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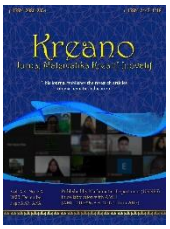

The Effectiveness of Discovery Learning on Mathematical Reflective Thinking Skills Based on the Level of Learning Independence of High School Students

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Abstract

Research on mathematical reflective thinking skills has generally focused on improving learning outcomes through various instructional models. However, limited attention has been given to examining the role of learning independence as a moderating variable that may influence the effectiveness of the discovery learning model, particularly in the topic of Systems of Linear Equations in Two Variables (SPLDV). Therefore, this study aims to evaluate the effectiveness of the discovery learning model in enhancing the mathematical reflective thinking skills of senior high school students by considering learning independence as a moderating variable. This study employed a quantitative approach using a non-equivalent control group design, involving an experimental group taught through the discovery learning model and a control group taught using the expository learning model. Data were collected using a mathematical reflective thinking test and a learning independence questionnaire, both of which were validated and demonstrated satisfactory reliability. Data analysis included descriptive statistics, normality and homogeneity tests, and hypothesis testing using an independent-samples *t*-test, N-gain analysis, and two-way ANOVA. The results showed that the two groups had comparable initial abilities. However, after the intervention, a statistically significant difference was found, with the experimental group achieving higher post-test scores and greater N-gain improvement than the control group. Furthermore, students' mathematical reflective thinking skills differed significantly according to their level of learning independence, with students demonstrating high levels of learning independence achieving better performance. The interaction between the learning model and the level of learning independence was also found to have a significant effect on students' mathematical reflective thinking skills. These findings indicate that the discovery learning model is an effective instructional strategy for enhancing students' mathematical reflective thinking skills, particularly when supported by high levels of learning independence. Therefore, future research is recommended to investigate other moderating variables and to examine the effectiveness of this model across different mathematical topics and educational levels.

Keywords: Discovery learning; mathematical reflective thinking; learning independence; mathematics learning.

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Abstrak

Penelitian mengenai kemampuan berpikir reflektif matematis umumnya berfokus pada peningkatan hasil belajar melalui berbagai model pembelajaran, namun masih terbatas dalam mengkaji peran kemandirian belajar sebagai variabel moderasi yang dapat mempengaruhi efektivitas model discovery learning, khususnya pada materi Sistem Persamaan Linear Dua Variabel (SPLDV). Oleh karena itu, penelitian ini bertujuan untuk menilai seberapa efektif model Discovery Learning dalam meningkatkan kemampuan berpikir reflektif matematika siswa SMA dengan mempertimbangkan kemandirian belajar sebagai variabel moderasi. Penelitian ini menggunakan pendekatan kuantitatif dengan desain kelompok kontrol non-ekuivalen, yang melibatkan kelompok eksperimen yang diajar melalui Discovery Learning dan kelompok kontrol yang diajar melalui model ekspositori. Data dikumpulkan menggunakan tes berpikir reflektif matematika dan kuesioner kemandirian belajar, yang keduanya telah divalidasi dan terbukti reliabel. Analisis data meliputi statistik deskriptif, uji normalitas dan homogenitas, serta pengujian hipotesis menggunakan uji t sampel independen, analisis N-gain, dan ANOVA dua arah. Hasil penelitian menunjukkan bahwa kedua kelas memiliki kesamaan dalam kemampuan awal, setelah diberi perlakuan, terdapat perbedaan signifikan dengan kelas eksperimen memperoleh skor post-test dan peningkatan N-Gain yang lebih tinggi daripada kelas kontrol. Selain itu, kemampuan berpikir reflektif matematika bervariasi secara signifikan berdasarkan tingkat kemandirian belajar, dengan siswa dengan kemandirian tinggi memperoleh hasil yang lebih unggul. Hubungan antara model pembelajaran dan tingkat kemandirian belajar pada keterampilan berpikir reflektif matematika juga terbukti signifikan. Penelitian ini menunjukkan bahwa model Discovery Learning efektif digunakan sebagai strategi pembelajaran untuk meningkatkan kemampuan berpikir reflektif matematis siswa. Oleh karena itu, penelitian selanjutnya disarankan untuk mengkaji variabel moderasi lainnya serta menguji penerapan model ini pada materi dan jenjang pendidikan yang berbeda.

INTRODUCTION

Mathematics as a subject plays a crucial role in developing students' methodical, logical, critical, and reflective thinking skills (Marfu'ah et al., 2022; Riswadi & Adirakasiwi, 2023). In addition to contributing to academic achievement, mastery of mathematics also serves as a crucial foundation for accelerating the development of science and technology in the context of a global society (Aminah & Rohayati, 2021; Januariawan et al., 2020). In the context of 21st-century learning, higher-order cognitive skills (HOTS), particularly reflective mathematical thinking skills, are often crucial indicators of academic success, as they enable students to analyze, evaluate, and find solutions to problems (Lilis Susanti et al., 2023; Susanna et al., 2026).

Mathematical reflective thinking skills not only contribute to academic success but are also relevant in everyday life because they support logical reasoning-based decision-making (Dewita & Ardiyanti, 2025; Olugbade et al., 2022). This is in line with (Hsia & Hwang, 2020), which shows that reflective thinking skills contribute to increased learning engagement awareness, allowing students to better monitor and evaluate their learning process independently. However, the results of the 2023 TIMSS international study showed that Indonesian students' problem-solving competency levels were below international standards. This finding exposes that students' mathematical reflective thinking capacity has not developed optimally and still requires further development. This condition is reinforced by students' tendency or habit of still being oriented towards completing various routine, procedural-based exercises, resulting in students experiencing difficulties when working on problems that require in-depth reasoning and reflection on the solution process (Rachmasari & Tirto Mada Wistama, 2020).

Based on the results of the study, in the form of initial observations at a senior high school (SMA) in Padarincang District, it showed that out of 30 students, only about 10 were able to solve problems that required

mathematical reflective thinking skills. This finding indicates a gap between competency expectations and empirical conditions in the field, so that updates in learning strategies are needed that can activate students' cognitive engagement independently, critically, and reflectively. One of the variables that plays a role is learning independence (Werdiningsih & Khoerunisa, 2021). Where students who demonstrate a high level of learning independence tend to have initiative, and are able to manage learning strategies, monitor the learning process, and have confidence in solving problems (Basyari et al., 2021; Lensi Hariyani et al., 2024; Noor et al., 2025; Saprizal & Nindiasari, 2021). (Losenno et al., 2020) emphasized that the ability to manage emotions, such as through cognitive reappraisal, contributes to the effectiveness of self-regulated learning. This explains why students with high independence show more optimal reflective performance.

In addition to internal student factors, external factors, such as learning models, also play a crucial role in optimizing reflective mathematical thinking skills. In educational institutions, the expository learning model is still predominantly used, tending to position students as recipients of information, thus providing little space for students to construct knowledge independently (Aisyah et al., 2025). Conversely, the discovery learning model encourages student involvement in discovering concepts and gathering information through exploration, analysis, and reflection (Khasinah, 2021). This has the potential to develop reflective mathematical thinking skills while simultaneously strengthening students' independence in the learning process (Cahyani et al., 2025).

Several previous studies have shown that the discovery learning model is effective in developing students' critical thinking skills and achieving mathematics learning outcomes (Akrom et al., 2023; Mahardika et al., 2024). Furthermore, several studies have examined the effectiveness of implementing the discovery learning model on mathematical reflective thinking competencies, considering that student learning independence is still very limited (Dewita & Ardiyanti, 2025; Husain et al., 2024). However, a synthesis of these findings shows that there are still limitations in research that integrates these three aspects simultaneously, namely the effectiveness of the discovery learning model on mathematical reflective thinking skills by considering learning independence as a moderating variable. Thus, a comprehensive study linking learning models, reflective thinking skills, and learning independence factors within a single analytical framework is not yet optimal.

Based on this explanation, this study aims to evaluate the effectiveness of the discovery learning model compared to the expository learning model in improving mathematical reflective thinking skills in the Two-Variable Linear Equation System (SPLDV) topic, considering student learning independence as a moderating variable.

This study emphasizes the integration of the discovery learning model, mathematical reflective thinking skills, and learning independence in a single quantitative experimental design, which has tended to be studied separately in previous research. This research's scientific contribution is expected to enrich the study of mathematics learning, particularly by providing empirical evidence regarding the importance of simultaneously considering cognitive and affective

factors in designing effective learning strategies. Furthermore, the results of this study are also expected to serve as a basis for developing more innovative mathematics learning practices oriented toward improving students' reflective thinking skills in accordance with the demands of 21st-century learning.

METHOD

The research framework employed a quasi-experimental design with a quantitative experimental approach to examine the effect of the Discovery Learning model on students' mathematical reflective thinking skills while considering the level of learning independence as a moderating variable. This approach is considered appropriate for providing an objective explanation of the causal relationship between the variables investigated (Selvira & Albina, 2025).

The study was conducted on participants selected based on sampling criteria that aligned with the research objectives. The participants shared comparable characteristics, including relatively equivalent prior mathematical ability (as evidenced by the pretest results), enrollment in schools with homogeneous academic conditions, and the same grade level. The sample was selected using a purposive sampling technique based on these characteristics, followed by a simple group assignment to determine the experimental and control classes.

Data were collected through three stages: pretest, treatment, and posttest. The experimental class received instruction using the Discovery Learning model, while the control class was taught using the expository learning model, as illustrated in Figure 1. In addition, a learning independence questionnaire was administered to classify students according to their level of learning independence. The overall research procedure is presented in the flowchart shown in Figure 2, illustrating each stage of the study, from sample selection to data analysis.

(a)

(b)

Figure 1. a). Implementation of the Discovery Learning Method in the Experimental Class, b). Implementation of the Expository Method in the Control Class

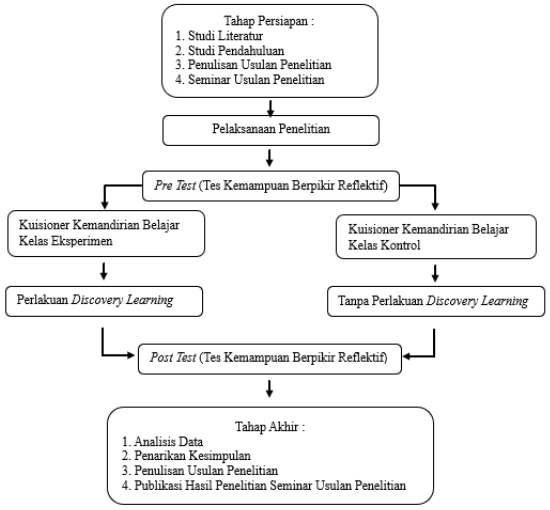

Figure 2. Research Flow

Data analysis was conducted in two stages: descriptive statistics and inferential statistics. Descriptive statistics

were used to summarize the data through the calculation of the mean, median, standard deviation, and range of scores. Subsequently, prerequisite tests were performed, including a normality test (Kolmogorov–Smirnov or Shapiro–Wilk) and a homogeneity test (Levene's test) at a significance level of 0.05. Hypothesis testing was carried out using an independent samples t-test to determine differences in mean scores between the two groups, N-gain analysis to measure the improvement in students' abilities, and Two-Way ANOVA to examine the effect of the learning model and its interaction with students' learning independence on their mathematical reflective thinking skills. All statistical analyses were performed using statistical software to ensure that the results were objective, accurate, and reliable.

RESULT AND DISCUSSION

Results

Overall, the findings indicate that the implementation of the Discovery Learning model was more effective in improving students' mathematical reflective thinking skills than the expository learning model. This improvement was reflected not only in the students' post-instruction performance but was also influenced by their level of learning independence, with students demonstrating high learning independence achieving better outcomes (Ab, 2020). These findings are consistent with those of (Waite et al., 2020), who reported that metacognitive awareness plays a crucial role in optimizing reflective processes and decision-making during learning. Furthermore, an interaction effect was identified between the learning model and students' learning independence, indicating that both factors jointly contributed to the improvement of mathematical reflective thinking skills.

The analysis stage included an evaluation of the quality of the research instrument through tests of validity, reliability, item difficulty, and discrimination index. The validity test was conducted to ensure that each test item appropriately measured students' mathematical reflective thinking skills, whereas the reliability test was used to determine the consistency of the measurement results (Kurniawati & Yuhana, 2024). The item difficulty analysis was performed to determine the proportional level of difficulty of each question, while the discrimination index analysis assessed the ability of each item to distinguish between students with high and low levels of ability (Rachmawati & Bayu Pradana, 2025).

The results of the instrument analysis showed that all test items met the required criteria for validity and reliability, possessed appropriate levels of difficulty, and demonstrated satisfactory discrimination indices. Therefore, the instrument was considered suitable for use in this study. These findings ensure that the collected data accurately represent the construct of mathematical reflective thinking skills being measured.

Descriptive statistical analysis was conducted to provide an overview of students' initial abilities (pretest) and learning outcomes after the instructional treatment (posttest) in both research groups. As presented in Table 1, the experimental group obtained a mean pretest score of 36.67, with a standard deviation of 16.17 and a variance of 261.471. Meanwhile, the control group achieved a mean pretest score of 42.17, with a standard deviation of 20.238 and a variance of 409.592. The relatively small difference in the mean scores indicates that the two groups had comparable

initial mathematical abilities, although the control group exhibited slightly greater variability in performance (Rachmawati & Bayu Pradana, 2025).

Following the instructional treatment, Table 2 shows that the experimental group's mean posttest score increased substantially to 71.50, with a standard deviation of 13.159 and a variance of 173.155. In comparison, the control group's mean posttest score improved to 57.20, with a standard deviation of 18.361 and a variance of 337.131. Although both groups demonstrated improvement after the intervention, the experimental group achieved a considerably greater increase in performance and exhibited a more homogeneous distribution of scores than the control group, suggesting the superior effectiveness of the Discovery Learning model in enhancing students' mathematical reflective thinking skills.

Table 1. Descriptive Statistics Based on Pretest Scores

Group	Descriptive Statistics			
	N	***Mean***	***Std. Dev.***	**Variance**
Experimental	30	36.67	16.17	261.471
Control	30	42.17	20.238	409.592

Table 1. Descriptive Statistics Based on Posttest Scores

Group	Descriptive Statistics			
	N	***Mean***	***Std. Dev.***	**Variance**
Experimental	30	71.50	13.159	173.155
Control	30	57.20	18.361	337.131

In addition, students' improvement in mathematical reflective thinking skills was analyzed using the N-Gain index. As presented in Table 3, the experimental group obtained an average N-Gain score of 57.78, which falls into the moderate category, whereas the control group achieved an average N-Gain score of 28.44, classified as low. These findings indicate that the instructional intervention implemented in the experimental group was more effective than the expository instruction used in the control group. The distribution of N-Gain categories presented in Table 4 further supports these findings. Most students in the experimental group were classified within the moderate to high improvement categories, while the majority of students in the control group remained within the low to moderate categories. These results are consistent with the findings reported by (Warda, 2018), which demonstrated that Discovery Learning is effective in promoting higher learning gains than conventional instructional approaches. Therefore, based on the descriptive statistical analysis, the Discovery Learning model contributed more substantially to improving students' mathematical reflective thinking skills, supporting the findings of previous studies (Amri et al., 2025).

Tabel 3. Statistika Deskriptif berdasarkan Nilai *N-Gain*

Class	N	Min	Max	Mean	Std. Deviation
E	30	29	100	57.78	15.746
C	30	0	50	28.44	15.110

E: Experiment
C: Control

Tabel 4. Klasifikasi Siswa berdasarkan Kriteria N-Gain

Kelas	Rendah	Sedang	Tinggi
Eksperimen	1	23	6
Kontrol	15	15	-

Prior to hypothesis testing, prerequisite analyses, including tests of normality and homogeneity, were conducted to ensure that the assumptions for parametric statistical analysis were satisfied. As shown in Table 5, the normality test of the pretest data yielded significance values of 0.139 for the experimental group and 0.089 for the control group. Since both values exceeded the significance threshold of 0.05, the pretest scores for both groups were considered to be normally distributed. Similarly, as presented in Table 6, the normality test of the posttest data produced significance values of 0.097 for the experimental group and 0.112 for the control group. As both values were greater than 0.05, the posttest data were also confirmed to follow a normal distribution. Therefore, the assumption of normality required for parametric analysis was satisfied.

Tabel 5. Hasil Uji Normalitas Data Pretest.

Kelas	*Sig.*	Keputusan	Keterangan
Eksperimen	0.139	H_o = *diterima*	Berdistribusi Normal
Kontrol	0.89	H_o = *diterima*	Berdistribusi Normal

Tabel 6. Hasil Uji Normalitas Data Postest

Kelas	*Sig.*	Keputusan	Keterangan
Eksperimen	0.097	H_o = *diterima*	Berdistribusi Normal
Kontrol	0.112	H_o = *diterima*	Berdistribusi Normal

Tabel 7. Hasil Uji Homogenitas Data Pretest

Levene	df1	df2	*Sig.*
1.328	1	58	0.254

Tabel 8. Hasil Uji Homogenitas Data Postest.

Levene	df1	df2	*Sig.*
2.121	1	58	0.151

In addition, a homogeneity test was conducted to examine the equality of variances between the two groups. Based on Table 7, the homogeneity test for the pretest data resulted in a significance value of 0.254 (> 0.05). Likewise, Table 8 shows that the homogeneity test for the posttest data produced a significance value of 0.151 (> 0.05). These findings indicate that the variances of the experimental and control groups were homogeneous. Since both the normality and homogeneity assumptions were fulfilled, the subsequent analyses were performed using parametric statistical procedures.

Hypothesis testing was conducted using an independent samples t-test to examine differences in the mean scores between the experimental and control groups. As shown in Table 9, the pretest comparison yielded a significance value of 0.250 (> 0.05), leading to the acceptance of the null hypothesis (H_0). This result indicates that there was no statistically significant difference in the students' initial mathematical reflective thinking skills between the experimental and control groups, confirming that both groups were equivalent before the intervention. However, following the instructional treatment, Table 10 shows that the significance value for the posttest comparison was 0.001 (< 0.05), resulting in the rejection of the null hypothesis (H_0). This finding indicates a

statistically significant difference in students' mathematical reflective thinking skills between the two groups. Specifically, students in the experimental group who were taught using the Discovery Learning model achieved significantly higher performance than those in the control group who received expository instruction.

A two-way ANOVA was conducted as a subsequent inferential analysis to examine the effects of the learning model, students' level of learning independence, and the interaction between these two factors on students' mathematical reflective thinking skills. As presented in Table 11, the significance value for the learning model factor was 0.001 (< 0.05), indicating a statistically significant difference between students who were taught using the Discovery Learning model and those who received expository instruction. The level of learning independence also had a significant effect, with a significance value of 0.000 (< 0.05). Students with a high level of learning independence demonstrated significantly better mathematical reflective thinking skills than those with moderate and low levels of learning independence. Furthermore, the interaction between the learning model and learning independence yielded a significance value of 0.042 (< 0.05), indicating a statistically significant interaction effect. This finding suggests that the effectiveness of the Discovery Learning model is influenced by students' readiness and level of learning independence. In other words, the impact of the learning model on students' mathematical reflective thinking skills varies according to their level of learning independence. Therefore, effective mathematics instruction should not only emphasize the selection of appropriate instructional strategies but also consider students' individual characteristics, particularly their learning independence, to maximize learning outcomes.

Discussion

The results of the data analysis revealed a significant difference in students' mathematical reflective thinking skills between those who learned through the Discovery Learning model and those who received expository instruction. The findings suggest that student-centered learning, which actively engages learners in the learning process, promotes the development of higher-order thinking skills, particularly mathematical reflective thinking. The greater improvement observed in the experimental group, as reflected in both the posttest scores and N-Gain values, indicates that the Discovery Learning model provides a more meaningful learning experience. Consequently, students not only acquire conceptual understanding but also develop the ability to critically reflect on their own thinking processes. Active participation throughout the learning process further facilitates the development of metacognitive skills through exploration, analysis, and evaluation of problem-solving strategies (Oktavia et al., 2024). In contrast, expository instruction, which is predominantly teacher-centered (Suweta, 2020), provides fewer opportunities for students to develop reflective thinking skills. These findings are consistent with constructivist learning theory and Bruner's theory of discovery learning, which emphasize that knowledge is actively constructed through the process of discovery. Accordingly, the results reinforce the superiority of discovery-based learning over transmissive instructional approaches in fostering students'

mathematical reflective thinking skills.

Students' level of learning independence also exhibited a clear pattern, with students possessing high learning independence demonstrating significantly better mathematical reflective thinking skills than those with moderate or low levels of learning independence. This finding suggests that learning independence, as an internal learner characteristic, plays a substantial role in students' academic success by enabling them to regulate and manage their own learning processes, in accordance with the principles of self-regulated learning. Furthermore, the two-way ANOVA revealed a significant interaction between the learning model and students' learning independence, corroborating the findings of previous studies (Maifi et al., 2021; Primantiko et al., 2021; Sitorus, 2021; Helena et al., 2025). This interaction indicates that the effectiveness of the Discovery Learning model depends on students' level of learning independence. Students with high learning independence benefited more substantially from discovery-based learning in developing mathematical reflective thinking and problem-solving skills (Cheng et al., 2025; Nugroho et al., 2021), whereas students with lower levels of learning independence required more intensive instructional guidance to achieve comparable learning outcomes.

Overall, the findings support all of the proposed research hypotheses. Specifically, the Discovery Learning model was found to be more effective than the expository learning model in enhancing students' mathematical reflective thinking skills. In addition, mathematical reflective thinking skills differed significantly across levels of learning independence, and a significant interaction effect was identified between the learning model and students' learning independence. These findings highlight that the effectiveness of instructional strategies depends not only on the learning model itself but also on individual learner characteristics, particularly learning independence. Therefore, mathematics instruction should integrate student-centered learning approaches while simultaneously fostering students' learning independence to maximize the development of mathematical reflective thinking skills.

The Effectiveness of the Discovery Learning Model in Improving Students' Mathematical Reflective Thinking Skills

The results of the data analysis indicate that students who participated in discovery learning demonstrated greater improvement in mathematical reflective thinking skills than those who received expository instruction. This finding is evidenced by the significant differences in both the posttest mean scores and N-gain values between the experimental and control groups. These results suggest that the implementation of the discovery learning model is effective in enhancing students' mathematical reflective thinking skills (Peranginangin, 2026; Versteeg et al., 2021; Lazonder & Harmsen, 2016).

From a theoretical perspective, these findings are consistent with Bruner's constructivist theory, which emphasizes that knowledge becomes more meaningful when learners acquire it through the process of discovery. Throughout the stages of observing, identifying problems, collecting and processing data, verifying findings, and drawing conclusions, students actively construct their own understanding. This learning process encourages them to reflect on their thinking strategies,

evaluate alternative solutions, and develop mathematical reflective thinking skills more effectively.

Differences in Mathematical Reflective Thinking Skills Based on Students' Learning Independence

The findings also reveal that students with a high level of learning independence demonstrated significantly better mathematical reflective thinking skills than those with moderate or low levels of learning independence.

Students with high learning independence tend to regulate, monitor, and evaluate their own learning processes more effectively. As a result, they become more actively engaged in understanding mathematical concepts, reflecting on their thinking processes, and evaluating problem-solving strategies. In contrast, students with low learning independence generally rely more heavily on teacher guidance, which limits their opportunities to engage in reflective thinking and self-evaluation during problem solving. These findings are in line with the self-regulated learning theory, which posits that the ability to regulate cognitive, motivational, and behavioral aspects of learning contributes substantially to academic achievement (Harris et al., 2015; Panadero, 2017). Therefore, fostering students' learning independence should be regarded as an essential component of effective mathematics instruction.

The Interaction Between Discovery Learning and Students' Learning Independence

HThe results of the two-way ANOVA revealed a statistically significant interaction between the implementation of the discovery learning model and students' level of learning independence in influencing mathematical reflective thinking skills. This finding indicates that the effectiveness of the instructional model depends not only on the learning approach itself but also on students' individual characteristics, particularly their level of learning independence.

Empirically, students with high learning independence experienced greater improvement in mathematical reflective thinking skills through discovery learning than through expository instruction. Nevertheless, students with moderate and low levels of learning independence also showed improvements, although their gains were smaller than those of students with high learning independence. These findings suggest that discovery learning is sufficiently flexible to accommodate differences in students' learning characteristics. However, students with lower levels of learning independence still require appropriate instructional scaffolding, such as structured guidance, guiding questions, and timely feedback throughout the learning process. Such support enables these students to participate more actively in discovery activities while gradually developing greater independence and stronger mathematical reflective thinking skills.

Implication

The findings of this study suggest that the implementation of the discovery learning model should be accompanied by adaptive and differentiated instructional strategies. Teachers should not only serve as facilitators but also as instructional guides who are able to adjust the level of support according to students' individual characteristics. (Tongchai, 2021) emphasized that teachers' professional development plays a crucial role in enhancing the quality of

instructional strategies that are responsive to students' diverse learning needs. Within the framework of Indonesia's Merdeka Curriculum, this approach is consistent with the principle of student-centered learning that accommodates individual differences among learners. Therefore, integrating the discovery learning model with the promotion of students' learning independence and the development of metacognitive skills is essential for optimizing students' mathematical reflective thinking skills (Stephanou & Mpiontini, 2017).

Limitation

This study has several limitations. First, the sample was limited to students from a single school and focused exclusively on the topic of SPLDV. Consequently, the generalizability of the findings should be interpreted with caution. Second, this study examined only learning independence as a moderating variable, whereas other factors that may influence students' mathematical reflective thinking skills were not investigated. Therefore, future research is recommended to involve more diverse and representative samples, examine different mathematical topics, and incorporate additional variables that may contribute to the development of mathematical reflective thinking skills.

CONCLUSION

The management of empirical data and the research findings indicate that the implementation of the discovery learning model is more effective than the expository learning model in improving students' mathematical reflective thinking skills on the topic of SPLDV. This is evidenced by the significant differences in the posttest results and the improvement in students' abilities (N-Gain) between the experimental and control groups. These findings confirm that learning which positions students as active participants in the process of discovering concepts is able to promote deeper understanding and optimize students' mathematical reflective thinking skills. Furthermore, learning independence was found to contribute to these skills, as students with a high level of learning independence demonstrated better mathematical reflective thinking skills than those with moderate and low levels of learning independence. This study also found a significant interaction between the discovery learning model and students' level of learning independence, indicating that the effectiveness of the discovery learning model is influenced by students' learning readiness and maturity. These findings strengthen the constructivist perspective in mathematics education by demonstrating that the effectiveness of discovery learning is determined not only by the characteristics of the learning model but also by students' internal factors, particularly learning independence. The integration of the learning model and learning independence in explaining students' mathematical reflective thinking skills represents the novelty of this study and expands the understanding of the relationship between cognitive and affective aspects in the learning process.

Practically, the findings of this study imply that teachers need to design learning activities that not only emphasize the discovery of concepts but also encourage the development of students' learning independence to achieve more optimal learning outcomes. Therefore, the implementation of discovery learning, supported by the enhancement of students' learning

independence, can serve as an effective strategy for improving the quality of mathematics education in accordance with the demands of the 21st century.

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