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Development of a Local Instruction Theory for Trigonometric Ratios

Abstract

3 Some students find trigonometric ratios challenging. Research on the application of Local Instruction Theory (LIT) in trigonometry is limited, particularly in secondary school understanding of trigonometric ratios. This study aims to develop an LIT for trigonometric ratios using Realistic Mathematics Education (RME). The researcher designed a learning pathway to help students grasp the fundamental concepts of trigonometric ratios. The study employs a research design methodology, developing a Hypothetical Learning Trajectory (HLT) to improve students' understanding. The development of LIT for Trigonometric Ratios follows three stages: initial design, teaching experiments, and retrospective analysis. Students demonstrated the ability to understand trigonometric ratios through the learning process. The findings suggest that the use of LIT-based instructional materials, incorporating RME principles, significantly enhances students' conceptual understanding of trigonometric ratios in high school.

Keywords: Local Instruction Theory; Hypothetical Learning Trajectory; Trigonometric Ratios; RME

Abstrak

Beberapa siswa menganggap materi perbandingan trigonometri cukup sulit. Penelitian tentang penerapan Local Instruction Theory (LIT) dalam pembelajaran trigonometri masih sangat terbatas, khususnya terkait dengan pemahaman rasio trigonometri di sekolah menengah. Penelitian ini bertujuan untuk mengembangkan Local Instruction Theory (LIT) perbandingan trigonometri dengan menggunakan Realistic Mathematic Education (RME). Dalam upaya membantu siswa membangun konsep dasar pada materi perbandingan trigonometri, peneliti mengembangkan alur LIT dengan menemukan jalur pembelajaran yang efektif. Pencapaian tujuan penelitian menggunakan desain penelitian. Serangkaian kegiatan mengembangkan Hypothetical Learning Trajectory (HLT) sehingga siswa sekolah menengah atas (SMA) memiliki pemahaman yang lebih baik tentang perbandingan trigonometri. Pengembangan Local Instruction Theory untuk Perbandingan Trigonometri meliputi tiga tahap yaitu 12 *mengembangkan desain awal, melakukan percobaan pengajaran, dan melaksanakan analisis retrospektif. Siswa mampu membangun pemahaman tentang perbandingan trigonometri selama proses pembelajaran berlangsung. Berdasarkan analisis kualitatif eksperimen pengajaran, penelitian ini berimplikasi pada penguatan Local Instruction Theory (LIT) perbandingan trigonometri dan pengembangan bahan ajar berbasis RME.*

INTRODUCTION

The topic of trigonometric ratios is fundamental in the application of advanced applied sciences and pure sciences. Various fields of study require the use of trigonometry as an essential basis for applying their concepts. For example, the utilization of trigonometric ratios supports the proof of Kepler's first law (Simha, 2021). The importance of understanding trigonometric ratios lies in its role as fundamental knowledge for comprehending pre-calculus and calculus, such as algebra, geometry, and the ability to create graphs (Weber, 2005). Trigonometric comparison is utilized in calculating the width or height of various shapes, such as a bridge, household tools, and bicycles (Urrutia et al., 2019). Indeed, students see the topic of trigonometric ratios as challenging (Blackett & Tall, 1991; Weber, 2005; Yiğit Koyunkaya, 2016; Karthikeyan, 2017; Urrutia et al., 2019; Nordlander, 2022).

During the initial stage of learning, students encounter difficulties in connecting triangle images to numerical relationships, as well as problems in manipulating the involved symbols (Blackett & Tall, 1991), and many students are unable to overcome these difficulties. While studying right triangles, students struggled to grasp the concept of trigonometric functions, such as sine, cosine, and tangent, which are defined as ratios (Weber, 2005a, 2008). According to Yiğit (2016), students exhibit a lack of flexibility when solving trigonometric comparison problems (Yiğit Koyunkaya, 2016). Students expressed that trigonometry was a subject that was highly stimulating and conceptual in comparison to other areas of mathematics (Karthikeyan, 2017). According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Urrutia et al., 2019). Typically, students struggle to distinguish the front side, side view, and slanted side of a right triangle even though this distinction greatly aids their comprehension of trigonometric ratios. The topic of trigonometric comparison is considered abstract for students (Karthikeyan, 2017).

From the preliminary study and literature review conducted by the researcher on textbooks, instructional videos, analysis of students' learning difficulties, and curriculum analysis, several aspects have caught the researcher's attention. One of these aspects is the emergence of real-world contexts in trigonometry materials in textbooks. Various examples have begun to present contextual situations, but teachers still need to learn how students might construct formal concepts from the presented contextual case. Batanero dan Diaz (2012) argue that the existing textbooks and curriculum documents need to sufficiently support learning, as the presented concepts are narrow in scope (Batanero & Díaz, 2012).

The typical pattern of learning that occurs in the delivery of contextual mathematics is the initial introduction and presentation of example problems, followed by the completion of practice exercises using general formulas. The typical pattern of learning still represents the teaching culture in carrying out the learning process. According to Weber (2005), mathematics learning in schools still follows the conventional sequence of explanation, example provision, and problem-solving practice. The learning process via direct application of general formulas can prevent students from developing their mathematical concepts independently (Weber, 2005b). Their thinking patterns need to be trained, resulting in a lack of familiarity with how these formulas are derived.

The application of real-life models to formal mathematical concepts has yet to be documented in supporting learning materials. The teacher must possess the ability to elucidate the models generated by students during the learning process, which can subsequently facilitate the comprehension of formal mathematics. Turmudi (2018) states that students should have a strong desire in the learning process to independently discover concepts, if teachers provide facilities, teaching materials, and learning resources (Turmudi, 2012).

According to Sandip's research, trigonometry is a challenging subject that is disconnected from everyday life (Adhikari & Subedi, 2021; Asomah et al., 2022; Dhungana et al., 2023). To bridge this gap, Realistic Mathematics Education serves as an alternative to teaching trigonometric ratios. Realistic Mathematics Education (RME) is a teaching and learning method in math education that follows the philosophy of Hans Freudenthal, who sees math as a human endeavor. (Freudenthal, 1973; Gravemeijer & Terwel, 2000; Treffers, 1987; Van den Heuvel-Panhuizen & Drijvers, 2020).

Realistic Mathematics Education is characterized by students experiencing a concrete learning process that is aligned with real-life situations and contextualized according to their everyday lives (Yuanita et al., 2018). Students undergo an informal process of constructing their understanding based on what they see, observe, and comprehend about their surrounding environment, be it at home, school, or wherever they learn (Meika, 2018).

The three primary ideas of Realistic Mathematics Education are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. (Gravemeijer, 1994). During the teaching and learning process, these three principles are elaborated into five characteristics of RME, namely: 1) Building and solidifying; 2) degree and frameworks; 3) contemplation and specific task; 4) societal environment and communication; and 5) organizing and intertwining (Gravemeijer, 1994; Lange, 1987). The significance of the Realistic Mathematics Education approach resides in its ability to empower students to develop concepts through interactive teaching and learning processes. The teachers have a crucial role in enabling students to explore concepts using both administrative and motivational methods (Meika, 2018).

The mistakes made by previous students should not be repeated in the following years. The provision of instructional materials is an effective solution for improving the situation and facilitating recovery. The focus of learning will be further narrowed down if the instructional materials used are appropriate. In this teaching material, steps are needed that can facilitate students' learning paths. According to Gravemeijer, to improve mathematics education, it is necessary to implement an instructional design that facilitates learning and enables students to progress from their current level of thinking to a more sophisticated level of mathematical thinking (Gravemeijer, 2004).

Local Instruction Theory (LIT) is a comprehensive theory that includes both the process of acquiring knowledge on a specific topic and the methods used to facilitate that learning (Liljekvist et al., 2017). LIT was developed in the 1990s as part of an ongoing research initiative into the adaptation of instructional strategies within mathematics education. LIT has been primarily applied in various educational contexts, particularly in mathematics classrooms across several countries, including the Netherlands, where the theory was initially developed. It is also relevant to the Indonesian educational context, as demonstrated in our study. Gravemeijer states that Local Instruction Theory is an exact theory that can assist students in developing mathematical thinking on a particular topic or material through detailed and gradual instructional medium (Gravemeijer, 2004). Researchers will develop a systematic learning path, also known as a learning trajectory, within the process of teaching and learning. This design is the one Gravemeijer mentioned.

LIT is used to describe the appropriate learning route or learning activities. LIT not only offers how learning takes place but also facilitates a range of specific learning activities that may work. The existence of LIT provides a basis for teachers to consider adaptations that may be needed in implementing new approaches. In addition, LIT offers a framework in which teachers can develop hypotheses about learning trajectories. LIT is tailored to the specific circumstances of the particular class and teacher. Teachers need to build HLT (Hypothetical Learning Trajectories) that are suitable for their own situations while using LIT as a foundational reference (Gravemeijer, 2004).

Understanding and applying (LIT) is essential for teachers as it serves as a bridge between theoretical frameworks and practical classroom instruction. LIT provides a structured approach to creating Hypothetical Learning Trajectories (HLTs), which help teachers anticipate and guide students' learning processes while adapting to their evolving needs. This process enables educators to align instructional activities with students' thinking and learning paths, ensuring a deeper and more meaningful understanding of mathematical concepts.

For example, guided reinvention and mathematization are central aspects of LIT. These strategies empower students to develop informal problem-solving methods that can be refined into formal mathematical reasoning, fostering their ability to connect classroom learning with real-world applications. Teachers' proficiency in LIT also facilitates the identification and addressing of misconceptions, promoting continuous conceptual development (Gravemeijer & Doorman, 1999; Van Den Heuvel-Panhuizen, 2002).

Furthermore, professional development programs incorporating LIT principles have demonstrated that teachers equipped with this knowledge are better prepared to design effective and flexible instructional sequences tailored to their students' needs. These programs emphasize the importance of using LIT to design activities that encourage reasoning, discussion, and reflection, which are key for fostering higher-order thinking skills (Gravemeijer, 2004).

The implementation of Local Instruction Theory (LIT) by teachers is often done through a contextual activity-based approach to help students understand mathematical concepts. For example, in teaching the concept of combination with the RME approach, teachers can use demonstrations such as handshakes or color mixing to connect abstract concepts to real-world situations, which have been shown to be effective in improving student understanding. However, teachers face challenges in adjusting learning designs to meet the needs of heterogeneous students, especially in aligning the Hypothetical Learning Trajectory (HLT) with classroom dynamics. Teachers have difficulty in designing learning activities that are contextual enough and engage students at a deeper level of engagement (Meika et al., 2019).

Based on preliminary research conducted by the researcher, no teacher at this school has ever implemented *Local Instruction Theory* (LIT) in teaching, particularly in the topic of trigonometric ratios. The lack of training and understanding of how to apply LIT to the curriculum is the main reason why teachers have not utilized this approach. As Gravemeijer notes, although LIT has proven effective in certain educational contexts, its implementation requires a deep understanding of its principles and methods. Teachers often face challenges when trying to apply innovative teaching strategies like LIT due to limited opportunities for professional development and curriculum constraints (Gravemeijer & Doorman, 1999). Furthermore, adapting LIT to specific topics, such as trigonometric ratios, can be a barrier if teachers do not have the necessary tools and support. Therefore, the lack of LIT implementation in this school makes it a relevant area for further research development.

HLT is developed for each learning activity and plays a crucial role in designing student learning tasks. The implementation of teaching is closely tied to the learning trajectory, which includes a planned sequence of learning content and the conceptual map students navigate during the learning process (Prahmana, 2016). In the subsequent phase, a Local Instruction Theory (LIT) for trigonometric ratios is developed.

Based on the explanation above, there is a need for the development and design of instructional materials that can minimize students' difficulties in developing thinking processes through Realistic Mathematics Education (RME) to construct an understanding of trigonometry content. These instructional materials are expected to be obtained through the design of LIT. This approach aims to address several challenges faced by students and to foster the development of their mathematical thinking, particularly in understanding trigonometric ratios through Realistic Mathematics Education.

According to the given definition, it is crucial to create and design instructional resources that can reduce students' challenges in developing cognitive processes through Realistic Mathematics Education to build comprehension of trigonometric concepts. The desired teaching materials are obtained through the LIT design. The LIT design is used to address the difficulties experienced by students and to enhance their mathematical thinking process, particularly their understanding of trigonometric ratios through Realistic Mathematics Education.

METHOD

In the book *Educational Design Research* by Koeno Gravemeijer, the conclusion highlights the importance of design research in bridging theory and practice within education. Design research allows for the development of innovative and contextually relevant learning tools that can be adapted and tested in real classroom environments. One of its key strengths is the ability to generate new theories based on real-world learning experiences, emphasizing the interaction between students, teachers, and learning materials (Nieveen et al., 2006).

Moreover, design research provides a framework for continuous reflection and improvement of teaching practices and learning strategies through iterative cycles of development, implementation, and evaluation. This ensures that the educational solutions developed are both effective and adaptable, ultimately enhancing the overall quality of education. Through this process, design research contributes to creating educational solutions that are both grounded in theory and practical in diverse educational settings.

A design research method based on Gravemeijer's approach is the research methodology used. This research method is focused on local instruction theory, or LIT, which was developed by researchers in collaboration with teachers to achieve better learning quality (Gravemeijer & van Eerde, 2009). A study paradigm known as "design research" tries to create a series of tasks and comprehend the workings of the learning process. This study aims to develop and construct a Local Instruction Theory (LIT) on the subject of trigonometric ratios, with a focus on Realistic Mathematics Education (RME).

Gravemeijer proposes a research design for the development of local instruction theory. The research design consists of three phases: 1) preliminary design (developing initial design), 2) teaching experiment (doing teaching experiment), and 3) retrospective analysis (performing retrospective analysis) (Gravemeijer, 2004). Preliminary design is the outcome of an initial learning plan aimed at implementing ideas about the development of instructional materials for LIT (Meika, 2018). This design is expected to support students in the construction of mathematical thinking from an informal form to a formal one. The construction of mathematical thinking is achieved through a guided reinvention process facilitated by mathematizing. Mathematizing refers to several ways of managing learning activities to demonstrate mathematical characteristics, such as generality, certainty, accuracy, and agility (Loc & Hao, 2016). The purpose of the teaching experiment is to evaluate the effectiveness of the instructional materials for LIT. Retrospective analysis is employed to examine the complete dataset of the teaching experiment outcomes.

HLT is applied in every learning activity and has a crucial role in designing learning activities for students. The execution of learning is indissociable from the learning trajectory, encompassing a deliberate progression of learning materials and idea maps that students traverse during the learning process (Prahmana, 2016). Next, in the subsequent stage, the trigonometric ratios LIT is produced (see Figure 1).

The research implementation is carried out in two stages, with each stage referring to the steps of design research (Gravemeijer, 2004) namely: The three main stages of the project include 1) formulating the initial concept, 2) executing the educational experiment, and 3) carrying out a retrospective study. During the teaching experiment phase, this study involved 30 tenth-grade students from SMAN 7 Bandung. The selection of Grade 10 at SMAN 7 Bandung was based on the rationale that this class had already engaged with the national curriculum currently implemented in Indonesia, where trigonometric ratios constitute a key topic. Additionally, the school was chosen because it represents senior high schools that consistently adhere to the implementation of the national curriculum. This selection provided an opportunity to explore the application of trigonometric ratios within a context relevant to Indonesia's existing educational framework.

An analysis was conducted on the activities performed in the initial phase, and subsequently, recommendations were given to enhance the activities in the second phase. The findings from the data analysis of the second phase provide the final recommendations of this research.

Preliminary findings indicate that no teachers at this school have implemented Local Instruction Theory (LIT) in their teaching practices, particularly in the context of trigonometric ratios. The primary reasons for this include limited training opportunities and a lack of understanding regarding LIT's application in the classroom. Gravemeijer (1999) highlights that while LIT has proven effective in various educational contexts, its implementation requires a comprehensive understanding of its principles and methodologies. Teachers often face challenges in adopting innovative teaching strategies like LIT due to insufficient professional develop-

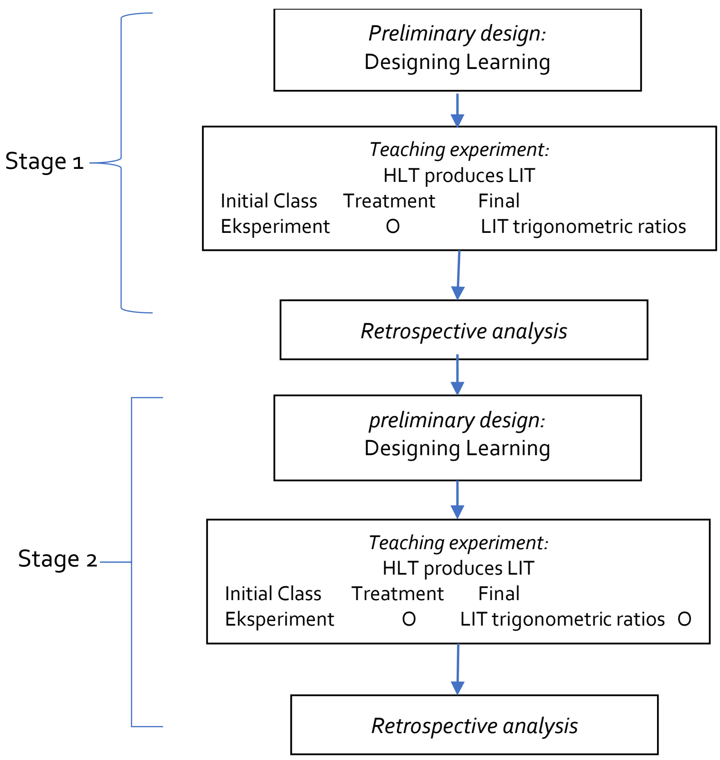

Figure 1. Research Design

ment opportunities and curriculum constraints. Additionally, adapting LIT to specific topics, such as trigonometric ratios, poses further challenges if educators lack adequate tools and support. Consequently, the absence of LIT's application for this topic at the school underscores the relevance and potential for further research and development in this area.

Data for this research was collected using multiple methods to gather comprehensive insights. First, written data were obtained from test results, tasks such as UBA-LIT and LKPD, and observation sheets during the teaching experiments. Second, documentation in the form of video recordings and photographs were used to observe classroom activities, student interactions, and the process of mathematization. Additionally, direct observation was conducted by the researcher, who also acted as the teacher, noting student responses during the learning process. Interviews with selected students were carried out to gather deeper insights into their experiences with UBA-LIT and LKPD. The data were analyzed qualitatively by comparing the observations and video recordings with the initial teaching design (HLT). Internal validity was ensured through data triangulation from various sources, while reliability was strengthened by testing the instruments and conducting cross-interpretation with experts. The conclusions were drawn from the retrospective analysis of the collected data, explaining how students grasped the concept of trigonometric ratios within the learning process using LIT.

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RESULTS AND DISCUSSION

Results

The research is carried out in three stages: 1) formulating the fundamental design, 2) executing the teaching experiment, and 3) implementing the retrospective analysis.

Elaborating on the Initial Design (preliminary design)

During this phase, the researcher assembles the educational materials and finalizes the instructional content. To meet the study aims, various actions are conducted, including (1) identifying the research subjects based on their diverse cognitive abilities. The researchers selected students from class X of SMAN 7 as appropriate research subjects; (2) examining and analyzing the curriculum to identify the compatibility of trigonometric comparison material with the curriculum currently being used in the school; (3) analyzing trigonometric comparison material to obtain an overview that "the achievement of Phase E-learning based on the derived geometry domain becomes the learning objective," in accordance with the research objectives. Next, the researcher developed a teaching material design in the form of a descriptive outline of trigonometric comparisons, also known as "Local Instruction Theory Descriptive Teaching Material (UBA-LIT)". The Realistic Mathematics Education (RME) technique enhances the learning process by employing diverse learning materials, including teaching modules and research tools. The study instrument is designed to incorporate the specific attributes of Realistic Mathematics Education (RME) and the curriculum used by the school. Additionally, the researchers analyze and assess the learning assistance tools. The result of the evaluation is referred to as the prototype. In the next stage, the research conducts an expert review, in which four experts validate the prototype based on its structure, content, and language.

Once the expert review procedure is finished, students who have been given trigonometric comparison information are tested on the prototype to get data on the readability and comprehension of the instructional material. The researchers engage in communication with the students during the testing process to anticipate any corrections or feedback regarding the instructional materials being tested by the students. The students write their input or suggestions on the prepared sheet. Students provide support for the discovery of formal concepts related to UBA-LIT through their comments. The researchers made improvements to the UBA-LIT after receiving feedback. After conducting expert validity testing and gathering comments from students, it was ultimately determined that the prototype design of the instructional material product generated by the researcher is valid at this level. The updated iteration of the prototype is referred to as the second prototype. Following that, the second prototype underwent testing in teaching experiment 1, including a total of 30 students from SMAN 7 Kota Bandung.

From a design research standpoint, the primary objective of the initial design is to develop a Local Instruction Theory (LIT) that can be visually portrayed as a portrayal of the process of acquiring and improving knowledge through teaching experiments (Gravemeijer & Cobb, 2006). During the preliminary study, the researcher designed the learning process activities based on a literature review. This set of educational exercises covers the beliefs held by students while they are building knowledge and the methods that will be used, which will be presented as an initial challenge, including advanced thinking skills. The contents of HLT are subject to change. The activities in HLT can be adapted and customized based on the circumstances and parameters of the ongoing educational experiment. HLT has a curriculum that students will adhere to when studying trigonometric ratios. The HLT curriculum comprises learning objectives, activity descriptions, and student thought conjectures that occur during the learning process. During the teaching experiment, HLT functions as a reference for teachers and researchers to determine the teaching emphasis, observations, and interviews. Moreover, HLT functions as a point of reference for the examination or analysis of a dataset in the context of a teaching experiment. Hypothetical Learning Trajectory (HLT) used in the teaching experiment is displayed in Table 1 (see appendix A).

Teaching Experiment

The primary objective of a teaching experiment is to systematically evaluate and enhance original ideas regarding Local Instruction Theory (LIT) while also striving to gain a more profound comprehension of the underlying mechanisms and functioning of LIT (Gravemeijer & Cobb, 2006).

Activity 1

The teacher requested three students to perform a demonstration using the provided tables/chairs and ropes.

The teacher requested one of the students to fasten the rope to the base of the table or chair provided. A student holds the end of the neat rope and moves back 1 meter from the base of the table/chair. After that, the student who is holding the end of the rapia rope shifts his grip to the end of the head. Other students are asked to draw sketches of the shapes that are occurring. The other students observed the demonstration requested by the teacher, depicted the requested sketch, and identified the positions of the angles, front side, side view, and oblique side of the formed triangle sketch. All students strive to comprehend the right-angled triangle sketch and identify the positions of angles, front side, side view, and hypotenuse on the sketch. The teacher provides a comprehensive understanding through the following activities (See Appendix B, Figure 2)

Activity 2

7 The teacher instructs the class to define the trigonometric ratios (sin θ, cos θ, and tan θ) with a demonstration involving the height of a shadow. This activity guides students in formally discovering the concept of trigonometric comparisons. The students were able to perform this activity proficiently after acquiring the idea of identifying angles and sides in the previous activity. The presented issues are contextual and aligned with the student's level of understanding. The LIT design is structured from the simplest to the most complex in order to provide a general solution. In this activity, the teacher instructs the students to face away from the light source so that a shadow appears in front of the standing students. The results of the student's practice are represented in the form of an image or sketch of a right-angled triangle. This sketch facilitates students in identifying the positions of angles, front side, side view, and inclined side of a right triangle based on the predetermined angle. Creating a sketch of a right triangle and determining angles helps students mathematize vertically and find formal solutions to define trigonometric ratios.

Figure 3 illustrates various outcomes of students' process variations in discovering formal mathematics from the concept of trigonometric ratios. The students' work begins with a demonstration of a contextual problem, allowing them to find solutions using various methods based on their prior knowledge. During the learning process, students are guided to discover formal mathematics, which is a process of mathematization based on the presented problems.

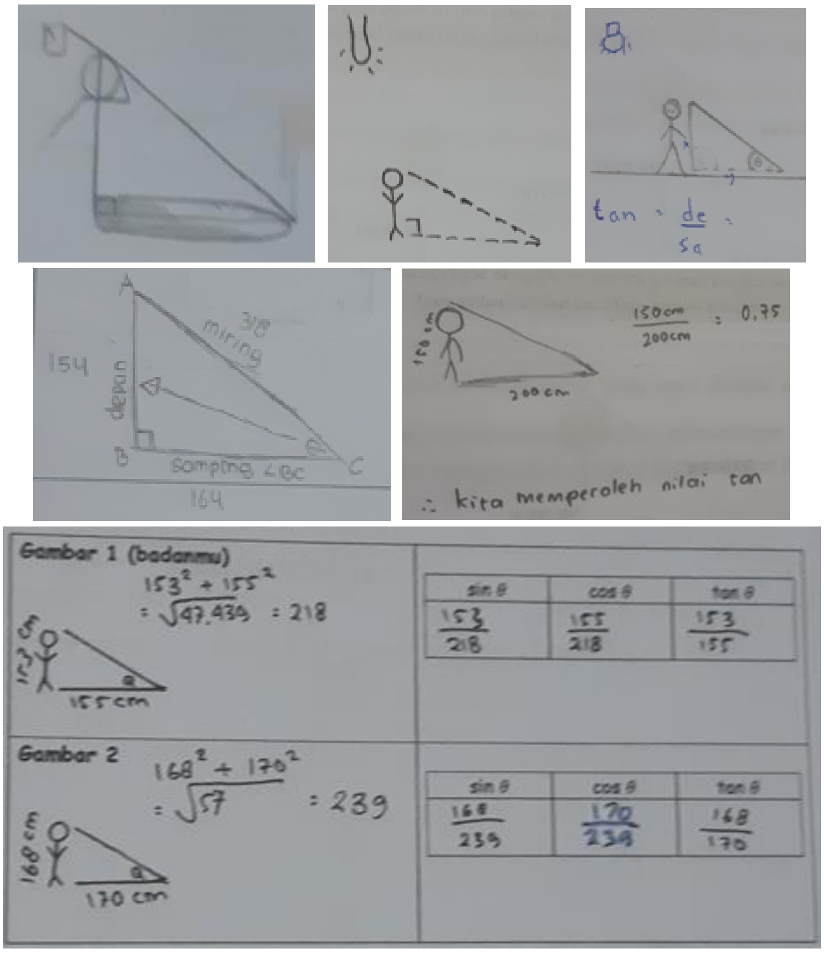


Figure 3. The vertical mathematical process of students in finding formal solutions to trigonometric ratios

Activity 3

Activity 3 begins with the teacher presenting problems based on everyday life situations. Everyday scenarios were employed to enable students to apply the notion of trigonometric ratios in practical circumstances. After the activity is finished, the teacher conducts a test as a final evaluation. One of the trigonometric comparison problems presented is as follows:

> *"Refer to the following image of a bed. What are the geometric shapes that occur between the stairs, bed, and floor? Sketch a model. (1) Does the shape formed constitute a right triangle? If so, mark one corner with a right angle. (2) If the angle between the stairs and the floor is unknown, and the distance from the bed to the stairs is 10 cm. Place the obtained data into the image. Subsequently, calculate the distance from the floor to the end of the staircase, which forms a right-*

angled triangle, using the tangent ratio. ($tan\,37^0 = 0{,}75$)"

Figure 4. Image of a bed as a model for the use of trigonometric ratios

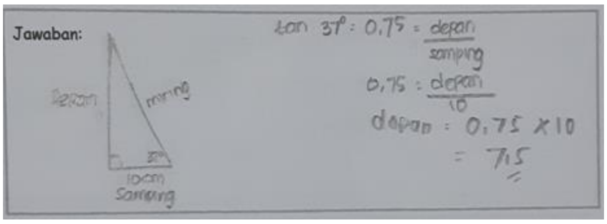


Figure 5. One of the student's responses was to solve the story problem

Figure 5 is an example of a student's response using the concept of trigonometric ratios to solve a word problem. The student solves the problem using the formal solution of trigonometric ratios that was previously constructed.

Retrospective Analysis

Researchers employ Hypothetical Learning Trajectory (HLT) as a first guide and reference when assessing a dataset during the data analysis stage of a teaching experiment. The HLT created is compared to the classroom learning process. The analysis includes descriptions of anticipated occurrences during the learning process and examples of behaviors that will occur among students. The conclusions drawn from the analysis will answer the research questions. A good design is not the main objective of the study but rather the reason for the study's success, which is the main objective (Gravemeijer & Cobb, 2006).

The many activities carried out during the learning process are represented in the flowchart of LIT before the learning process on the topic of trigonometric ratios. The flowchart is presented in Figure 6 (See Appendix C). The image shows the differences in stages or steps during the learning process. The difference is evident in the red-colored box. Before the learning process begins, the design aims to facilitate activities, experiments, observations, group discussions, and presentations as processes that run parallel. Students will go through these stages simultaneously.

Meanwhile, after the learning process takes place, a revision of the initial design is made. The stage of differentiation involves an initial discussion process with students prior to the demonstration (*See Appendix D, Figure 7*). This stage provides students with an introduction to understand the concept of trigonometric ratios in relation to the prior knowledge they have already constructed. This stage is considered crucial because linking it to the previously acquired knowledge is necessary for students to comprehend the fundamental concepts of trigonometric ratios.

The next difference is in the "responding" stage, which students go through after conducting a demonstration and before the "observing" stage. The act of "responding" is the first step for students to participate actively in the learning process. The students will provide their responses or feedback to the demonstration presented. Their knowledge will guide and stimulate the students to continue the learning process via the "observing" stage.

Before the learning process begins, the steps "activity/experiment," "observe," "group discussion," and "presentation" have the same level of suitabilityThere is no text provided. However, the post-learning phases in these four things exhibit distinct variations. The "responding" stage initiates the steps of learning. Next, the process begins with the "observation" stage, followed by the "group discussion" step, and concludes with the "presentation" step. The students go through the four steps of the learning process in different stages.

The appropriate use of the LIT framework, as shown in Figure 7 *(See Appendix D)*, allows students to construct an understanding of the concept of trigonometric ratios through everyday problems. They can identify trigonometric ratios and solve real-life issues accordingly, because of analyzing the HLT and conducting teaching experiments. Based on the results of the student's work using the LIT design for trigonometric comparison, the number of students capable of completing the learning activities is determined. The students are becoming accustomed to the learning process, which involves determining mathematics formally based on pre-established models. The utilization of Pythagorean triples, the Pythagorean theorem, models (diagrams), and measuring tools can aid in achieving vertical mathematization. The students may effectively solve common problems by utilizing trigonometric ratios.

Some students found the Local Instruction Theory (LIT) approach challenging as it shifted their understanding from merely applying formulas to understanding the deeper concepts. For example, one student expressed initial difficulty with LIT's emphasis on understanding the relationships between concepts rather than just following procedural steps. However, after grasping the underlying logic, the student felt satisfied with being able to understand the reasoning behind trigonometric ratios, rather than just memorizing formulas.

On the other hand, some students struggled to adapt to this new teaching design, especially those who were accustomed to more traditional, structured methods. A few students felt overwhelmed by the responsibility of independent learning and actively solving problems. This feedback suggests that while LIT can provide a deeper and more engaging learning experience, its effectiveness depends heavily on how well students are supported in transitioning to this new method.

Additionally, student feedback highlighted the benefits of the collaborative learning aspect of LIT. Students appreciated the group discussions and the opportunity to explore different approaches to problem-solving, which provided a more shared learning experience. However, some also criticized the method, feeling that they lacked the foundational knowledge to fully engage in the collaborative or reflective aspects of LIT, emphasizing the need for careful planning when implementing this method. These student responses offer valuable insights for refining the design and application of LIT in teaching.

The findings indicate that some students lack the prerequisite knowledge they should have acquired, which is influenced by factors such as mental unpreparedness, teacher didactics, and inadequate teaching materials (Kurniasih et al., 2020; Puspita et al., 2023). In particular, the emphasis on rote memorization of formulas, such as trigonometric ratios, without a geometric explanation, leads to a lack of motivation and understanding among students (Andika et al., 2017; Urrutia et al., 2019). Teacher-centered methods that rely solely on memorization often result in short-term retention of knowledge (Orhun, 2002). To overcome this, teachers need a comprehensive understanding of trigonometry to connect various concepts, enhancing students' learning (Andiani et al., 2020, 2024; Rittle-Johnson & Schneider, 2014). Additionally, incorporating real-life contexts into lessons, such as through models and diagrams, makes learning more relevant and aids in the long-term retention of knowledge (Dhungana et al., 2023). The use of Local Instruction Theory (LIT) and Realistic Mathematics Education (RME) approaches has been shown to improve students' ability to construct their understanding of trigonometric ratios, as evidenced by their improved learning outcomes.

Discussion

The introduction of Local Instruction Theory (LIT) in this study revealed significant improvements in students' conceptual understanding of trigonometric ratios. Unlike traditional teacher-centered approaches, LIT facilitated deeper connections between abstract mathematical concepts and real-life applications. This was evident in the students' increased ability to relate geometric properties to trigonometric functions.

During the teaching experiment, students exhibited greater engagement with the material, particularly when real-life problems were introduced. Many students showed improved motivation and understanding, especially those who had struggled with rote memorization in previous lessons. Their ability to visualize trigonometric ratios in practical contexts, such as through diagramming angles and distances in real-world scenarios, was significantly enhanced.

Previous studies on Local Instruction Theory have largely focused on its implementation in general mathematical contexts, but this study fills a gap by specifically applying LIT to the teaching of trigonometric ratios. By incorporating real-world contexts and geometrical reasoning, this research extends the application of LIT to improve both conceptual understanding and long-term retention in mathematics education.

Before the intervention, students struggled with memorizing trigonometric formulas and showed limited understanding of their geometric foundations. After the intervention, students were able to connect these formulas with real-world contexts, as evidenced by their improved problem-solving abilities and retention rates.

Implication of Research

The design of this Local Instruction Theory (LIT) for trigonometric ratios incorporates the syntax of the Realistic Mathematics Education (RME) approach. The foundation of this LIT is a conjecture based on the learning trajectory using RME. Consequently, the designed learning trajectory needs to be further developed to align with the specific content objectives. Researchers intending to implement a learning trajectory are advised to master various instructional strategies or approaches for designing problems. By mastering problem design, a wide array of solutions both formal and informal can be found. Additionally, exploring variations in students' experiences, influenced by factors such as prior knowledge, learning styles, or socio-economic backgrounds, could provide alternative perspectives when applying LIT.

Limitation

This study was conducted in two phases, each of which aligns with the steps outlined in design research (Gravemeijer, 2004), namely: 1) developing the preliminary design, 2) conducting the teaching experiment, and 3) performing retrospective analysis. There are two reasons for selecting only two phases in this research. First, the researcher's time and resources were limited. Second, the material in question is only implemented once a year. To address the limitations of time and resources, the researcher recommends extending the duration of the teaching experiments and incorporating digital resources. These approaches would enhance the efficiency and effectiveness of both time and resource utilization in the study. The extended timeframe allows for more comprehensive data collection, while digital resources provide flexible, accessible learning tools that can alleviate constraints related to physical materials (Means et al., 2014). Furthermore, leveraging technology enables more dynamic and scalable teaching practices, supporting diverse learning needs (Çelik & Baturay, 2024).

CONCLUSION

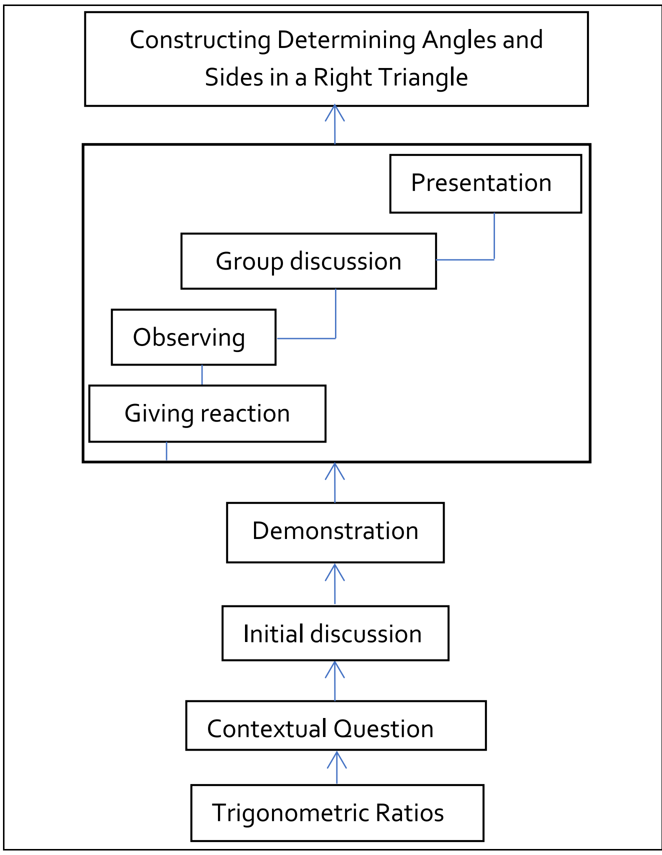
The outcome of the learning trajectory in this study is a Comparative Trigonometry LIT developed through the Realistic Mathematics Education (RME) approach. This theory encompasses several steps that students must go through in constructing their knowledge related to trigonometric comparisons. These steps nonetheless adhere to the syntax of the Realistic Mathematics Education (RME) approach.

1 Appendix A of article entitled Development of A Local Instruction Theory for Trigonometric Ratios

Table 1. HLT illustration for learning the concept of trigonometry comparisons

Activity	Learning Objectives	Activity Description	Conjecture
Students Demonstrate the activity of identifying by body height.	The students are able to identify the front, side, and diagonal positions by referring to specific angles of a right-angled triangle.	The teacher instructed three students to engage in a practical exercise involving the formation of a right-angled triangle using a table base, a measuring tape, and their height. Each group carefully observes the demonstration and discusses the position of the corner, front, side, and slant.	Some students may be able to illustrate a sketch of a right triangle from the demonstration activity. Some students may quickly recognize the positions of the corner, front side, side side, and hypotenuse.
Shows high shadow intensity activity	The students can articulate the trigonometric ratios (sin θ, cos θ, and tan θ) by employing the notion of ratios within a right triangle.	The teacher instructs the students to face away from the light until a shadow appears in front of the standing student. The sketch formed is a right-angled triangle. Students can determine the front side, side view, and diagonal side of a right triangle sketch based on the determining angle. Students will be *guided* in *rediscovering the concept* of trigonometric ratios.	- Some students may use tables as a tool to solve problems. - Certain students may employ finger manipulation methods to solve difficulties. - Some students may utilize measuring instruments to solve problems. - Some students may use the number parity method to solve problems. - Certain students may possess the ability to represent this activity in a visual manner. They initiated the construction of a model and engaged in horizontal and vertical mathematization to uncover the notion of trigonometric ratios through the shown examples. - Some students may be able to relate the demonstration process to the concept of trigonometric ratios.
Problem-Solving	Students can solve real-life problems related to right triangles using trigonometric ratios	Students use trigonometric ratios to solve real-life problems by performing calculations.	- Certain students possess the ability to perceive the relationship between sine and cosine values in mathematical calculations and are capable of streamlining the calculation process. - Some students may be able to remember or repeat knowledge they have previously acquired to solve problems. - Some students may use tables, finger methods, odd-even tricks, or measuring tools to solve problems. - Some students may directly apply trigonometric ratio concepts.

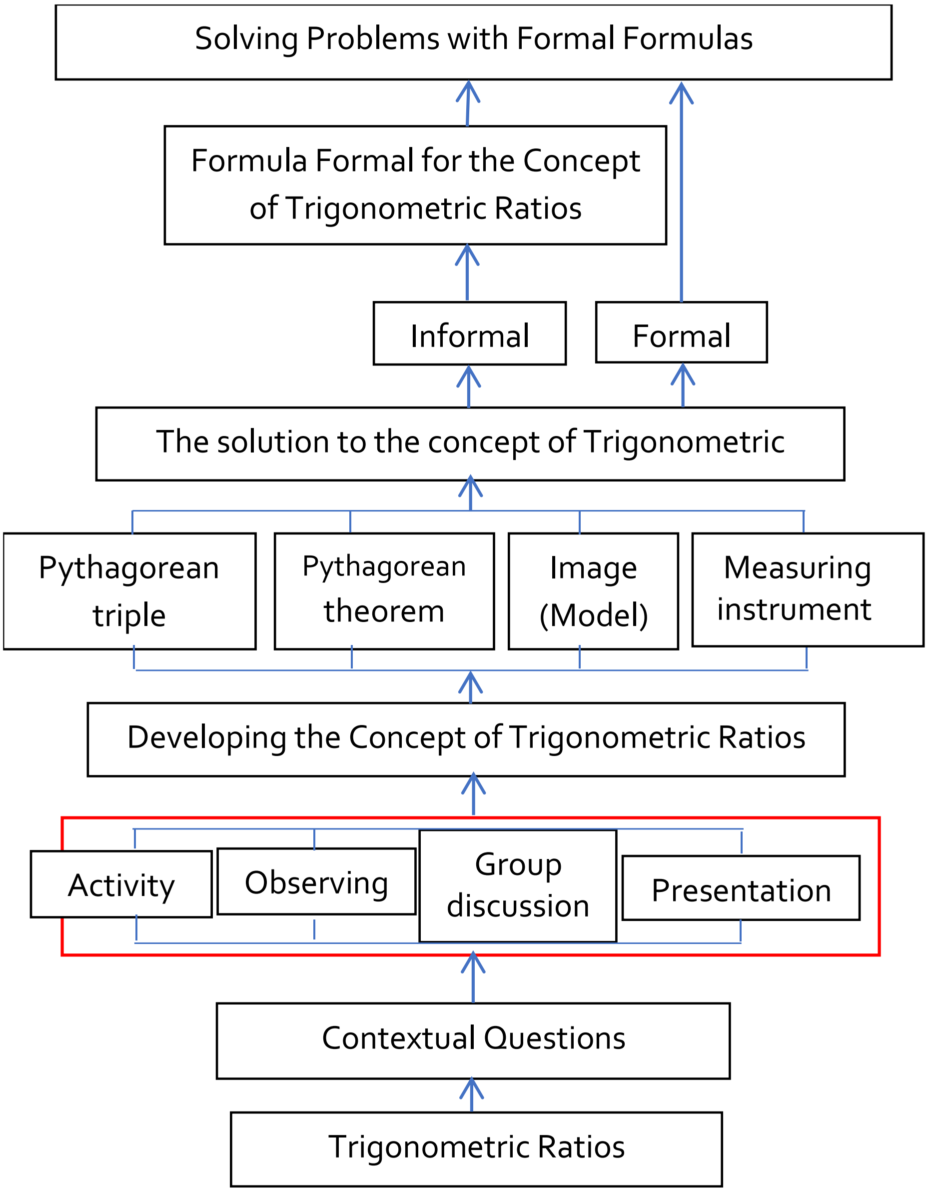
Appendix B of article entitled Development of A Local Instruction Theory for Trigonometric Ratios



The student model illustrated the activity of identification by using height as part of the learning process for understanding trigonometric ratios.

Figure 2. Demonstration of the flow of LIT and height in learning comparative trigonometry

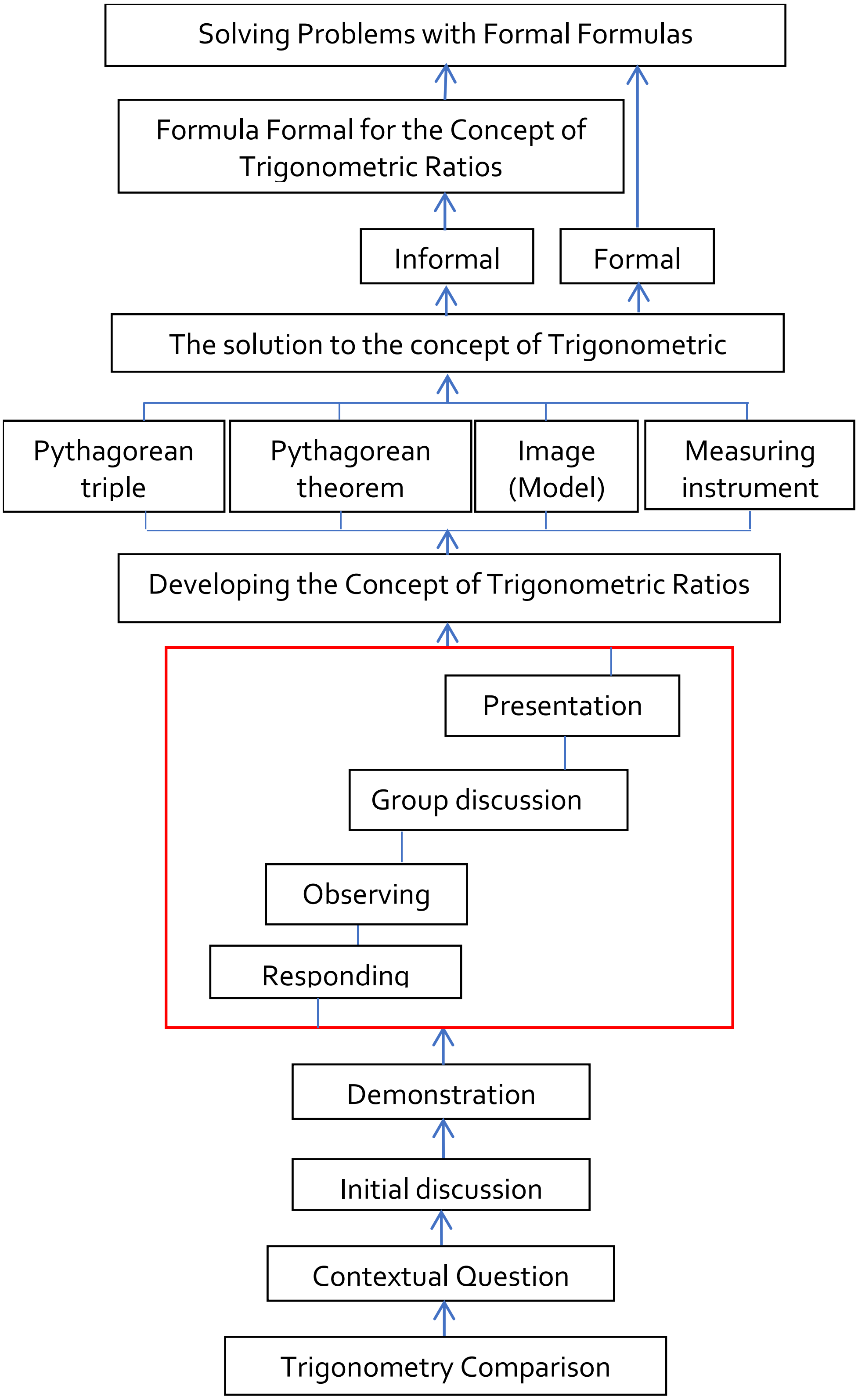
1 Appendix C of article entitled Development of A Local Instruction Theory for Trigonometric Ratios



Before the learning process begins

Figure 6. Flowchart of LIT on the topic of Trigonometric Ratios before the lesson

1 Appendix D of article entitled Development of A Local Instruction Theory for Trigonometric Ratios



After the learning process is complete

Figure 7. Flowchart of LIT on the topic of Trigonometric Ratios after the lesson