



Students' Learning Obstacles in Solving Mathematical Proficiency Tasks: A Hermeneutic Phenomenological Study Focused on Algebra

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Abstract

In the context of linear equations in one variable, students often struggle in basic algebra including variable degrees, variable concepts, coefficients, and constants. However, proficiency in understanding algebraic concepts is significantly lower compared to other areas and skills. As a consequence, this research aimed to explore students' learning obstacles in solving linear equations in one variable in secondary schools based on mathematical proficiency. The research design using the hermeneutic phenomenological approach. This research is a framework of the Didactical Design Research (DDR). DDR is based on two paradigms: the interpretive and the critical paradigms to empower students in the context of generating new knowledge as justified true belief. In this study, the researcher delves into the interpretive paradigm. The research subjects involved 46 seventh-grade secondary school students from five different schools across four distinct provinces in Indonesia. The instruments used were test and non-test. The test was done by giving a diagnostic test about linear equation problems, while interviews used the non-test technique. The study findings revealed that students met three types of obstacles in linear equation material, namely ontogenic, epistemological and didactical obstacle. Ontogenic obstacles occur due to insufficient prerequisite knowledge and cognitive limitations. Epistemological obstacles arise when students' understanding works in certain contexts but fails in others, often due to misinterpretations of algebraic principles. Didactical obstacles emerge when instructional materials or teaching methods are inadequate. Therefore, future research can focus on developing a Didactical Design to reduce this obstacle.

Keywords: Didactical Design Research; Learning obstacles; Mathematical proficiency; Linear equation; Hermeneutic phenomenology.

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Abstrak

Dalam konteks persamaan linear satu variabel, peserta didik sering mengalami kesulitan dalam pembelajaran aljabar, contohnya kesulitan dalam memahami konsep variabel, koefisien, dan konstanta. Selain itu, pemahaman peserta didik terhadap konsep aljabar jauh lebih rendah dibandingkan dengan bidang dan mata pelajaran lainnya. Oleh karena itu, penelitian ini bertujuan untuk mengeksplorasi hambatan belajar peserta didik dalam menyelesaikan persamaan linear satu variabel di sekolah menengah berdasarkan mathematical proficiency. Desain penelitian ini bersifat kualitatif dengan pendekatan fenomenologi hermeneutik. Penelitian ini merupakan bagian besar dari framework Didactical Design Research (DDR). DDR berpijak pada dua paradigma yaitu paradigma interpretif dan kritis dan bertujuan untuk memandirikan peserta didik dalam konteks menghasilkan pengetahuan baru sebagai pengetahuan yang justified true belief. Dalam penelitian ini, peneliti melakukan pendalaman dalam paradigma interpretif. Subjek penelitian ini yaitu 46 peserta didik kelas VII dari 5 sekolah yang berasal dari empat provinsi yang berbeda di Indonesia. Instrumen yang digunakan adalah tes dan non-tes. Instrumen tes dalam penelitian ini berupa asesmen diagnostik tentang persamaan linear satu variabel untuk mengidentifikasi learning obstacle peserta didik, sedangkan instrumen non-tes terdiri dari pedoman wawancara dengan peserta didik. Hasil penelitian mengungkapkan bahwa peserta didik mengalami tiga jenis hambatan belajar pada materi persamaan linear satu variabel, yaitu hambatan ontogenik, hambatan epistemologis, dan hambatan didaktis. Hambatan ontogenik muncul karena kurangnya pengetahuan dasar dan keterbatasan kognitif. Hambatan epistemologis terjadi ketika pemahaman siswa berhasil dalam konteks tertentu namun gagal jika diaplikasikan kedalam konteks yang berbeda. Hambatan didaktik muncul karena sajab bahan ajar atau metode pengajaran yang tidak memadai. Oleh karena itu, penelitian mendatang direkomendasikan untuk berfokus pada pengembangan Desain Didaktis untuk mengurangi hambatan belajar dalam pembelajaran aljabar.

INTRODUCTION

Mathematics is an essential subject taught at various educational levels, from elementary school to high school and even at the university level (Vilianti et al., 2018). Mathematics is divided into three main subjects: algebra, analysis, and geometry (Suherman et al., 2001). However, algebra stands as the most foundational, particularly at the junior high school level, where it plays a critical role in the development of mathematical skills for future learning (Fardian et al., 2024; Jupri et al., 2014). As part of the 2013 Curriculum, developing strong algebraic skills has been prioritized to ensure students' readiness for more advanced mathematical concepts (Putri, Juandi, & Turmudi, 2024). Despite its importance, students often demonstrate lower proficiency in algebra compared to other areas of mathematics, with particular struggles observed in linear equations (Al-Mutawah et al., 2019).

Algebra is considered a gateway subject for higher education and professional careers in mathematics, yet it is also seen as one of the most challenging areas due to its abstract nature (Thorpe,

2018). Among algebraic topics, linear equations are especially difficult, as they lie at the intersection of arithmetic and symbolic, formal mathematics (Sulastri & Arhasy, 2017). Mastery of linear equations is not only essential for progressing to calculus and statistical analysis but also for understanding nonlinear functions in advanced algebra (Casey, 2015; Nagle & Moore-Russo, 2013). Without a thorough understanding of linear equations, students face difficulties in grasping more complex mathematical concepts, limiting their ability to solve real-world problems and engage in future studies (Jupri et al., 2024; Saraswati et al., 2016).

Despite its importance, students face persistent obstacles in mastering linear equations. One of the most pressing issues is students' difficulty in solving word problems involving linear equations. These problems are complex because they require translating real-world situations into mathematical models, a task that many students find challenging (Khoshaim, 2020; Reynders et al., 2014; Siregar et al., 2022). In particular, identifying the correct variables, operations, and

formulating equations from narrative text poses a significant hurdle, hindering their ability to apply abstract concepts to concrete situations (Nashiru et al., 2018; Putri, Juandi, Turmudi, et al., 2024). Moreover, students often struggle with misconceptions in basic algebra including variable degrees, variable concepts, coefficients, and constants (Makonye, 2015). These misconceptions lead to errors in identifying the general form of linear equations in one variable (Arnawa et al., 2019). Misunderstandings about linear equations often lead to repeated errors and misconceptions, creating a barrier to deeper learning (Yansa et al., 2021).

Despite extensive research on linear equations, there remains a significant gap in understanding the specific learning obstacles students face in secondary education, particularly concerning their mathematical proficiency. A learning obstacle represents a condition that restricts students' acquisition of new knowledge during the learning process, potentially causing difficulties in their learning process (Suryadi, 2019a). The diffusion and acquisition of knowledge are likely to take place within a scientific community setting due to transitions occurring between different institutions (Puspita & Kustiawan, 2024). Learning obstacles can be observed based on the relationship between teacher-student-material (Suryadi, 2023). However, any disruptions in these relationships can contribute to the emergence of learning obstacles. Brousseau (2002) classifies learning obstacles into three categories based on their origins: ontogenic obstacles, didactical obstacles, and epistemological obstacles.

An ontogenic obstacle emerges in the student's cultivation of intellectual capabilities (Utami et al., 2022). Students may experience this obstacle if the problems presented by the teacher have difficulty levels either below the actual ability

or above the potential ability of the students (Bakar et al., 2019). The identification of ontogenic obstacles occurs when there is an imbalance between the learning demands and the mental readiness of the student (Islamiyah et al., 2023). Suryadi (2019b) classifies ontogenic obstacles into three categories: psychological ontogenic obstacles, instrumental ontogenic obstacles, and conceptual ontogenic obstacles.

A didactical obstacle refers to a difficulty in learning that is linked to the instructional stages implemented by the teacher (Prabowo et al., 2022). However, the extent of detail in presenting teaching material can impact the student's learning process (Fardian et al., 2024). Didactical obstacles can be identified when the actual learning process and the theoretically expected content are dissimilar. Since the primary aspect of this obstacle can be observed through the content, stages of material presentation, this obstacle may originate from the teaching materials prepared and implemented by the teacher in the learning process.

An epistemological obstacle represents a difficulty in learning that refers to how students acquire knowledge (Suryadi, 2023). This obstacle is defined by the limited understanding of students related to a specific context of knowledge. Consequently, epistemological obstacles can be identified when students' knowledge of a mathematical object they have learned works well in a specific problem context but is then inappropriately applied to another context. In essence, students have an incomplete understanding of that mathematical object.

Mathematical proficiency is defined as possessing suitable or sufficient mathematics skills, knowledge, or experience (Corrêa & Haslam, 2020; Hendriyanto et al., 2024). Proficiency in mathematics

provides learners with expertise, competence, knowledge, and facility in the subject, facilitating successful learning (Fardian & Dasari, 2023; Findell et al., 2001). Acknowledging a student's starting level and proficiency in specific mathematical content forms the basis for creating successful learning environments (Altarawneh & Marei, 2021). Mathematical proficiency is typically characterized by five strands, namely conceptual understanding, adaptive reasoning, procedural fluency, strategic competence and productive disposition. The five strands serve as a framework for examining the knowledge, skills, abilities, and beliefs essential to mathematical proficiency (Junpeng et al., 2020). It is important to recognize that these strands are interconnected and interdependent, playing a collaborative role in the cultivation of mathematical proficiency (Algarni & Lortie-Forgues, 2023; Yunus et al., 2012). The framework aims to identify the components essential for successful learning in mathematics.

Consequently, this proficiency allows learners to navigate the mathematical challenges encountered in everyday life and supports their continued pursuit of mathematical studies through high school and beyond.

While the literature on learning obstacles in algebra is growing, few studies have focused on the impact of conceptual understanding, adaptive reasoning, and strategic competence in overcoming these barriers (Brousseau, 2002; Findell et al., 2001). Furthermore, much of the existing research fails to provide a comprehensive analysis of how these obstacles relate to students' ability to solve linear equations and apply mathematical reasoning effectively. To further emphasize the importance of this research, the researcher conducted a bibliometric analysis of algebra-related publications over the past five years, as shown in Figure 1.

Figure 1 illustrates that there is no direct connection between linear equations and mathematical proficiency. This

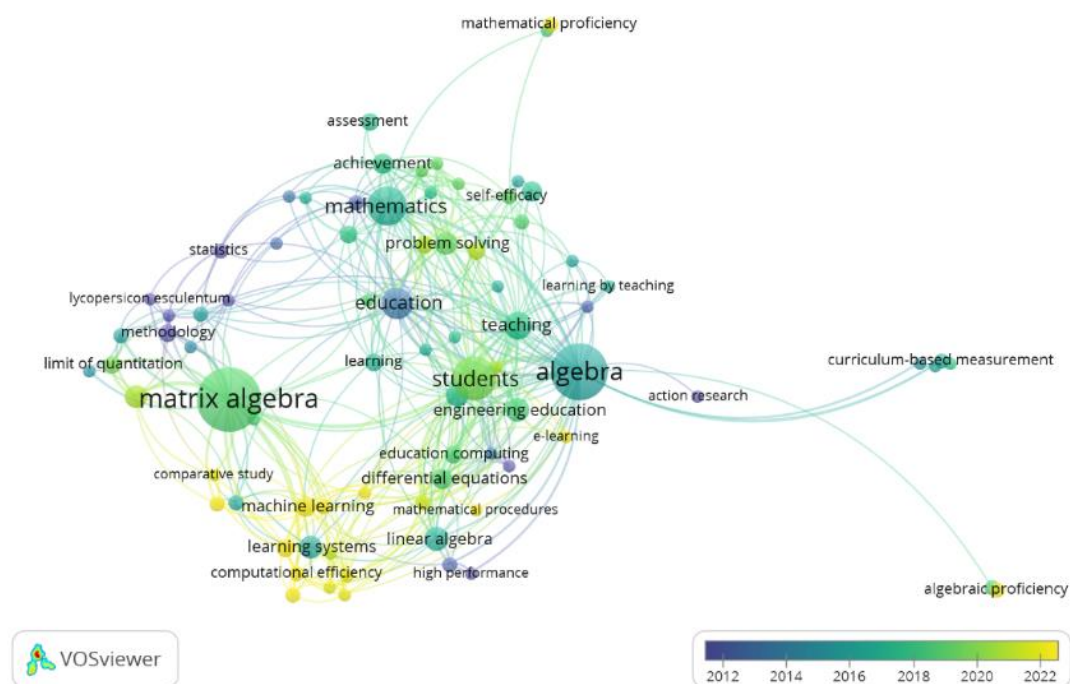


Figure 1. Research trend on algebra over the past five years

suggests a significant research gap in the existing literature. Furthermore, the data shows that mathematical proficiency has been relatively underexplored compared to other topics, as indicated by the smaller number of publications in this area within the bibliometric analysis. The yellow color in the figure highlights that mathematical proficiency has emerged as a hot topic in the last five years, drawing increasing attention from researchers. However, this growing interest contrasts with the limited exploration of its relationship with specific mathematical topics like linear equations, reinforcing the need for further research in this domain.

To provide an overview of previous research on early algebra, Figure 2 summarizes key studies conducted in the field, focusing exclusively on Scopus Q1-indexed articles.

Based on figure 2, the orange nodes in the visualization represent significant

areas of focus in existing research, highlighting the six major themes in early algebra studies. The blue nodes, on the other hand, represent the authors who have contributed to these themes. However, the green node, labeled "Mathematical Proficiency", emphasizes a crucial void in the literature. Despite the extensive focus across the six themes, none explicitly analyzes the role of mathematical proficiency in the context of early algebra. This lack of attention to mathematical proficiency, particularly in relation to key algebraic concepts like linear equations, presents a clear research gap. This focus could pave the way for more effective teaching practices and contribute significantly to mathematics education reform.

The importance of this research is underscored by the findings from the PISA 2022 assessment, which reveal that 61% of OECD countries have mathematics performance below the average, with

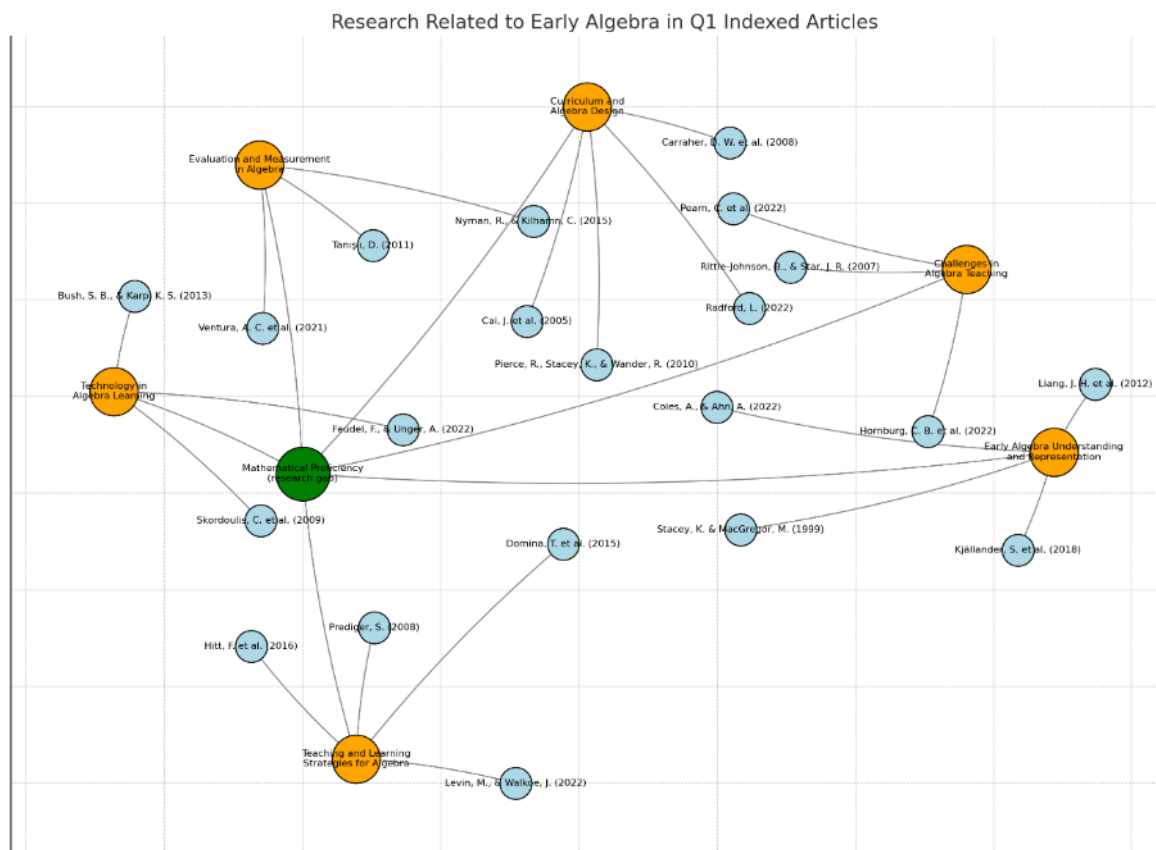


Figure 2. Research related on algebra in the Q1-indexed articles

31% of students demonstrating proficiency below Level 2 (Fardian & Dasari, 2023). This level of proficiency is critical for engaging in active societal participation, making it clear that a substantial portion of students lack the foundational mathematical skills necessary for success in both education and life (Lee *et al.*, 2023).

This study seeks to address this gap by investigating the obstacles students encounter while solving linear equations, emphasizing mathematical proficiency. Through this investigation, we aim to shed light on the specific obstacles that hinder students' progress, including their understanding of basic algebraic principles, their ability to reason adaptively, and their strategic competence in solving real-world problems. By focusing on these critical areas, the research will contribute to a clearer understanding of how these obstacles manifest and how they can be overcome in the classroom.

For this reason, the researcher considers it crucial to identify student learning obstacles in understanding linear equations in terms of mathematical proficiency. Through this identification, it is hoped that it will prompt teachers to establish effective learning environments. This study aims to investigate the challenges that middle school students encounter when solving linear equations.

METHOD

The research design using hermeneutics phenomenological method (Moustakas, 1994). This research is a significant part of the Didactical Design Research (DDR) framework. DDR constitutes research to identify learning difficulties within the educational process, focusing on proactively tackling and eliminating these obstacles (Suryadi & Itoh, 2023). This conceptual framework aligns with the objective of the

current study, which is to explore learning obstacles related to linear equations in secondary schools. DDR is based on two paradigms: the interpretive and the critical paradigms, and it aims to empower students in the context of generating new knowledge as justified true belief (Suryadi, 2019a). In this study, the researcher delves into the interpretive paradigm. The interpretive paradigm aims to analyze the impact of existing didactic designs on students' thinking patterns in the process of acquiring new knowledge, thus identifying learning obstacles.

A total of 46 seventh-grade students were selected as participants, representing five schools across four distinct provinces in Indonesia. The participant distributions are as follows: 7 students from a middle school in West Sulawesi, 24 students from a middle school in Central Sulawesi, 5 students from a middle school in Gorontalo, and 10 students from two middle schools in South Sulawesi. The differences in the selected schools in the research provide important variations in the subjects represented. These schools were chosen with different variations, because: (1) geographical diversity: selecting schools from different provinces allows the research to encompass students from various geographical backgrounds. This can broaden the scope of the research findings and strengthen conclusions about potential learning obstacles. (2) differences in education systems: Each province has slightly different educational approaches, including curriculum, teaching methods, and learning approaches. By selecting schools from various provinces, the research can explore how these differences affect students' understanding and learning, as well as how they influence learning obstacles in the context of linear equations with one variable. Considering these factors, selecting schools from different provinces can help ensure that the

research covers important diversity in the represented student population, thus enabling stronger conclusions about potential learning obstacles in the field of linear equations in one variable. Moreover, the researcher selected for this particular group due to the relevance of the material under investigation to seventh-grade middle school students.

The researchers took several steps

to minimize bias in participant selection and ensure that the sample was representative of the broader population. First, all selected schools were of comparable educational levels, meaning that the schools were chosen based on their similarity in terms of the grade level being studied (seventh-grade) and the overall academic standards. This step was crucial to control for any age-related cognitive

Table 1. The instrument of mathematical proficiency tasks

| Strands | Indicators | Test Instrument | Non-test Instrument |
|-------------------------------|--|---|---|
| Conceptual Understanding (CU) | Unable to solve mathematical concepts | Solve the following equations by writing the steps of the solution: a. $3x - 2 = 10$ | The researcher asks the student to solve the equation $3x - 2 = 10$, explaining each step taken. |
| | Unable to solve mathematical operation | Solve the following equations by writing the steps of the solution: b. $\frac{3x+1}{14} - \frac{1-2x}{2} = 2$ | The researcher asks the student to describe the steps needed to solve the fractional operations in this equation. |
| Adaptive Reasoning (AR) | Unable to think logically about the relationship between concepts and situations | Mirna is 30 years younger than her father. Five years later, the sum of their ages will be 46 years. How old are Mirna and her father now? | The researcher asks the student to read the problem about Mirna and her father, then explain how they use the information in the problem to create an equation. |
| | Unable to justify the conclusions | The price of a chicken is Rp25.000 each, and the price of a goat is Rp700.000 each. Fauzan wants to buy two goats, but he currently only has Rp125.000. To have enough money to buy the two goats, Fauzan decides to sell his chickens. How many chickens does Fauzan need to sell? | The researcher asks the student to justify the steps they took in calculating the number of chickens to sell. |
| Strategic Competence (SC) | Unable to formulate mathematical problems | Risda and Arlinda are running on the same track. Risda runs at a speed of 240 meters per minute and starts 100 meters ahead of Arlinda. If Arlinda runs at a speed of 290 meters per minute, after how many minutes will Risda and Arlinda be at the same position? | The researcher asks the student to read the problem about Risda and Arlinda, then explain how they start formulating the mathematical problem from the given situation. |
| | Unable to represent and solve mathematical problems | After running, Risda goes to buy some drinks. Risda buys 2 bottles of drinks and 2 snacks with Rp50.000 and receives Rp8.000 in change. The price of one bottle of drink is twice the price of one snack. What is the price of one bottle of drink and one snack? | The researcher asks the student to read the problem about the price of drinks and snacks, then explain how they choose variables to represent the price of each item. |

differences that might otherwise influence the results. To further reduce potential bias, the study intentionally included a mix of public and private schools. This strategy was used to account for the variations in resources, teaching methods, and educational systems that may exist between public and private institutions. Additionally, the researchers employed stratified sampling to ensure that the sample was proportionally representative of each region's population. Stratified sampling involves dividing the population into subgroups (regions) and then selecting participants from each subgroup in a way that mirrors the region's actual demographic distribution.

The research employed both test and non-test instruments. The test was done by giving questions about linear equation problems, while interviews used the non-test technique. The current study focuses exclusively on three strands, specifically conceptual understanding, adaptive reasoning, and strategic competence, with several justifications guiding this selection. Firstly, it is important to emphasize that the size and scope of the strands should not be confused, as the model aims to provide a comprehensive depiction of holistic mathematics proficiency strands. Certain strands may hold greater significance at specific age levels than others. For instance, while conceptual understanding and procedural fluency may be fully developed for 14-year-old students, their adaptive reasoning might still be somewhat limited (Findell *et al.*, 2001). Table 1 represent the test and non-test instrument of mathematical proficiency tasks.

Each indicator, category, and code assigned to the student responses are processed using ATLAS.ti 9 software. To mitigate coder bias in the qualitative data analysis, the data coding process is carried out by multiple researchers. This reduces

the risk of individual interpretation errors and ensures consistency across the coding process. Additionally, member checking used to verify the interpretations of student responses, ensuring that the analysis accurately reflects the participants' understanding and experiences. The code label "1A – CU1" indicates the issue found in question 1A (1A), category conceptual understanding (CU), the first code (1), which is "unable to solve mathematical concept.

The data analysis employed in this study used the Miles and Huberman model, which was conducted in three stages: (1) data reduction, where the researchers recorded all student questions responses related to linear equations; (2) data presentation, during which the researchers systematically categorized and identified various types of student responses based on their obstacles; (3) drawing conclusions, wherein the researchers analyzed the identified types of learning obstacles in linear equations among seventh-grade middle school students, drawing insights based on the theoretical framework of learning obstacles (Miles & Huberman, 1994). Figure 3 represents the data analysis process diagram.

RESULTS AND DISCUSSION

Results

The analysis was conducted by examining the components of the obstacle aspect to present the number of students who had obstacles based on the mean score for each set of questions. Subsequently, the analysis progressed to identify the percentage of students facing obstacles in each aspect. The researchers generally observed a uniform distribution of obstacles across conceptual understanding, procedural fluency, and strategic competence.

The result shows that the main obstacles were discovered in the domain of linear equation material, particularly in strategic competence. Strategic competence is the proficiency to formulate, illustrate, and solve mathematical problems (Schulz, 2023). This aspect aligns with what has been termed as problem-solving and problem formulation in the field of mathematics education literature (55.44%), followed by adaptive reasoning, which is the capacity to navigate through a lot of facts, procedures, concepts, and solution methods and discern their inter-connected coherence and meaningfulness (48.92%).

Conceptual understanding of linear equation in one variable

On average, 44.57% of students met this initial problem, with a slight standard deviation 5.09. This research indicates that

the data is homogeneous (see table 1). Based on Table 1, most students had difficulties in comprehending and performing operations related to linear equations in one variable.

Table 1. Percentage of students who met indicators on the first obstacle aspect

| Indicators | Question Number | The Number of Students | Percentage |
|--|-----------------|------------------------|------------|
| Unable to solve mathematical concepts | 1a | 7 | 15.21% |
| Unable to solve mathematical operation | 1b | 34 | 73.91% |

To further illustrate the challenges faced by students in solving the problem, Figure 4 provides a visual representation of the patterns of difficulty, particularly in terms of conceptual understanding and performing mathematical operations related to linear equations in one variable.

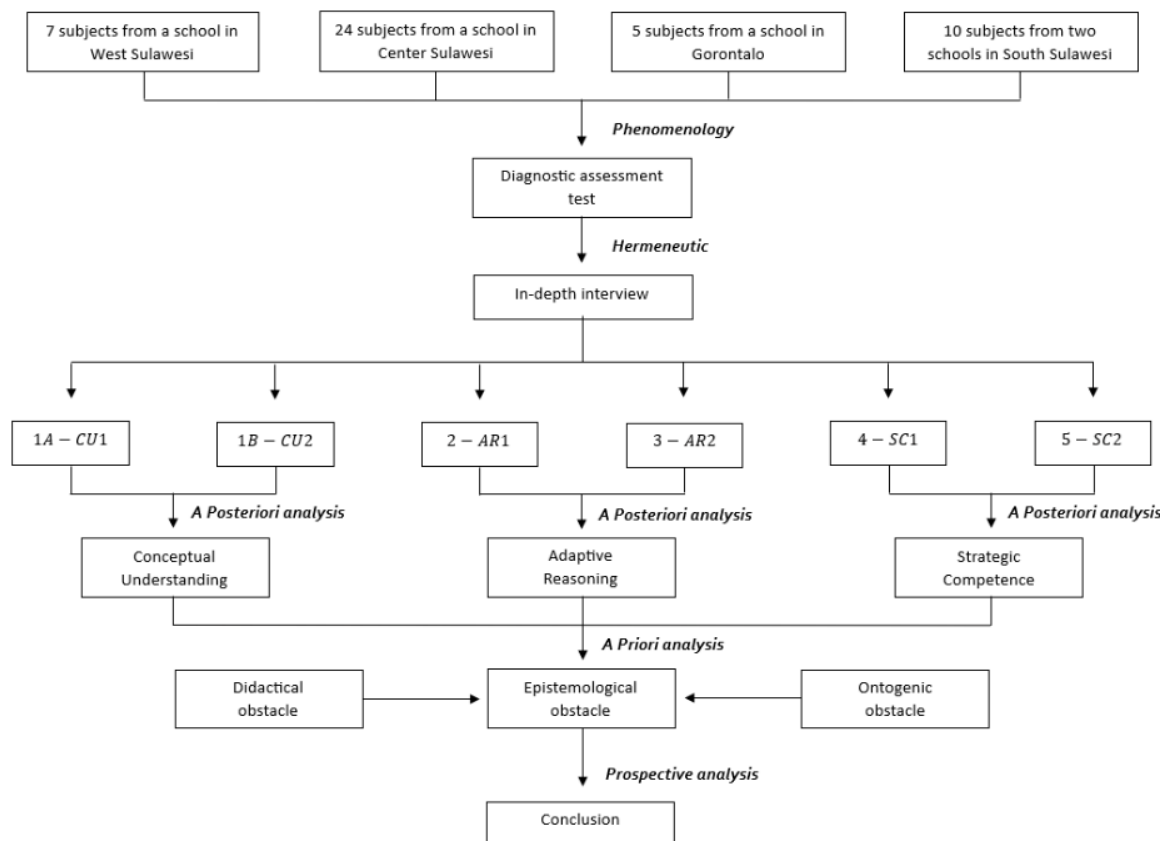


Figure 3. The data analysis process diagram

1.

① $3x - 2 = 10$
 $3x - 2 = 10$
 ~~$3x - 2 = 10$~~
 ~~$3x = 10 + 2$~~
 ~~$3x = 12$~~
 $3x - 2 + 2 = 10 + 2$
 $3x = 12$
 $1x = 12 - 3 = 9$

(a)

1 | $3x - 2 = 10$
 $3x - 2 + 2 = 10 + 2$
 $3x = 12$
 $\frac{3x}{3} = \frac{12}{3}$
 $1x = 6$
 $= 6$

(b)

Figure 4. Answers by subject 1 (a) and subject 2 (b)

According to the book of seventh-grade mathematics curriculum "Merdeka" developed by the Ministry of Education, Culture, Research and Technology in Indonesia, when solving linear equation problems, students must move terms from one side to the other (transpose). However, there are obstacles experienced by subject 1 and subject 2 (see Figure 1). Subject 1 shifts the coefficient from the left-hand side to the right-hand side, causing the sign in front of the coefficient to change inversely. In 1A – CU1, subject is directed to solve the equation $3x - 2 = 10$. Initially, it appears that the students understand the concept of transposing, which is subtracting the same number from both sides. However, the problem occurs when Subject 1 reaches $3x = 12$, and the student moves the number 3 (red colour) to the other side, resulting in the equation $1x = 12 - 3$.

To examine the underlying meaning behind this student error, the researcher conducted further analysis from a hermeneutic perspective. Hermeneutics, as a methodology, focuses on interpreting and understanding the meaning behind human experiences and actions within

their context. Based on in-depth interview, the error made by subject 1 in transposing the coefficient reflects a misunderstanding of the conceptual framework underlying the operation of moving terms in linear equations. From a hermeneutic standpoint, this error can be seen because of the student's limited contextual knowledge and their inability to fully grasp the deeper implications of the transposing rule. The lack of appropriate examples provided by the teacher, as well as the absence of a comprehensive explanation in the mathematics textbook, means that students may interpret the rule based on surface-level understanding, leading to errors like incorrectly shifting the coefficient without adjusting its sign. Hermeneutic analysis suggests that these kinds of mistakes are not just about cognitive limitations, but about misinterpretations shaped by the absence of meaningful guidance and contextual grounding. Moreover, since one of the sources of this misunderstanding originates from the textbook, the researcher examined the 'Merdeka' curriculum mathematics textbook. Upon review, the researcher found that the textbook presents a mis-interpretation of the concept of transpose, which likely contributes to the students' confusion. Figure 5 illustrates this mis-representation, showing how the textbook may inadvertently encourage a simplistic understanding of transpose, leading students to make errors like the one demonstrated by subject 1.

Students original answer

Kita belajar dari halaman sebelumnya, bahwa dalam persamaan kita dapat memindahkan suku-suku dari satu sisi ke sisi yang lain. Hal ini disebut *mentranspos* atau memindahkan suku-suku.

Diagram illustrating the concept of transposing (moving terms) in equations. It shows two examples:

- Example 1: $x - 9 = 3$. An arrow points from -9 to the right side, resulting in $x = 3 + 9$. The text below the arrow says "memindahkan suku".
- Example 2: $2x = 6 + x$. An arrow points from $+x$ to the left side, resulting in $2x - x = 6$. The text below the arrow says "memindahkan suku".

Ingat, ketika sebuah suku berpindah sisi, tanda yang ada di depannya berubah menjadi kebalikannya.

(a)

Translation

We learned from the previous page that in an equation, we can move terms from one side to the other. This is called transposing or moving terms.



Remember, when a term moves to the other side, the sign in front of it changes to its opposite

(b)

Figure 5. The presentation in the textbook related to the concept of transpose

Based on Figure 5, if students internalize this concept, they will encounter difficulties when they move on to the topic of inequalities. In the concept of inequalities, if both sides are divided by a negative number, the inequality sign will change. For example, $-3x < 12$. If students apply the concept they learned, the inequality will become $x < -4$. However, in the context of inequalities, dividing both sides by a negative number should change the inequality sign, resulting in $x > -4$.

Furthermore, in 1A – CU1, subject 2 selects an arbitrary coefficient and divides both sides by that coefficient rather than dividing by the coefficient associated with x . In the assesment diagnostic test, the student was able to solve the initial problem smoothly and understood the concept of transposing, which involves subtracting the same number from both sides. However, the problem arose when subject 2 reached $3x = 12$. Instead of dividing both sides by the coefficient 3, the student mistakenly divided both sides by 2 (red colour), resulting in the equation $\frac{3}{2}x = \frac{12}{2}$. When researcher asked why the subject 2 divided by 2 instead of the coefficient 3, subject explained that they misunderstood the structure of the equation. Initially, they understood the transposition process, which involves subtracting the same number from both sides. However,

when they reached the equation $3x - 2 = 10$, the student explained that they saw the number 2 (red colour) and mistakenly thought it was the "coefficient" to divide by, rather than the number directly associated with x (which is 3). The obstacle occurs due to the limited understanding regarding the concept of numerical operations and the concept of the coefficient in linear equations.

While Subject 2's misunderstanding stemmed from confusion about the role of coefficients in the equation, subject 3 faced a different type of challenge in solving the problem. Figure 6 illustrates the errors made by subject 3 during this process.

Students original answer

Jawaban
a. jika $x = 4$ maka $3x - 2 = 10$
 $(3(4)) - 2 = 12 - 2 = 10$

Translation

Answer

a. If $x = 4$, then $3x - 2 = 10$
 $(3(4)) - 2 = 12 - 2$
 $= 10$

Figure 6. Subject 3's answer

Based on Figure 6, instead of applying a structured approach to isolate x , subject 3 resorted to a trial-and-error method, searching for a value that satisfied the equation without fully understanding the meaning of x in the context of the problem. Subject 3 substituted the value 4 to fulfil the equation $3x - 2 = 10$. Although the chosen number was correct, when the researcher directly asked about the value of x that fulfilled the equation, the students saw confusion in explaining their answer. The student interpreted that the value of x is something that can be replaced with any number without understanding the meaning of x . This led the

subject to use a trial-and-error system, replacing the value of x with any number without considering the equality in the problem.

Based on the hermeneutic analysis, it is revealed that the error experienced by subject 3 stemmed from an inability to distinguish between the concept of the equal sign in algebra and the equal sign in equations. In equations, the equal sign represents a relationship of equivalence between the left and right sides, whereas in algebra, the equal sign is perceived as a command to solve the problem. The student was still operating within an algebraic thinking framework, which was shaped by their previous exposure to algebraic concepts in Chapter 2, before being introduced to equations in Chapter 3. This early learning stage led the student to internalize the concept that the equal sign represents an instruction to solve, rather than recognizing it as a symbol of equivalence that must be respected in the context of solving an equation.

In 1B – CU2, the student experienced difficulties when solving the second problem. Figure 7 represents the student's struggles in this process.

$$\begin{aligned}
 & \text{b. } \frac{3x+1}{14} - \frac{1-2x}{2} = 2 \\
 & \frac{2 \cdot (3x+1) - 14 \cdot (1-2x)}{14 \cdot 2} = 2 \\
 & \frac{6x + 2 - 14 + 28x}{28} = 2 \\
 & \frac{34x - 12}{28} = 2 \\
 & = 56
 \end{aligned}$$

Figure 7. Subject 4's answer

Based on Figure 7, it is evident that the subject has mastered basic arithmetic concepts such as multiplication and division, as evidenced by their responses when solving the initial problem. However, subject 4 encountered difficulties

when reaching the equation $\frac{34x-12}{28} = 2$, where the subject performed cross-multiplication to eliminate the fraction. Despite this, the student was unable to continue solving the equation to completion because they did not understand the technical concept of algebraic manipulation, which made it difficult for the student to fully follow the progression of the learning process. To justify this phenomenon, the researcher conducted an in-depth interview with the subject, as presented in the following interview excerpt

- Researcher : I want to ask about question 1b, the problem you previously worked on. The question is $\frac{3x+1}{14} - \frac{1-2x}{2} = 2$. In the first step, what does $2(3x + 1)$ mean?
- Subject 4 : 2 times $3x$
- Researcher : Just 2 times $3x$?
- Subject 4 : 2 times $3x$ plus 1
- Researcher : Where did you get 2?
- Subject 4 : From here (pointing at student work), because it is multiplied
- Researcher : Multiplied by what? Why multiplied?
- Subject 4 : Because of the cross
- Researcher : What? Cross-multiplication, right?
- Subject 4 : (silent), because of cross-multiplication
- Researcher : Okay, then, so where does this 14 come from? (pointing at student work)
- Subject 4 : From here (pointing at student work)
- Researcher : Alright, so this 14 is from here, and this 2 (pointing at student work). So, you use a cross-multiply method?
- Subject 4 : Yes
- Researcher : Okay, so what can be operated on are the ones with the same?
- Subject 4 : Like terms
- Researcher : Yeah, like terms or those with the same x , right?
- Subject 4 : Yes
- Researcher : Okay, good job. Now, is the final result 56?
- Subject 4 : (silent)
- Researcher : Where did you get 56, dear?
- Subject 4 : 28 times 2
- Researcher : Why is that?

Subject 4 : I don't know anymore

Based on the interview results conducted by the researcher, subject 4 had a learning obstacle. During the phase of moving terms in the form of variables to the left side and terms in the form of numbers to the right side, the subject used cross-multiplication by multiplying the denominators on the left side with the constant on the right side. In other words, subject 4 applied the cross-multiplication method that they acquired in the fraction operation material to linear equation problems. From a hermeneutic perspective, this error can be understood because of the student's previous experiences and the way they interpreted new knowledge based on those experiences. The student's prior knowledge of cross-multiplication in fractions influenced how they approached the linear equation problem. The lack of proper contextual understanding, shaped by their earlier learning experiences, prevented them from interpreting the equation accurately. This error occurred because the student lacked sufficient prerequisite knowledge, which made it difficult to adapt to the process of constructing new knowledge. Additionally, this misunderstanding was also influenced by the teacher's insufficient explanation of the material, which did not provide the necessary clarity for the student to fully grasp the concept.

Adaptive reasoning for linear equation in one variable

In this component, two questions were employed to analyze students' comprehension of the operational procedures for linear equations, featuring two indicators to identify any obstacles in this aspect. The outcomes revealed that, on average, 48.92% of students encountered chal-

lenges in this aspect, with 71.73% of students unable to estimate the results of the problem related to linear equations (see table 2).

Table 2. Percentage of students who met indicators on the second obstacle aspect

| Indicators | Question Number | The Number of Students | Percentage |
|---|-----------------|------------------------|------------|
| Unable to estimate the results of a procedure | 2 | 33 | 71.73% |
| Unable to solve the problem based on procedures | 3 | 12 | 26.08% |

To further illustrate the challenges faced by students in solving the problem, Figure 8 provides a visual representation of the patterns of difficulty, particularly in terms of adaptive reasoning related to linear equations in one variable (2 – AR1).

Students original answer

Translation

2. Children's age = $46 - 30 - 5$
 $= 46 - 30 - 5$
 $= 11$
 Father's age = $46 - 11$
 $= 46 - 11$
 $= 36$

Figure 8. Subject 5's answer

Based on the student's answer in Figure 8, it is evident that subject 5 only interpreted the story problem within the framework of arithmetic thinking without undergoing the process of transforming it into algebraic form first. This was identified because the student did not initially

represent the child's age as x and the father's age as y . The student directly worked through the problem arithmetically. Based on the interview results, it was revealed that this phenomena arose because the problem presented was beyond the student's actual ability, making it difficult for them to solve it correctly. The word problem required the student to apply abstract algebraic thinking, but the student had not yet fully developed the cognitive skills needed for such tasks. As a result, the student struggled to move beyond simple arithmetic operations and was unable to recognize the need to represent the child's age as x and the father's age as y . Additionally, student's struggle and fear of making mistakes when interpreting the information provided in a word problem. The complexity of the problem, combined with the lack of foundational understanding in translating real-world situations into algebraic expressions, led to the student's reliance on arithmetic operations rather than algebraic thinking. This suggests that the student was not able to properly deconstruct and translate the problem into an algebraic form, which is a necessary skill in solving such types of problems. The obstacle indicates that the student's thought pattern remains in the field of arithmetic thinking and has not transitioned to algebraic thinking.

After conducting an in-depth interview, it can be concluded that subject 6 does not understand the meaning of variables in linear equations. This is what causes subject 5 to struggle in transforming word problems into algebraic form. Figure 9 visualizes the errors of student in comprehending the concept of variable.

Students original answer

$$\begin{array}{l}
 2. \text{ Mirna} + \text{Ayah} = 36 \\
 \text{Ayah} - 30 + \text{Ayah} = 36 \\
 \text{Ayah} + \text{Ayah} = 36 + 30 \\
 2 \text{ Ayah} = 66 \\
 \text{Ayah} = 66 : 2 \quad \text{Jawab} \\
 \text{Ayah} = 33 \quad \text{Jawab}
 \end{array}$$

(a)

Translation

| |
|--|
| <p>2. Mirna + Dad = 36 Dad - 30 + Dad = 30 Dad + Dad = 30 + 30 2 Dad = 66 Dad = 66 : 2 Dad = 33</p> |
|--|

(b)

Figure 9. Subject 6's answer

Based on Figure 9, it can be clearly seen that subject 6 interpreted the variables in the problem as "Mirna" and "Father" literally, rather than as symbols representing "Mirna's Age" and "Father's Age." Procedurally, this misinterpretation may not directly affect the student's final answer. However, from an epistemological standpoint, this misunderstanding has the potential to create ambiguity in defining mathematical statements within equations. When students view variables as mere labels or names, they may struggle to recognize them as abstract symbols that represent unknown quantities. Based on the results of the in-depth interview, the subject acknowledged that the concepts they hold are derived from the mathematics book used in the "Kurikulum Merdeka" curriculum, which is applied in classroom learning.

After analyzing the secondary school mathematics book from the Ministry of Education, Culture, Research and Technology in Indonesia for the seventh-grade with the "Merdeka" curriculum, there are obstacles found in the teaching materials prepared and implemented by the teacher in the classroom.

"A rabbit hutch is made from a square fence. Using a wire fence with a length of 24 m, what is the length of the side fence so that the length of the front fence is 3 m longer than the side fence?"

To solve the problem, the first step is to transform the word problem into algebraic form:

$$2x + (x + 3) = 24$$

However, a learning obstacle identified in the secondary school mathematics book from the Ministry of Education, Culture, Research and Technology in Indonesia for the seventh-grade with the "Merdeka" curriculum. In this context, the equation is expressed in the sentence, "two times the side plus the front equals the total length". The equation $2x + (x + 3) = 24$ should ideally be expressed as "two times the length of the side plus the length of the front equals the total length."

Strategic competence in the field of linear equation in one variable

Most obstacles occurred in the material related to linear equations, particularly in strategic competence (55.44%). More than 70% of students could not represent and solve story problems related to linear equations, and approximately 36% struggled to formulate story problems as represent in table 3.

Table 3. Percentage of students who met indicators on the third obstacle aspect

| Indicators | Question Number | The Number of Students | Percentage |
|---|-----------------|------------------------|------------|
| Unable to formulate mathematical problems | 4 | 17 | 36.95% |
| Unable to represent and solve | 5 | 34 | 73.91% |

| Indicators | Question Number | The Number of Students | Percentage |
|-----------------------|-----------------|------------------------|------------|
| mathematical problems | | | |

To further illustrate the challenges faced by students in solving the problem, Figure 10 provides a visual representation of the patterns of difficulty, particularly in terms of strategic competence related to linear equations in one variable (4 – SC1).

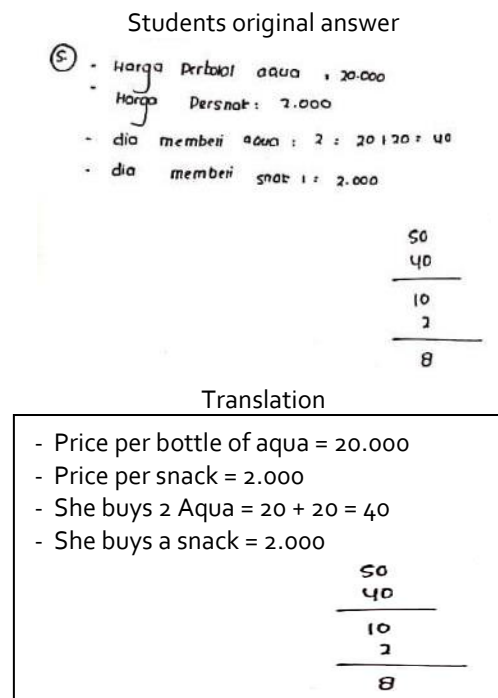


Figure 10. Subject 7's answer

Based on Figure 10, it can be concluded that subject 7 does not understand the sentence "the price of 1 drink bottle is twice that of 1 snack". This misunderstanding occurs due to the limited problem-solving abilities of the student. The inability to properly interpret the relationship between the prices of the drink bottle and the snack suggests that the student struggles with applying mathematical reasoning to real-life situations, which may hinder their ability to solve problems involving ratios or proportional relationships effectively.

Based on in-depth interview, subject's misinterpretation stems from a limited ability to identify and apply key mathematical relationships within the problem. More precisely, the student struggles with proportional reasoning, which is foundational to understanding linear equations. The sentence itself implies a ratio-based relationship (2:1) between the prices of the two items, yet the student either did not grasp this or struggled to translate this into a solvable equation. In other words, many students fail to recognize how to translate real-life situations or verbal descriptions into mathematical relationships. For example, the phrase "the price of 1 drink bottle is twice that of 1 snack" should prompt students to create a simple algebraic expression like $P_{drink} = 2P_{snack}$, but many students do not make this connection. This often happens because word problems involve multiple layers of interpretation that require careful analysis. A lack of number sense can make it difficult for students to connect the verbal description to the mathematical structure. Subject 7's answer, where the price of a drink bottle is assumed to be Rp20.000 and a snack Rp2.000, points to a failure in logical deduction rather than a basic arithmetic error. This suggests that subject 7 may be over-relying on heuristics or assumptions that are not grounded in the problem's structure. For instance, the student may have arbitrarily chosen a set of prices that seem reasonable to them, rather than systematically deriving these from the relationships stated in the problem.

Based on an interview with a teacher, it was revealed that the challenges students encounter in understanding linear equations are not solely attributed to difficulties in applying mathematical concepts but are also influenced by significant disparities in their educational backgrounds. Specifically, some

students, particularly those from elementary schools with more basic curricula, have not been adequately exposed to prerequisite materials, such as the concepts of ratios or proportions, which are essential for a comprehensive understanding of linear equations. As a result, the teacher, being familiar with the characteristics of the students, recommended that the teaching of linear equations should not begin directly with problem-solving tasks. Instead, it is essential to first provide a stimulus, or a review of prerequisite knowledge related to algebra. This approach would help activate students' prior understanding and bridge the gap before introducing more complex concepts, such as linear equations.

Discussion

Mathematical proficiency can be described as possessing the necessary skills, knowledge, or experience in mathematics to a sufficient level (Go, 2023). The strands of mathematical proficiency are referred to as conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Findell et al., 2001). However, the current study focuses exclusively on three strands, specifically conceptual understanding, adaptive reasoning, and strategic competence.

In the conceptual understanding perspective, several phenomena were identified when students studied linear equations. Figure 11 summarizes the overall phenomena and the underlying meanings behind the phenomena identified in students when solving the indicators of conceptual understanding. The white node represents the phenomena identified when students solve problems, while the orange node indicates the meaning behind the identified phenomena (hermeneutic) as students work

through conceptual understanding. The green node denotes the type of learning obstacle encountered during the problem-solving process.

Based on Figure 11, there are six phenomena captured when the subject is working on conceptual understanding tasks. These phenomena included errors made by the students in performing basic arithmetic operations such as addition, subtraction, multiplication, and division. Additionally, students struggled with applying the transpose method correctly, and they found it challenging to recognize patterns or relationships within mathematical expressions. Many students also experienced difficulty with algebraic manipulations, particularly in terms of simplifying or rearranging equations. There was also a limited understanding of the concept of coefficients, and students did not fully comprehend the meaning of the

equal sign in the context of linear equations.

Based on an in-depth investigation from a hermeneutic perspective (Figure 11), it was identified that all phenomenon is caused by seven factors: (1) the subject had weak prerequisite abilities; (2) the presentation of the mathematics textbooks is less comprehensive; (3) the lack of comprehensive examples provided by the teacher; (4) the students' reliance on a trial-and-error method; (5) subject lacked an understanding of the underlying principles of algebraic manipulation (distributive, associative, and commutative properties); (6) subject interprets the "=" sign as an instruction to solve the equation, and (7) subject interprets the concept of transpose as a method of moving terms from one side of the equation to the other.

In the first factor, it was identified

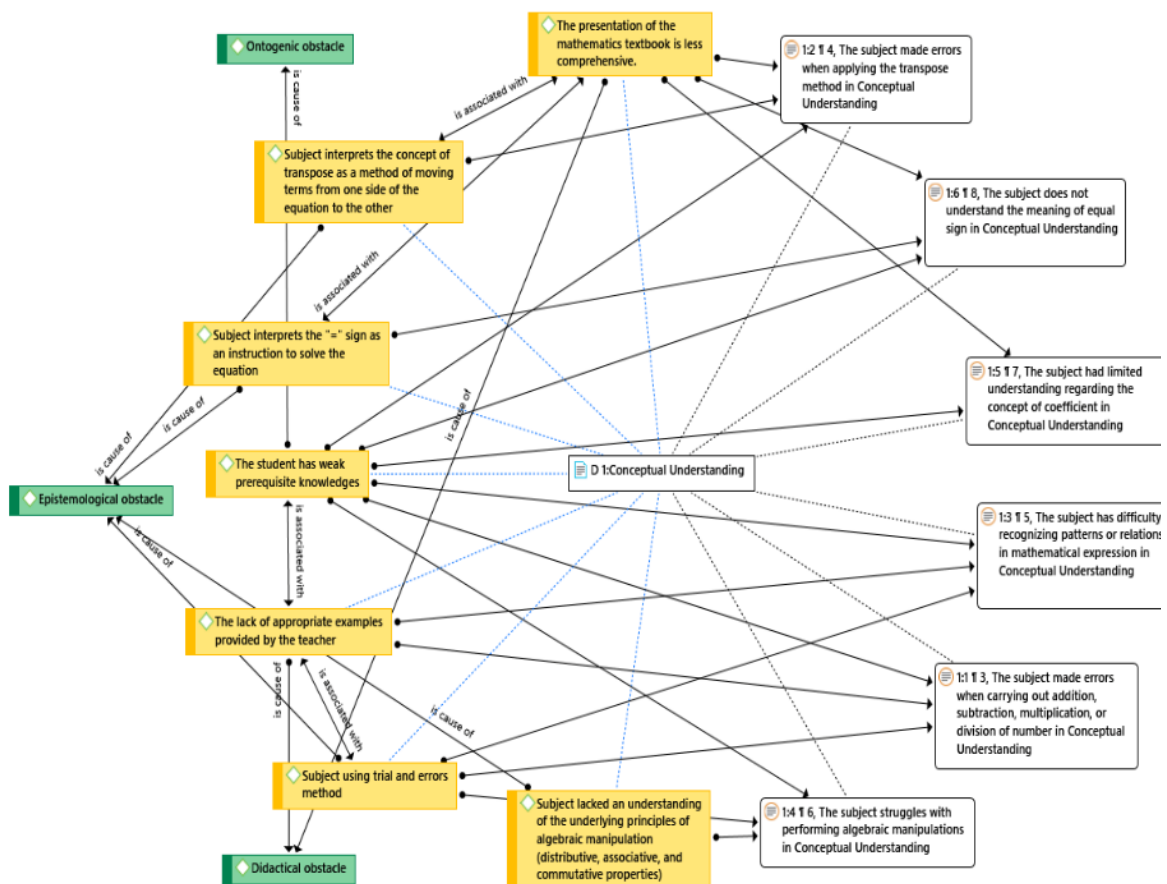


Figure 11. Overall phenomena when students solving the conceptual understanding task

that students have weak prerequisite knowledge related to algebra concepts. According to Brousseau (2002), the mismatch between an individual's cognitive processes and inadequate prerequisite abilities hinders the adaptation of new knowledge, leading to the emergence of ontogenic obstacles. Suryadi (2019b) categorizes these difficulties as conceptual ontogenic obstacles, wherein the conceptual level within didactic design does not align with the students' prior learning experiences. This is in line with Karlsson and Kilborn (2023), which stated that ontogenic obstacles arise when students are unable to connect new concepts to their existing knowledge. Additionally, Utami *et al* (2023) highlight that such obstacles occur due to students' insufficient understanding of prerequisite concepts, such as variables. Students struggle to make sense of learning due to limitations in their thinking processes (Pratiwi *et al.*, 2019).

The second and third factors reveal that errors arise due to the inadequate presentation of the mathematics textbooks and the insufficient examples provided by the teacher. According to Brousseau (2002), errors resulting from the design of the instructional process or the teacher's interventions are classified as didactical obstacles. This aligns with the study by Hannah *et al* (2016). When interviewed about the conceptual knowledge they had worked on, some respondents still gave incorrect answers. This suggests that they understand the material better when it is based on examples provided. Consequently, this will likely pose a problem if they encounter different types of mathematical questions. Magfiroh *et al* (2024) stated that this issue primarily stems from how the textbook content is presented, with insufficient examples limiting students' ability to apply their knowledge in varied contexts.

According to Brousseau (2002), the fourth through seventh factors, namely, the students' reliance on trial-and-error methods, insufficient understanding of algebraic principles (such as the distributive, associative, and commutative properties), misinterpretation of the '=' sign, and misunderstanding of the concept of transpose, can be classified as epistemological obstacles. This epistemological obstacle is identified when students' understanding of a mathematical concept works effectively in a certain context, but when they are faced with a different situation, that understanding becomes difficult or ineffective to apply. These difficulties, however, are not purely the result of external factors but are also linked to ontogenic obstacles, which stem from the students' individual cognitive development and learning experiences. One notable ontogenic obstacle is the students' lack of sufficient prerequisite knowledge. Due to this gap, students often rely on trial-and-error methods to solve problems. This strategy, though sometimes effective in simple contexts, fails when the problem's context changes. The results supported by Pincheira and Alsina (2021), who observed that students often approach algebraic tasks primarily by trial and error, neglecting the structure of the problem and the underlying knowledge required to develop algebraic thinking. Similarly, Edo and Tasik (2022) found that students tend to solve problems based on instinct, trial and error, and logic, rather than relying on a deep understanding of the concepts involved.

From the strands of adaptive reasoning, several key phenomena were observed as students worked on linear equations. Figure 11 provides an overview of these phenomena in relation to the indicators of adaptive reasoning. The white nodes represent the phenomena identi-

fied during problem-solving, while the orange nodes signify the underlying meanings (hermeneutic) associated with students' conceptual understanding. The green nodes highlight the types of learning obstacles encountered throughout the problem-solving process.

Based on Figure 12, four key phenomena were observed as the subject worked on tasks involving adaptive reasoning. First, the subject relied on an arithmetic approach instead of using an algebraic method to solve the problem. This suggests a fundamental difficulty in transitioning from basic arithmetic to more abstract algebraic thinking. Second, the subject displayed a lack of understanding regarding the meaning of a variable, which is crucial for solving algebraic equations. Third, the subject struggled to transform the word problem into an algebraic form, highlighting a challenge in abstracting real-world situations into math-

emathical representations. Finally, the subject tended to focus on procedural steps without grasping the underlying rationale behind them, indicating a reliance on rote procedures rather than developing a deep understanding of the mathematical concepts involved.

Based on an in-depth investigation from a hermeneutic perspective (Figure 12), it was identified that all phenomenon is caused by eight factors: (1) the subject does not understand the concept of equation; (2) the subject interpreted the variables in the problem as "Mirna" and "Father" literally, rather than as symbols representing "Mirna's Age" and "Father's Age"; (3) the available textbook presentation is not comprehensive; (4) the explanation by the teacher is lacking in comprehensiveness; (5) the problem presented was beyond the student's actual ability; (6) the subject's fear of making mistakes when answering questions; (7) previous

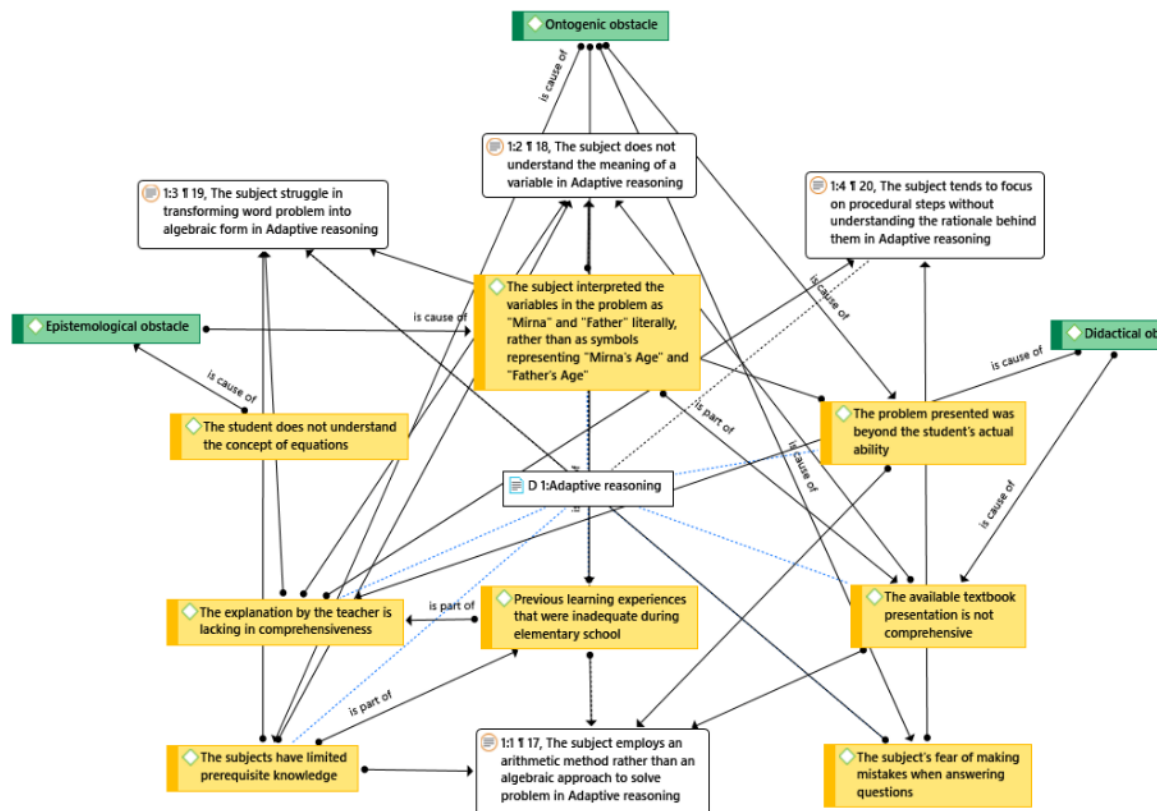


Figure 12. Overall phenomena when students solving the adaptive reasoning task

learning experiences that were inadequate during elementary school, and (8) the subjects have limited prerequisite knowledge.

In the first and second factors, although the students had previously studied linear equations in Chapter III, they failed to apply this knowledge when solving the problem. Instead, they approached the problem from an arithmetic perspective, using basic operations rather than algebraic methods. This reflects a limitation in the students' ability to transfer their understanding to new contexts. While students can solve problems within the same context, they encounter significant obstacles when faced with problems presented in different contexts. Additionally, the students struggled with understanding the concept of variables in equations. According to Brousseau (2002), errors arising from limitations in the context in which a concept is initially learned are classified as epistemological obstacles. These obstacles occur when a student's understanding works well in a specific context but becomes ineffective or difficult to apply in different situations. Li (2010) highlighted that many students' errors and misconceptions arise specifically in areas like variables, equations, and functions, which are foundational to algebraic thinking.

According to Brousseau (2002), errors that result from the design of the instructional process or are caused by an explanation from the teacher that lacks comprehensiveness, as identified in the third and fourth factors, are categorized as didactical obstacles. These obstacles arise when the teaching methods, materials or explanations provided do not fully support the students' understanding or problem-solving abilities, hindering their ability to grasp the concept effectively. Moreover, these didactical obstacles con-

tribute to the emergence of epistemological obstacles for students. The lack of comprehensive material presentation impacts the students' understanding of key concepts, such as equations and variables, preventing them from applying these concepts effectively in problem-solving tasks. This is consistent with the findings of Utami and Prabawanto (2023), who argued that didactical obstacles might be one of the two primary learning obstacles. The second type, epistemological obstacles, occurs when students become confused because the knowledge they have acquired applies only to specific contexts and cannot be transferred to different problems, even though they belong to the same concept. The third type, ontogenic obstacles, is related to the individual development of students and their personal growth in understanding.

The fifth to eighth factors, according to Brousseau (2002), contribute to the emergence of ontogenic obstacles. The complexity of the problem, which exceeded the student's current capabilities, is categorized by Suryadi (2019b) as a conceptual ontogenic obstacle. This type of obstacle occurs when students struggle to understand or apply the underlying concepts needed to solve a problem (Pertiwi *et al.*, 2023). The student's fear of making mistakes when responding to questions, as identified in the sixth factor, is classified by Suryadi (2019b) as a psychological ontogenic obstacle. This fear inhibits the student's confidence and ability to approach the problem effectively. Additionally, the inadequate prior learning experiences during elementary school and the limited prerequisite knowledge, prevented the students from grasping the technical aspects of solving the problem. According to Suryadi (2019b), this is categorized as an instrumental ontogenic obstacle, where students lack the necessary skills or techniques to solve the problem.

correctly. This aligns with the stages of cognitive development proposed by Piaget (1970), students in the age range of 12-13 years (seventh-grade) are still in the cognitive shift from the concrete operational stage to the formal operational stage. As a consequence of this transition, students may encounter challenges when faced with problems that require abstract reasoning, hypothetical thinking, or the application of formal rules.

In examining the strands of strategic competence, several key phenomena emerged as students worked on tasks involving linear equations. Figure 13 offers a summary of these phenomena in relation to the indicators of strategic competence. The white nodes illustrate the phenomena observed during problem-solving, while the orange nodes represent the deeper meanings (hermeneutic) tied to the students' conceptual understanding. The green nodes indicate the types of learning obstacles that students faced during the process of solving the problems.

Based on Figure 13, three phenomena were identified as the subject engaged in tasks requiring strategic competence. First, the subject struggled to understand the information presented in the problem. Second, subject had difficulty selecting the appropriate methods or

strategies to solve the problems. Lastly, the subject demonstrated limited problem-solving skills, which hindered their ability to effectively tackle the tasks. Based on an in-depth investigation from a hermeneutic perspective (Figure 13), it was identified that all phenomenon is caused by six factors: (1) A lack of student number sense; (2) the subjects have limited prerequisite knowledge; (3) the subject's over-reliance on heuristics or assumptions that are not grounded in the problem's structure; (4) the subject does not understand the sentence "the price of 1 drink bottle is twice that of 1 snack"; (5) the subject struggles with proportional reasoning, and (6) failure of students in logical deduction.

In the first, second and third factors, it is clear that these issues stem from the students' lack of problem-solving skills. This barrier occurs due to technical difficulties, such as a limited understanding of fundamental mathematical concepts or restricted cognitive abilities in breaking down complex problems, which makes it difficult for students to fully keep up with the learning process. According to Brousseau (2002), such mistakes can be categorized as ontogenic obstacles, which are inherent difficulties that arise from the student's own cognitive and developmental limitations in learning. In addition, in the

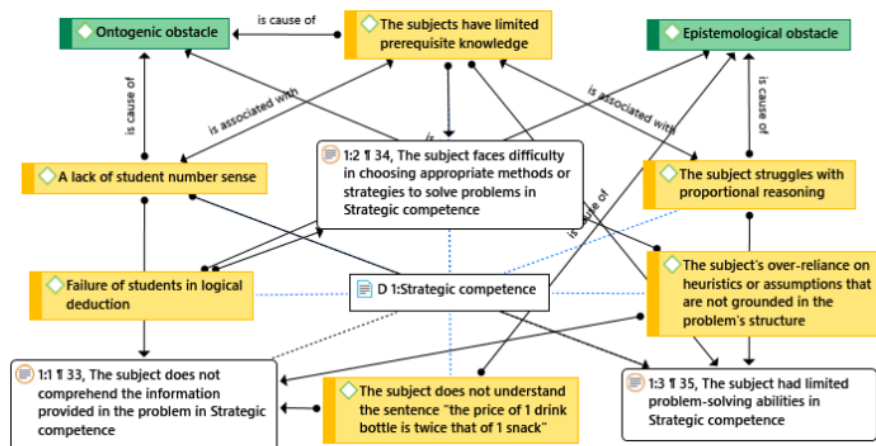


Figure 13. Overall phenomena when students solving the strategic competence task

fourth, fifth, and sixth factors, it is evident that these challenges are attributed to the limited context that students have in their mathematics learning. According to Brousseau (2002), knowledge that was once relevant and effective in specific contexts, including school settings, but later becomes outdated or inadequate, is categorized as epistemological obstacles. This results in line with Swan (2014), which stated that this strand encompasses the capability to formulate, illustrate, and resolve mathematical problem. It aligns with what has been referred to as problem-solving and problem formulation within the field of mathematics education literature. The primary challenge faced by students in strategic competence concerning linear equations is particularly apparent when they attempt to translate information from story problems into mathematical models. However, the results are in line with the research by Astuti *et al* (2017), where students make mistakes when they restate a concept, classify items according to their composition, calculation operations, and apply concepts or problem-solving algorithms.

In summary, the novelty of this study lies in the detailed categorization of learning obstacles (ontogenic, didactical, and epistemological) within the context of linear equations, highlighting both cognitive and psychological obstacles that students face. The research further provides new insights into students' misinterpretations of key algebraic principles, analyzed through the three strands of mathematical proficiency: conceptual understanding, adaptive reasoning, and strategic competence. It also examines how teaching practices can contribute to the emergence of these challenges, offering a nuanced view of the complex factors influencing students' learning. These find-

ings offer valuable implications for educators and researchers, emphasizing the need for more comprehensive textbooks, clearer teacher explanations, and a deeper understanding of cognitive development stages in algebra instruction. By focusing on the foundational knowledge gaps, improving instructional materials and methods, fostering deeper conceptual understanding, supporting students' psychological well-being, and providing appropriate challenges, educators can effectively address the various learning obstacles students face in mastering algebra. These strategies will not only improve students' proficiency in linear equations but also contribute to their overall mathematical development.

Implication of Research

The implications of the findings on learning obstacles can significantly benefit researchers, educators, and policymakers in designing didactic frameworks based on justified true belief. By thoroughly understanding the challenges that students encounter, these stakeholders can develop more effective and precise educational strategies. This, in turn, supports the creation of a learning environment that not only enhances student comprehension and retention but also fosters critical thinking and the application of knowledge. Consequently, this research can lead to a more robust and resilient educational system that is better equipped to meet diverse learner needs and adapt to evolving educational demands.

Limitation

Didactical Design Research relies on two paradigms: the interpretive and the critical paradigms, with the goal of empowering students to generate new knowledge as justified true belief. However, this

study only focuses on the interpretive paradigm. Further research is needed from a critical paradigm perspective to perceptually and memorially examine the learning obstacles identified from previous research results, thus yielding a hypothetical didactic design. Additionally, this research only reveals the potential learning obstacles of learners through the method of hermeneutic phenomenology. Further research is needed on the potential learning obstacles obtained through the analysis of the Theory of Didactic Situations, praxeology, and didactic transposition. Another limitation of this study is that it does not analyze two other strands of mathematical proficiency: procedural fluency and productive disposition. By focusing on only three strands: conceptual understanding, adaptive reasoning, and strategic competence, this research may overlook critical elements of students' mathematical development. Procedural fluency, which involves the ability to perform mathematical operations accurately and efficiently, and productive disposition, which refers to students' attitudes and beliefs about learning mathematics, are both essential in forming a complete picture of students' mathematical proficiency. The exclusion of these strands may limit the overall scope of the findings and their applicability to all areas of mathematical learning.

CONCLUSION

Based on the discussion above, it can be concluded that students identified three types of learning obstacles in linear equation material, namely ontogenic obstacle, epistemological obstacle and didactical obstacle. However, students had limited knowledge, ranging from basic concepts of mathematical operation, arithmetic operations, and solving linear equations in one variable problem.

Ontogenic obstacles occur due to insufficient prerequisite knowledge and cognitive limitations, which prevent students from connecting prior learning to new concepts like variables or algebraic manipulation. Epistemological obstacles arise when students' understanding works in certain contexts but fails in others, often due to misinterpretations of algebraic principles, such as the role of variables or the equal sign. Many students rely on trial-and-error methods instead of developing a deeper understanding of problem-solving strategies, preventing them from effectively transferring knowledge to more complex situations. Didactical obstacles emerge when instructional materials or teaching methods are inadequate. When textbooks are not comprehensive or teachers fail to provide sufficient examples, students struggle to fully understand the concepts, as they lack the necessary support to apply their knowledge effectively. Therefore, it is recommended that teachers construct a Didactical Design to minimize this obstacle.

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