

Problem-Solving in Geometry Teaching for Pre-service Mathematics Teacher Students from a Computational Thinking Perspective

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Abstract

Computational thinking and problem-solving skills are necessary for the future careers of pre-service mathematics teacher students in the 21st century. This study aims to analyse problem-solving activities in geometry teaching and corresponding assessment for pre-service mathematics teacher students from a computational thinking perspective. To do so, we carried out a qualitative case study through teaching and learning observations involving 41 pre-service mathematics teacher students from one of the state universities in West Java, Indonesia. In this study, 6 x 50 minutes of geometry teaching were observed, and written work containing problem-solving processes retrieved from the formative assessment was analysed from the computational thinking perspective. The results showed that two types of problem-solving activities are identified from the teaching and learning processes and the corresponding assessment, i.e., problem to find and problem to prove. We view that a problem to prove can typically be considered as a structured problem to find. Both types of problems can be fruitfully analysed using Polya's problem-solving strategy from the computational thinking perspective. For future research, we recommend investigating each type of problem using more specific characteristics of computational thinking.

Keywords: *Computational Thinking; Geometry Teaching and Learning; Pre-service Mathematics Teacher Students; Problem-Solving*

Information of Article

Subject classification	97G10 Comprehensive works on geometry education
Submitted	28 November 2023
Review Start	11 February 2024
Round 1 Finish	25 February 2024 (Reviewer A)
Resubmitted	20 June 2024
Round 2 Finish	20 June 2024 (Reviewer B)
Round 3 Finish	23 November 2024 (Reviewer C)
Accepted	26 November 2024
Scheduled online	1 December 2024
Similarity Check	4%

Abstrak

Keterampilan berpikir komputasi (computational thinking) dan pemecahan masalah diperlukan untuk karir masa depan mahasiswa calon guru matematika di abad ke-21. Penelitian ini bertujuan untuk menganalisis aktivitas pemecahan masalah dalam proses pembelajaran geometri dan asesmennya untuk mahasiswa calon guru matematika dari perspektif berpikir komputasi. Untuk itu, kami melakukan studi kasus kualitatif melalui observasi proses pembelajaran yang melibatkan 41 mahasiswa calon guru matematika dari salah satu universitas negeri di Jawa Barat, Indonesia. Dalam penelitian ini, kami mengobservasi pembelajaran geometri untuk calon guru matematika selama 6 x 50 menit, dan hasil tes tertulisnya yang berisi proses pemecahan masalah yang dianalisis dari perspektif berpikir komputasi. Hasil penelitian menemukan adanya dua jenis tipe pemecahan masalah, yang diidentifikasi dari pengamatan proses pembelajaran dan penilaian formative, yaitu masalah tipe menemukan dan masalah tipe membuktikan. Kami memandang bahwa masalah tipe membuktikan sebagai masalah terstruktur dari tipe menemukan. Kedua tipe masalah tersebut dapat dianalisis dengan baik menggunakan strategi pemecahan masalah Polya dari perspektif berpikir komputasi. Untuk penelitian mendatang, kami merekomendasikan untuk menelaah lebih lanjut setiap tipe masalah dengan menggunakan karakteristik berpikir komputasi yang lebih spesifik.

INTRODUCTION

Problem-solving is considered one of the skills that need to be mastered to live in the 21st century (English & Sriraman, 2009; Kemdikbud, 2017). Pre-service mathematics teacher students need to master problem-solving skills as preparation for their future careers, both in the academic world as mathematics teachers and in social life as citizens (De Lange, 2006; Widana, 2017). This problem-solving skill can be developed, among others, through learning geometry topics (Budhi & Kartasmita, 2015; Jupri & Rosjanuardi, 2020; Jupri et al., 2020).

However, problem-solving skills in geometry topics are still not fully mastered by most pre-service mathematics teacher students in Indonesia. Previous research results showed that the problem-solving abilities of pre-service mathematics teacher students in Indonesia in geometry still need to be improved (Jupri, et al., 2020). Yuwono (2016) found that pre-service mathematics teacher students still had difficulty solving problems regarding the use of the concept of triangle congruence and the process of proving the area of triangles and quadrilaterals. Samo (2017) found that the problem-solving proficiencies of pre-service teacher students in geometry are relatively di-

verse and tend to be low for most students. In other research, pre-service mathematics teacher students were considered to have relative difficulty in solving simple geometric problems (Masfingatin et al., 2018).

The findings above are an indication of the necessity to enhance the quality of geometry teaching for pre-service mathematics teacher students. Enhancing the quality of geometry teaching can be conducted, for instance, in the form of improving the quality of teaching materials for students and improving the teaching process which provides wider opportunities for students to do problem-solving processes. The latest research results suggest that aspects of problem-solving proficiencies can be developed better, in line with developments in the current industrial revolution 4.0 era, by integrating computational thinking (CT) skills both in the development of teaching materials and their implementation in learning and teaching processes (Kallia et al., 2021; Van Borkulo et al., 2021; Wing, 2006; 2010). Computational thinking is a process and way of thinking in formulating problems so that the solution can be shown in a form that can be understood by humans, computers, or both (Denning, 2009; Wing, 2006; 2010). This means that CT emphasizes problem-solving processes that can be carried out by humans themselves or

with the help of computers, such as through the use of software, or a combination of the two.

Considering that the meaning of CT is not restricted only to the field of computer science, experts in the field of mathematics education (such as Van Borkulo *et al.*, 2021; Kallia *et al.*, 2021; Sung *et al.*, 2017) have attempted to integrate the idea of CT in the field of mathematics education. For example, Calculus teaching in the Netherlands has been implemented by integrating CT aspects into the learning process. The integration process is done by using the GeoGebra software (Van Borkulo *et al.*, 2021). This integration process was carried out to make CT accessible to teachers and students, and in the future can be applied in everyday situations, the academic world, or the world of work.

Several aspects of CT which are characteristics of a way of thinking and can be integrated into mathematics education include problem decomposition, pattern recognition, abstraction, and algorithms (Kallia *et al.*, 2021; Voogt, 2015). Decomposition is a way of thinking in describing a problem so that it can be separated into smaller sub-problems, and then a relatively more efficient solution can be found. Pattern recognition is a way of thinking in identifying similarities, regularities in data, or recurring phenomena. Abstraction is a way of thinking in generalizing a principle into a formula or rule by modelling the patterns found. The abstraction process can also be interpreted as an activity of looking for the essential parts of a problem and neglecting the unimportant ones, making it easier to focus on the solution to the problem. Finally, the algorithm is a thinking process in compiling steps to solve a problem so that it becomes more systematic and efficient. If examined carefully, these four aspects of Computational Thinking (CT) are in line

with the steps of problem-solving proposed by Polya (1973).

The steps of the problem-solving process according to Polya (1973) include understanding the problem, devising a plan, carrying out the plan, and looking back. In the first step, understanding the problem, we need to specify the known data, the unknown, and the condition of the problem. In the second step, devising a plan, we should relate between the unknown and the known data and should provide problem-solving strategies to solve the problem. From a CT perspective, the first and second steps can be supported by the process of decomposition, pattern recognition, and abstraction. In the third step, carrying out the plan, we should resolve the problem by applying problem-solving strategies provided in the aforementioned step. Finally, in the fourth step, looking back, we need to consider whether the solution that we found makes sense or not, whether we can solve the problem using other problem-solving strategies, and whether we can generalize the problem-solving process for other similar problems. From a CT perspective, the third and fourth steps can be supported by the process of algorithms, abstraction, and decomposition.

Taking the previous description into account, this research aims to analyse problem-solving activities in a geometry learning and teaching process for pre-service mathematics teacher students from the computational thinking perspective. Through this research, it is expected that the results will be useful for improving the quality of the process and results of geometry teaching which can develop the problem-solving abilities of pre-service mathematics teacher students.

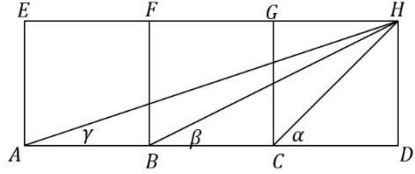
METHOD

To investigate problem-solving activities in a geometry teaching and learning process, we conducted a qualitative case study through classroom observations on the teaching and learning processes with the following three steps. Firstly, we carried out classroom observations in a geometry course for two meetings (6x50 minutes), involving 41 pre-service mathematics teacher students in one of the state universities in West Java, Indonesia. The participants were in the second year of the mathematics education program. They had learned the foundation of mathematics, calculus, linear algebra, number theory, and statistics. Therefore, we assume that they have already enough experience in doing proof and problem-solving in mathematics. In the observations, we investigated the learning and teaching process for the topic of similarity of triangles, its application in the problem-solving of geometry, and its corresponding written assessment. The focus of teaching observations was on the learning process and the problem-solving activities performed by the lecturer and pre-service mathematics teacher students during the teaching and learning.

Secondly, we administered a formative individual written assessment after the teaching and learning processes. For this formative assessment, two geometry tasks were used (see Table 1). Both tasks ask for proving processes using the concepts of similarity of triangles. The individual written test was administered for about 40 minutes to 41 pre-service mathematics teacher students.

Table 1. Tasks used in the assessment

No	Tasks
1.	Draw a right triangle $\triangle ABC$. $\angle ACB$ is a right angle, $BC = a$, $AC = b$, $AB = c$ and \overline{CD} is the altitude of the triangle. Prove each of the following statements:

No	Tasks
	$\triangle ABC \sim \triangle ACD$ $\triangle ABC \sim \triangle CBD$ $a^2 + b^2 = c^2$.
2.	<p>Given a three series of the same squares as shown. Prove that $\alpha = \beta + \gamma$.</p> 

Thirdly, data analysis was carried out. For analysing the learning and teaching process data, we used the framework of problem-solving heuristic strategies (Polya, 1973) from the perspective of Computational Thinking (CT). Similarly, for analysing the written test data, we also used the framework of Polya's problem-solving strategy from a CT perspective.

RESULTS AND DISCUSSION

Results of Learning and Teaching Observations

The process of learning and teaching the topic of similarity of two triangles was started by the lecturer by asking pre-service mathematics teacher students about this topic which had been studied when they were studying at junior high school level. After questions and answers, the lecturer wrote and explained the definition of the similarity of two triangles, wrote and explained the theorems of the similarity of two triangles, namely the SAS (Side-Angle-Side) theorem, the AAA (Angle-Angle-Angle) theorem, and the SSS theorem (Side-Side-Side). As an illustration of the use of the rules for the similarity of two triangles, the lecturer gave two example problems. The first example problem discussed in the learning process is as follows: "Given an acute triangle ABC . The point P is on \overline{AC} and Q is on \overline{BC} such

that $\overline{PQ} \parallel \overline{AB}$. Prove that $\frac{CP}{PA} = \frac{CQ}{QB}$."

The process of proving, guided by the lecturer through questions and answers as well as explanation, was carried out as follows. Let the acute triangle be ABC . With some information given in the problem, the triangle is constructed as in Figure 1.

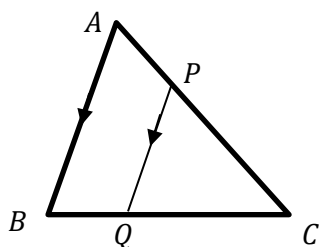


Figure 1. The triangle ABC and its information

Since $\overline{PQ} \parallel \overline{AB}$, then we have $\angle CPQ \cong \angle CAB$, and $\angle CQP \cong \angle CBA$. Also, it is obvious that $\angle PCQ \cong \angle ACB$. As a consequence, according to the theorem of AAA (Angle-Angle-Angle), we conclude $\triangle CPQ \sim \triangle CAB$. Because $\triangle CPQ \sim \triangle CAB$, then we have the following relation:

$$\begin{aligned} \frac{CP}{CA} &= \frac{CQ}{CB} \\ \Leftrightarrow \frac{CP}{CP+PA} &= \frac{CQ}{CQ+QB} \\ \Leftrightarrow \frac{CP+PA}{CP} &= \frac{CQ+QB}{CQ} \\ \Leftrightarrow 1 + \frac{PA}{CP} &= 1 + \frac{QB}{CQ} \\ \Leftrightarrow \frac{PA}{CP} &= \frac{QB}{CQ} \\ \Leftrightarrow \frac{CP}{PA} &= \frac{CQ}{QB} \end{aligned}$$

Based on the observation of the teaching episode above, we have the following notes. Regarding the explanation process for proving the first example problem above, we view that the lecturer explains the steps for the proving process according to Polya's problem-solving

strategy and the perspective of Computational Thinking (Van Borkulo et al., 2021; Gadanidis et al., 2017; Kallia et al., 2021; Polya, 1973; Sung et al., 2017). The ability to understand and identify problems can be seen as the application of the decomposition aspect. The ability to devise a plan in the form of creating appropriate visualizations, by describing the information and information given in the problem correctly, can be seen as the application of aspects of pattern recognition and abstraction. The ability to apply rules of the similarity of triangles, as well as simple algebraic manipulation, are aspects of algorithms. Thus, again it can be said that the process of problem-solving in the form of a proving process is an implementation of the aspects of Computational Thinking (CT).

The second example problem discussed during the teaching and learning process is as follows: "Given a right triangle PQR where $\angle QPR$ is the right angle. If $PQ = 3$ cm, $PR = 4$ cm, and PS is perpendicular to QR , then find the lengths of PS , QS , and RS ."

The problem-solving process for the second example problem was also guided by the lecturer as follows. From the given information, we can construct Figure 2 below.

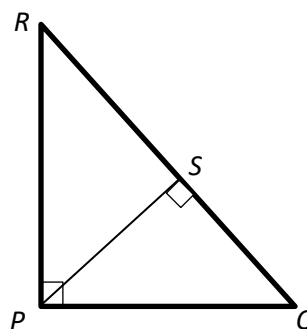


Figure 2. The triangle PQR and its information

Using the Pythagoras' theorem on the triangle PQR , we find that $QR = 5$ cm. Next, by using the AAA (Angle-Angle-Angle) theorem the lecturer verbally showed

that $\Delta PQR \sim \Delta SPR$, $\Delta PQR \sim \Delta SQP$, and $\Delta SQP \sim \Delta SPR$.

From $\Delta PQR \sim \Delta SPR$, we have the relation $\frac{PQ}{SP} = \frac{QR}{PR}$. This implies that $SP = \frac{PQ \cdot PR}{QR}$. By substitutions of the given information, we have $SP = PS = \frac{12}{5}$ cm.

From $\Delta SQP \sim \Delta SPR$, we have the relation $\frac{SQ}{SP} = \frac{QP}{PR}$. This implies that $SQ = \frac{QP \cdot SP}{PR}$. By substitutions of the lengths from the given information and the previous result, we have $SQ = QS = \frac{9}{5}$ cm.

From $\Delta PQR \sim \Delta SPR$, we have the relation $\frac{PR}{SR} = \frac{QR}{PR}$. This implies that $SR = \frac{PR \cdot PR}{QR}$. By substitutions of the lengths from the given information, we have $SR = RS = \frac{16}{5}$ cm.

Concerning the observation of the second example problem above, we have the following notes. We clearly view that the lecturer in guiding the problem-solving has used Polya's strategy and computational thinking characteristics. When the lecturer showed verbally the similarity between two triangles repeatedly to obtain $\Delta PQR \sim \Delta SQP$, $\Delta PQR \sim \Delta SPR$, and $\Delta SQP \sim \Delta SPR$, he decomposed the problem into three similarities of triangles (Kallia et al., 2021; Van Borkulo et al., 2023). Next, when he repeatedly uses the proportional relations from the similarity of triangles to find the lengths of segments, this means he has used algorithmic thinking meaningfully (Gadanidis et al., 2017; Van Borkulo et al., 2023). Also, when the lecturer creates the visualization from the given problem, it concerns the application of aspects of pattern recognition and abstraction (Gadanidis et al., 2017; Kallia et al., 2021; Van Borkulo et al., 2023). Therefore, we identify that the process of problem-solving in the form of applying the similarity of triangles concepts is the implementation of computational

thinking.

Whereas the first example concerns the proving problem of the similarity of triangles, the second example concerns the problem-solving by applying the concepts of the similarity of triangles. In our view, both types of problems need problem-solving strategies which can be seen from the perspective of computational thinking (CT). In this case, the aspects of CT play crucial roles in each step of the problem-solving heuristics strategy.

Results of Written Assessment

Table 2 presents the findings from pre-service mathematics teacher students' written work on solving proving problems for the topic of the similarity of triangles. The results for Task 1 are relatively better than for Task 2. In general, however, we view that both tasks are relatively difficult for most of the participants.

Table 2. Results of the assessment (n =41)

Tasks	#Correct(%)
1. Draw a right triangle ΔABC . $\angle ACB$ is a right angle, $BC = a$, $AC = b$, $AB = c$ and \overline{CD} is the altitude of the triangle. Prove each of the following statements: (a) $\Delta ABC \sim \Delta ACD$ (b) $\Delta ABC \sim \Delta CBD$ (c) $a^2 + b^2 = c^2$.	(a). 32 (78.1) (b). 33 (80.5) (c). 18 (43.9)
2. Given a three series of the same squares as shown. Prove that $\alpha = \beta + \gamma$.	15(36.6)

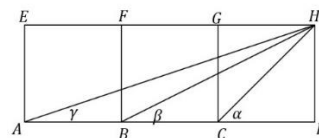


Figure 3 (in Appendix A) shows an example of correct answers for Task 1 parts (a), (b), and (c). The main objective for Task 1 is proving the Pythagoras' theorem using the concepts of the similarity of triangles. To reach this objective, this

task has been divided into three questions. Parts (a) and (b) are used to get part (c). From a CT perspective, this task directly guides students through decomposing the problem into parts (a) and (b) (Kallia et al., 2021; Zhang & Nouri, 2019).

Tasks 1(a) and 1(b) concern finding and proving two similar triangles. The first step that must be done is drawing a right triangle and the given information. We found that all participants did this correctly. The second step for solving either part (a) or (b) is applying the AAA (Angle-Angle-Angle) theorem to prove $\triangle ABC \sim \triangle ACD$, $\triangle ABC \sim \triangle CBD$, and $\triangle ACD \sim \triangle CBD$. Proving each of these similarities concerns the use of the step-by-step process, which from the CT perspective is included within the algorithm aspect (Kallia et al., 2021; Katai, 2020; Van Borkulo et al., 2023). We found that the difficulties encountered by the participants in dealing with Task 1 part (a) or (b) include finding corresponding angles between two triangles that have the same measures. As a consequence, they were not able to prove the similarities of $\triangle ABC \sim \triangle ACD$, $\triangle ABC \sim \triangle CBD$, and $\triangle ACD \sim \triangle CBD$.

Task 1(c) requires students to prove Pythagoras' theorem using the concept of similarities of triangles. To do so, parts 1(a) and 1(b) should be used for solving Task 1(c) as follows. Because $\triangle ABC \sim \triangle ACD$, then we have the relation $\frac{AC}{AD} = \frac{AB}{AC}$ or $AC^2 = AB \cdot AD$. Because $\triangle ABC \sim \triangle CBD$, then we have the relation $\frac{BC}{BD} = \frac{AB}{CB}$ or $BC^2 = AB \cdot BD$. By adding these last two results, we obtain $BC^2 + AC^2 = AB \cdot BD + AB \cdot AD = AB^2$. From a CT perspective, to be able to prove the Pythagoras' theorem in Task 1(c), we need abstractive skills that synthesize previous results (Lv et al., 2022; Van Borkulo et al., 2023).

Figure 4 (in Appendix B) illustrates a

correct answer for Task 2. Similar to the previous task, Task 2 also asks for participants to do a proving process. However, as indicated by the written test results in Table 2, this task is more challenging than Task 1. The difficulties found in written student work mainly concerns, for instance, finding similarities between triangles for proving the required property of $\alpha = \beta + \gamma$.

In our view, from the CT perspective, the ability to see and prove $\triangle ACH \sim \triangle HCB$ using the SAS (Side-Angle-Side) theorem (see Figure 4) needs not only algorithmic skills but also abstractive skills (Kallia et al., 2021; Sanford, 2018).

Discussion

From the results section, we note the following three points to discuss. First, problem-solving activities for the topic of the similarity of triangles for pre-service mathematics teacher students that can be identified from teaching observations can be classified into problem-solving to prove and problem-solving to find (see Polya, 1973). Problem-solving to prove means the activity of proving statements, properties, or theorems. Problem-solving to find means the activity of finding a solution to a non-routine problem. We view that problem-solving to prove can be seen as a structured problem-solving to find. From a CT perspective, both types of problem-solving involve CT aspects, including decomposition, abstraction, and algorithm (Kallia et al. 2021; Van Borkulo et al., 2023). The CT aspects involve the implementation of problem-solving heuristic strategies, for instance in the steps of devising a plan, carrying out the solution process, and looking back.

Second, the teaching and learning process for pre-service mathematics teacher students for the topic of similarity

of triangles can be classified into a deductive teaching approach (Ndemo et al., 2017; Prince & Felder, 2006). The process starts from general ideas of the topic in the form of addressing definitions and proving theorems to more specific ideas in the form of discussing example problems through questions and answers. In our view, even if the deductive characteristics are dominant, still the lecturer provides room for students to be involved during the teaching and learning process through questions and answers.

Third, difficulties encountered by the participants that emerged in written work include identifying similarities between two triangles, applying the similarities theorems for the proving process, and executing algebraic manipulations to obtain the required results. From a CT perspective, these difficulties indicate that pre-service mathematics teacher students are struggling with decomposing, synthesising, abstracting, and thinking algorithmically when solving geometry problems (see Futschek, 2006; Kallia et al., 2021; Kynigos & Grizioti, 2018).

Implication of Research

Regarding the finding of two types of problem-solving activities, we wonder whether these two types can be compared to which one is more challenging and requires computational thinking aspects during the problem-solving processes. This might be an opportunity to be investigated in the learning and teaching processes and future research.

Concerning the usage of the deductive teaching approach in the learning and teaching process, which has a more lecturer-centred approach, for future research we wonder if the student-centred approach would produce better results for improving the skills of problem-solving

and computational thinking. Many studies (such as Hino, 2007; Ridlon, 2009) suggest that a student-centred approach can enhance problem-solving skills. In addition, we do wonder whether the main aspects of CT can be applied to enhance problem-solving abilities, particularly in the topic of geometry.

With regard to encountered difficulties during problem-solving activities for the topic of similarity of triangles, we do wonder a more comprehensive repertoire on this. For future studies, it is worth exploring possible difficulties encountered by pre-service mathematics teacher students in dealing with problem-solving on the topic of the similarity of triangles.

Limitation

We admit that this study has several limitations. First, since this study focused only on the results of classroom observations for the topic of the similarity of triangles and written work from the formative assessment, we are aware that the results of the study might be more comprehensive if additional interview data are incorporated. Also, more comprehensive results might be obtained if observations are carried out for more than one cohort of pre-service mathematics teacher students.

Second, three main CT aspects, namely decomposition, abstraction, and algorithm played crucial roles in interpreting problem-solving activities from both observations and written work. However, the aspect of pattern recognition does not emerge explicitly. This probably is caused by the fact that problem-solving in the forms of problem to prove, which are apparently dominant in the observations, does not provide an explicit opportunity to recognize patterns. Probably, in the future, it is necessary to add problems to find or problems to prove that require explicit recognition of patterns when doing

problem-solving activities. Other CT aspects that might emerge and should be exploited include mathematical modelling which plays a crucial role in dealing with problem-solving on word or contextual problems.

CONCLUSION

From the previous sections, we draw the following conclusions. Based on the observations of the learning and teaching process for pre-service mathematics teacher students on the topic of the similarity of triangles, we found two types of problem-solving activities. These types include problems to prove and problems to find. We consider that both types of problems can be fruitfully analysed using Polya's problem-solving strategy from the computational thinking perspective. Three main CT aspects play crucial roles in the interpretation of problem-solving activities, including decomposition, abstraction, and algorithm aspects. These aspects are used to interpret problem-solving heuristic strategies used in the learning and teaching process and written student work. In the future, we consider investigating other CT aspects for interpreting problem-solving activities for either problem to prove or problem to find.

ACKNOWLEDGMENTS

This study was funded by the Faculty of Mathematics and Science Education, *Universitas Pendidikan Indonesia*, year 2023, No: 1170/UN40/PT.01/2023.

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Appendix A of article entitled Problem-Solving in Geometry Teaching for Pre-service Mathematics Teacher Students from a Computational Thinking Perspective

Gambarkanlah $\triangle ABC$ siku-siku di C , dengan $\overline{BC} = a$, $\overline{AC} = b$, $\overline{AB} = c$ dan \overline{CD} adalah garis tinggi. Buktikan bahwa

(a) $\triangle ABC \sim \triangle ACD$
 (b) $\triangle ABC \sim \triangle CBD$
 (c) $a^2 + b^2 = c^2$

bukti

(a) Perhatikan bahwa
 $m\angle ACB \cong m\angle ADC$ (siku-siku)
 $m\angle CAB \cong m\angle DAC$ (berimpit)
 $m\angle ABC \cong m\angle ACD$ (besarannya sama hasil dari $180^\circ - (90^\circ + \angle A)$)
 Menurut aturan AAA, maka terbukti
 $\triangle ABC \sim \triangle ACD$

(b) Perhatikan bahwa
 $m\angle ACB \cong m\angle CDB$ (siku-siku)
 $m\angle ABC \cong m\angle CBD$ (berimpit)
 $m\angle CAB \cong m\angle DCB$ (besarannya sama hasil dari $180^\circ - (90^\circ + \angle B)$)
 Menurut aturan AAA, maka terbukti $\triangle ABC \sim \triangle CBD$

(c) Karena $\triangle ABC \sim \triangle ACD$, maka

$$\frac{AC}{AD} = \frac{AB}{AC}$$

$$AC^2 = AB \cdot AD \dots (i)$$

Dan karena $\triangle ABC \sim \triangle CBD$, maka

$$\frac{BC}{BD} = \frac{AB}{CB}$$

$$BC^2 = AB \cdot BD \dots (ii)$$

Dari persamaan $AC^2 = AB \cdot AD \dots (i)$ dan $BC^2 = AB \cdot BD \dots (ii)$, maka diperoleh

$$AC^2 + BC^2 = (AB \cdot AD) + (AB \cdot BD)$$

$$= AB(AD + BD)$$

$$= AB(AB)$$

$$= AB^2$$

Maka, dapat disimpulkan bahwa $AC^2 + BC^2 = AB^2$ atau $a^2 + b^2 = c^2$
 Jadi, terbukti $a^2 + b^2 = c^2$

Figure 3. Solution to Task 1 from written student work

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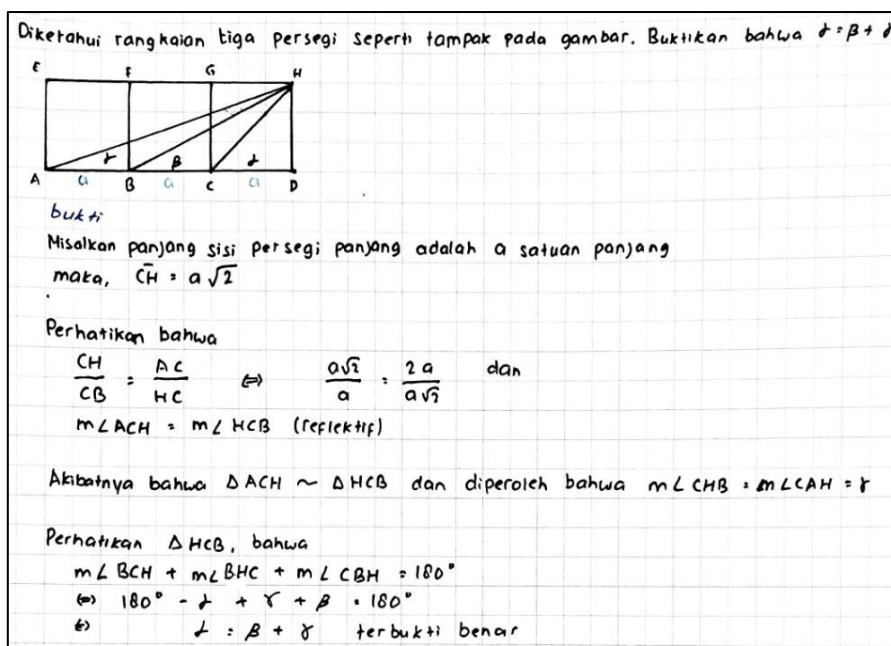


Figure 4. Solution to Task 2 from written student work