) or explored the benefits of visual media separately (Cheong et al., 2023; Herrera et al., 2024) this study directly connects empirical needs analysis with the Gravemeijer & Cobb design research framework. This explicit linkage has not been addressed in earlier studies and therefore represents the main novelty of the present work. By integrating students' conceptual difficulties, their technological needs, and the principles of guided reinvention, this study provides a systematic foundation for designing, implementing, and validating GeoGebra-supported Multivariable Calculus materials in the Indonesian higher education context.

Implication of Research

The findings of this study have important implications in three main domains. Theoretically, these results extend the application of the Gravemeijer & Cobb framework in learning Multivariable Calculus, confirming that needs analysis can serve as a foundation for *preliminary design* relevant to students' real-world difficulties. Practically, the questionnaire data indicate the need to develop GeoGebra-based teaching materials that provide interactive visualizations and facilitate exploration activities, mini-projects, and links to real-world applications. This study provides an initial foundation for further research cycles in the form of implementation (*teaching experiments*) and retrospective analysis, which are expected to

produce valid, instructionally meaningful, and generalizable teaching materials for advanced mathematics learning in higher education.

Limitation

This study has several limitations that should be noted. First, there were only eight respondents, so the data obtained is not representative of the broader student population. Second, the online questionnaire instrument provided a general overview of students' difficulties, perceptions, and expectations. However, it did not delve deeply into their learning experiences through interviews or direct observation. Third, this study is only at the needs analysis stage, so the instructional impact of the intervention in the form of GeoGebra-based teaching materials cannot yet be verified. These limitations open up opportunities for further research involving larger samples, incorporating qualitative data collection methods, and testing the validity of the learning design through classroom experiments and retrospective analysis.

CONCLUSION

This study highlights that students' primary challenges in Multivariable Calculus lie in visualizing multivariable functions, interpreting partial derivatives, and integrating multiple mathematical representations. These findings indicate that traditional instructional approaches, which often emphasize procedural fluency over conceptual understanding, may not fully address students' cognitive needs. The positive perceptions toward GeoGebra suggest that technology mediated tools, when thoughtfully integrated, can enhance conceptual comprehension and support active, exploratory learning. The main

contribution of this research is the demonstration that a needs-based analysis, combined with the Gravemeijer & Cobb design research framework, provides a systematic foundation for developing instructional materials tailored to students' conceptual and representational challenges. Building on these results, future research should focus on iterative teaching experiments using GeoGebra-based materials, assess their meaningful in improving spatial reasoning and conceptual understanding, and refine blended learning strategies to optimize engagement and learning outcomes in Multivariable Calculus.

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