Physics Communication
Volume 9 Issue 2 Year 2025 Pages 74-82

P-ISSN 2528-5971 | E-ISSN 2528-598X
https://journal.unnes.ac.id/journals/pc
https://doi.org/10.15294/pc.v9i2.21721

Expansion of the Double Pendulum System and Its Decomposition Model: Systematic Literature Review

Anisa Zulfa Nadia*, Joko Siswanto, Joko Saefan

Physics Education Study Program, Faculty of Mathematics, Natural Sciences, and Information Technology, Universitas PGRI Semarang, Indonesia

*Correspondence to: *anisazulfanadia2003@gmail.com*

Abstract: Double pendulum is a mechanical system that exhibits complex dynamic behavior, often used to study nonlinear dynamics and chaos. Along with the development of research, this system has been extended with the addition of several new constraints. This research aims to conduct a Systematic Literature Review (SLR) related to the extension of the double pendulum system and the formulation of the relevant equations of motion. The article search was conducted systematically, resulting in 27 documents that met the inclusion criteria. The results of the analysis show that the extension of the gantry pendulum crane system with a payload is the main focus in this study. In addition, the most frequently used decomposition model is a mathematical model with a Lagrangian approach. These findings provide deep insights for further development of mathematical models as well as their applications in the fields of robotics, system control, and dynamic structural analysis.

Keywords: Systematics Literature Review; Double Pendulum; Mechanical System

Submitted: 2025-02-24. **Revised:** 2025-06-10. **Accepted:** 2025-08-7.

Introduction

The double pendulum is a simple mechanical system that exhibits complex dynamic behavior, including the emergence of chaos even under slightly different initial conditions (Strogatz, 2018). It is often used as a model to study nonlinear dynamics, chaotic behavior, and the interactions between mechanical components. In practical applications, the double pendulum plays an important role in various fields such as robotics, system control, and dynamic structural analysis (Baker & Gollub, 1996).

Building upon this fundamental understanding, researchers have extended the double pendulum system to explore its behavior under increasingly complex conditions. These extensions involve the addition of elements such as variable masses, springs, damping effects, or external interactions. Lagrangian-based analytical methods constitute one of the main approaches in modeling and analyzing the dynamics of these systems, as the Lagrangian formulation provides a strong theoretical foundation for deriving the equations of motion (Chai et al., 2021).

Based on the analysis conducted in this article, some research questions (RQ) that can be raised to deepen the understanding of the double pendulum system are as follows:

RQ 1: What are the recent extensions of the double pendulum system that have been studied in the literature, and how do they affect the understanding of nonlinear dynamics and chaos?

RQ 2: To what extent does the physical model contribute to understanding the dynamic behavior of the double pendulum system compared to the mathematical model?

Accordingly, this article aims to conduct a Systematic Literature Review (SLR) of studies related to the double pendulum system, including its extensions and modeling strategies. The focus is on identifying research trends, significant contributions, and open challenges, particularly in relation to decomposition models that refer to modular analysis, segmentation of system dynamics, or separation of degrees of freedom. This approach is expected to provide in-depth insights into the development of mathematical models and their applications across various disciplines.

Double Pendulum System Expansion

Double pendulum systems have long been an object of study in nonlinear dynamics. As research progresses, many system extensions have been proposed to broaden the understanding and practical applications of this system. A key extension involves modifications to the system configuration, including the addition of external forces or additional pendulums. For example, several studies have introduced double pendulum systems that incorporate additional electromagnetic or gravitational forces to observe the system's behavior under more complex conditions (Cunha et al., 2015). The inclusion of more than two pendulums has also been explored to extend the model into multi-body systems, making them more applicable to robotics and applied mechanics (Nenuwe, 2019).

In addition, friction and damping factors--often neglected in idealized double pendulum models--are now considered critical components in contemporary models. Air resistance, mechanical friction, and temperature variations can significantly influence the pendulum's motion, necessitating more realistic mathematical formulations. Models incorporating damping effects have been developed to simulate the system more accurately, allowing for results that better represent real-world dynamics (Kurcbart et al., 2009). In such cases, the mathematical modeling approach commonly employed is based on Lagrangian and Hamiltonian mechanics, which offer more efficient formulations of the equations of motion, even when additional complex elements are introduced.

In addition to applications in mechanics theory, double pendulum systems are also widely used in various technological applications, especially in the fields of robotics and control systems. For example, double pendulums are used to model the motion of robot arms with high degrees of freedom, or to understand system stability and control in the context of autonomous (Nenuwe, 2019). Further research in chaos dynamics and bifurcations has also shown how these systems can exhibit behavior that is highly sensitive to initial conditions, resulting in chaos effects that are often difficult to predict. This knowledge is crucial in designing robust and adaptive control systems, especially in technological applications that require high precision (Kurcbart et al., 2009).

With these various extensions, the double pendulum remains a relevant model for studying the dynamics of non-linear systems, both in theoretical and applied contexts. The development of more advanced numerical and analytical techniques allows for a more in-depth study of these systems, with a focus on an increasingly wide range of practical applications in various scientific and engineering fields. However, despite the growing interest and numerous studies on the double pendulum, there is a noticeable gap in synthesizing this body of research through a comprehensive and systematic literature review. Many existing works focus on specific aspects or applications without providing a holistic overview of the methodologies, findings, and emerging trends. This gap highlights the necessity of conducting a Systematic Literature Review to map out the current state of research, identify overlooked areas, and propose future directions.

Mathematic Model

Mathematical models play an important role in analyzing and understanding the dynamic behavior of physical systems, including the double pendulum system. In this context, mathematical models are used to describe the interactions between the components of the system and formulate equations of motion that reflect the dynamics of the system. There are several approaches commonly used in the development of mathematical models for double pendulum systems, including the Lagrangian, Hamiltonian, and Equation of Motion approaches.

The Lagrangian approach, which is based on Lagrange's principle, relates the kinetic energy and potential energy of the system. By using generalized variables, this approach allows researchers to express the equations of motion in a simpler and more elegant form, making them easier to analyze (Yu et al., 2018). This approach is very useful in capturing nonlinear dynamics and interactions between components, which are the main characteristics of the double pendulum system. Research by Nenuwe (2019) shows that the Lagrangian approach can be used to analyze more complex systems by adding new elements, such as external forces or interactions between pendulums.

Meanwhile, the Hamiltonian approach focuses on the total energy of the system and is often used to analyze the conservative properties of dynamic systems. In this approach, the system is expressed in terms of a Hamiltonian function, which is the sum of kinetic energy and potential energy. The Hamiltonian approach provides additional insight into system behavior, especially in stability and chaos analysis (Nenuwe, 2019). Research Maraslidis et al. (2022) shows how this approach can be applied to design more effective controls in systems involving inverted pendulums.

On the other hand, the Equation of Motion approach involves directly writing the equations based on Newton's laws or basic principles of dynamics. Using this method, researchers can explicitly write down the relationship between the forces acting on the system and the resulting motion. This approach is often used in practical applications because it provides a clear understanding of how external and internal forces affect system dynamics (Orsini, 2021). Research by Zhang et al. (2020) shows that this approach can be used to develop better control strategies in crane systems involving the double pendulum effect.

Methods

Systematic data collection using the PRISMA (Preferred Reporting Items for Systematic Reviews and Meta-Analyses) reference procedure, which includes selected scientific searches, inclusion and exclusion criteria used, and eligibility met.

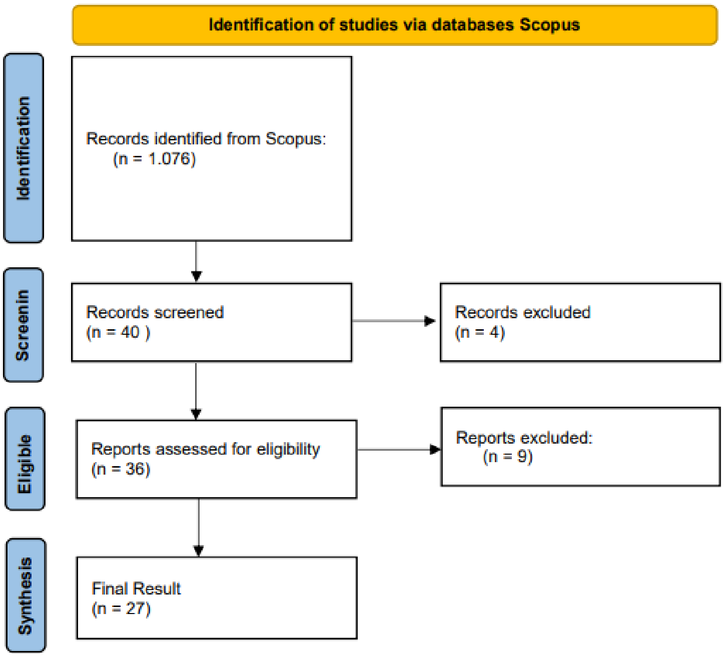

Figure 1. Flow of Search and Selection Procedure

Literature Search

Literature search was conducted using the Scopus database. The search was limited to article titles containing the words "double pendulum" to ensure that the documents obtained were truly relevant and specifically discussed double pendulum systems. This search was conducted using the advanced query "TITLE (double AND pendulum)," resulting in 1,076 documents.

Although limiting the search to titles can increase the specificity of the search results, further studies are recommended to expand the scope of the search to include abstracts (ABS) and keywords (KEY) in order to capture relevant studies that do not explicitly include the term in the title. In this study, advanced search techniques such as backward snowballing and forward snowballing were not applied because the scope of the research had been systematically determined through database queries.

Inclusion and Exclusion Criteria

The selected time frame for the literature search was four years, from 2020 to 2023. The inclusion criteria covered all types of articles, with the exception of conference papers, book chapters, and errata. To simplify the synthesis process in the next stage, articles with restricted access were also excluded. Thus, only articles with free access were included in the analysis, as outlined in Table 1. The selection of open-access articles is intended to ensure that all articles can be accessed and reviewed comprehensively, given the limited access to closed publications.

Table 1. Inclusion and Exclusion Criteria

Criteria	Inclusion	Exclusion
Year	2020-2023	<2020 or >2023
Document Type	Article	Note, Review, Letter, Erratum
Open access	All	Green, Hybrid Gold, bronze

This combination of search identification and criteria can be expressed with the advanced query "TITLE (double AND pendulum) AND PUBYEAR > 2019 AND PUBYEAR < 2024 AND (LIMIT-TO (DOCTYPE , "ar")) AND (LIMIT-TO (LANGUAGE, "English")) AND (LIMIT-TO (OA, "all"))". We obtained 40 documents for restriction with these criteria

Eligible

The next step was to verify the availability of documents for access. From the verification results, 36 documents were successfully accessed and met the eligibility criteria. Meanwhile, 4 other documents were not included in the analysis because their full text was not accessible. The successfully accessed documents were then used in the synthesis process in the next stage.

Synthesis

The synthesis process was carried out on 36 documents that had been obtained, focusing on the specified categories, namely the development of a double pendulum system and its decomposition model. From the synthesis results, 9 articles were not included because they did not meet the criteria, namely they did not discuss the development of a double pendulum system. Thus, 27 articles were obtained that met the synthesis criteria and were relevant for further analysis.

Results and Discussion

Double Pendulum System Expansion

Based on an analysis of 27 articles that met the research criteria, 15 variations or developments of the double pendulum system were found. Of these 15 variations, there were three types of systems that were most dominant based on the number of articles discussing them. Eleven articles discussed the gantry pendulum crane system with payload, two articles discussed the pendulum bridge crane system with distributed mass payload, and two other articles investigated the quadrotor system carrying a double pendulum.

From the results of this analysis, it can be concluded that the gantry pendulum crane system with payload is one of the most significant topics in the study of double pendulum system development, as shown in Table 2.

Thematic analysis reveals significant differences in characteristics between each system. Crane-based systems, such as gantry and bridge cranes, generally involve larger masses and emphasize load stability during lifting and movement processes. These systems have fewer degrees of freedom but present significant challenges in controlling load oscillations and external disturbances.

Conversely, quadrotor-based systems have a higher degree of freedom and exhibit greater sensitivity to chaotic behavior due to lighter loads and dynamic flight conditions. Therefore, control requirements for quadrotor systems are far more complex and necessitate adaptive control strategies to maintain stability during transportation.

Based on the analysis results, the most widely discussed double pendulum system expansion is the "pendulum gantry crane with load." This system is designed to lift and move heavy loads using two connected pendulums, where one pendulum functions as a lifting arm and the other as a support pendulum, as illustrated in Figure 2. This gantry crane is commonly used in industrial applications, such as in ports for moving containers and in construction for lifting heavy materials, where precision and stability are critical.

Table 2. Inclusion and Exclusion Criteria

System Expansion	Freq	Reference
Pendulum gantry crane with payload	11	(Alhassan et al., 2023; Amer et al., 2025; Chai et al., 2021; Jaafar et al., 2021; Orsini, 2021; Qiang et al., 2021; Shi et al., 2022; T. Wang et al., 2021; Wu et al., 2022; M. Zhang et al., 2020b; W. Zhang et al., 2021a)
Crane bridge pendulum with payload mass distributed	2	(Lizuain et al., 2020; Vrabel, 2021)
Quadrotor that transports a double pendulum	2	(Estevez et al., 2021; Qi et al., 2022)
Pendulum double with varying rod lengths and a center of mass moving in an elliptical orbit	1	(de Menezes Neto et al., 2022)
Installed boom bolster on double pendulum suspensions	1	(Chen et al., 2012)
Spatial double pendulum with axis connection cylinder non-parallel.	1	(Smirnov & Smolnikov, 2021)
Double-Inverted Pendulum (DIP) mounted on the train	1	(Maraslidis et al., 2022)
Double pendulum used in the context of swing control on a jib crane mounted in a ship.	1	(J. Wang et al., 2024)
Double pendulum used in crane control rotary	1	(Ouyang et al., 2021)
Two rotating pendulums. This set is mounted on the LDS V780 shaker head, and both pendulum bodies are made of aluminum beam	1	(Wojewoda, 2023)
Active suspension of spray boom based on the double pendulum system	1	(Li et al., 2023)
Compound double pendulum	1	(Williams, 2023)

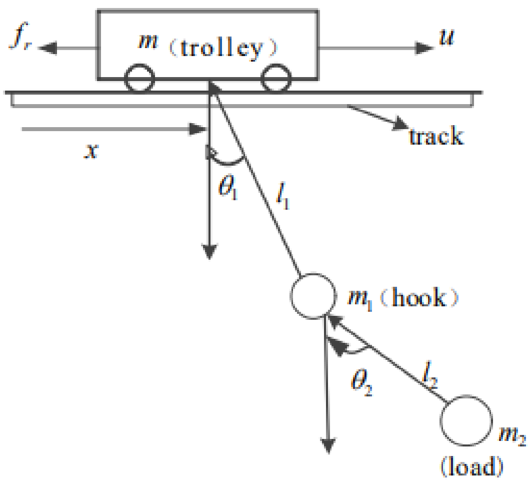

Figure 2. Double Pendulum Crane Model

In solving the double pendulum system that has been developed, it is necessary to select the appropriate model. One commonly used approach is the decomposition model, which is a method that aims to break down complex systems into simpler components so that analysis and solution become easier. By applying decomposition, a system that was previously difficult to analyze as a whole can be understood through its separate parts, both in terms of dynamics and control.

Based on the study in this research, the decomposition model used can be categorized into several types of models, which are described in detail in Figure 3.

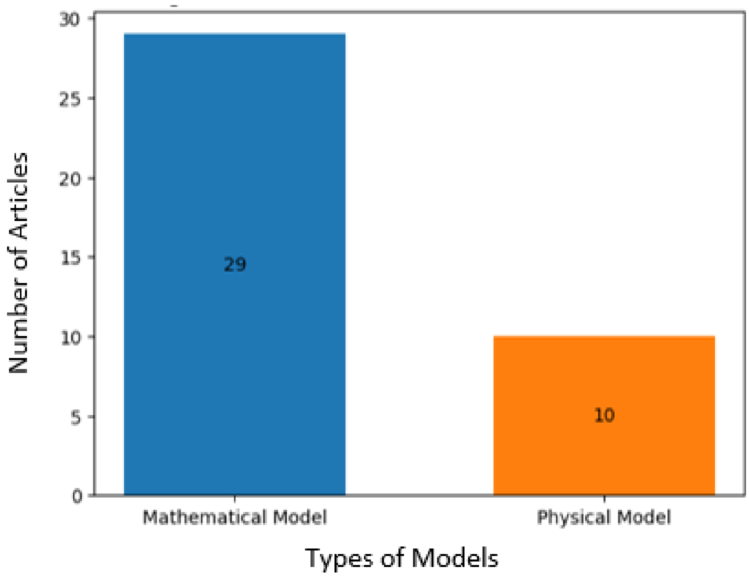

Figure 3. Categories of Models Used in Double Pendulum Studies

There are two categories of models used in this study, namely mathematical models and physical models. Mathematical models are abstract representations of systems that use mathematical equations to describe dynamic behavior and interactions between components within a system. These models generally employ three main approaches: Lagrangian, Hamiltonian, and the Equation of Motion, which are then analyzed numerically.

The Lagrangian approach focuses on kinetic and potential energy within the system, enabling a simpler and more elegant analysis of nonlinear dynamics. The Hamiltonian approach, on the other hand, emphasizes the total energy of the system and is often used to analyze the conservative properties of a dynamic system. Meanwhile, the Equation of Motion approach directly derives the equations of motion based on Newton's laws, providing a more concrete understanding of the influence of forces on the system's motion.

Based on the study results, 29 articles used mathematical models, with 7 articles using the Lagrangian approach, 2 articles using the Hamiltonian approach, and 20 articles using the Equation of Motion approach. Meanwhile, 10 articles applied physical models. Physical models are real representations of systems that can be tested and observed directly, either in the form of prototypes or physical simulations. These models allow researchers to observe the behavior of systems under real conditions and test hypotheses derived from mathematical models.

It should be noted that not all articles use only one type of model; some articles combine two to three models simultaneously to obtain more comprehensive analysis results. Based on the data obtained, the majority of studies primarily use mathematical models, particularly the Equation of Motion approach, as the main method for analyzing system dynamics. One dominant example of application is in the gantry pendulum crane with payload system, where the Equation of Motion is the primary approach in analyzing the dynamics of that system.

The use of a combination of mathematical and physical models provides a more comprehensive understanding of system behavior and enables the design of more effective solutions in practical implementation.

From the physical system in Figure 2, 3 degrees of freedom obtained:

$$\ddot{x} = -l_1\ddot{\theta}_1 + l_2\ddot{\theta}_2 + g\theta_2$$

$$\ddot{\theta}_1 = -\frac{x_z}{g} - \frac{m_1 l_2}{(m_1+m_2)g^2}x_z$$

$$\ddot{\theta}_2 = -\frac{x_z}{g}$$

After obtaining 3 degrees of freedom, this system is solved using several approaches, namely Lagrangian, Hamiltonian, and Equation of Motion. It was found that there were 10 that used the Equation of Motion approach, namely Zhang et al. (2021), Chai et al. (2021), Jaafar et al. (2021), Orsini (2021), Qiang et al. (2021), Wang et al. (2021), Wu et al. (2022), Shi et al. (2022), Fan et al. (2023), and Alhassan et al. (2023). There are also those who use the Lagrangian approach, namely Zhang et al. (2020).

After being solved using a mathematical model, the system is then visualized using numerical methods. Among those who use Matlab are Chai et al. (2021), Jaafar et al. (2021), Orsini (2021), Zhang et al. (2021), Shi et al., (2022), Alhassan et al. (2023), and Fan et al. (2023). There are also those who use the Runge Kutta method, namely Zhang et al. (2020) and Alhassan et al. (2023).

The research results show various variations and extensions of the double pendulum system that are the focus of scientific study. One of the most prominent topics is the development of a "pendulum gantry crane with load," which reflects a keen interest in practical applications and load control technology. This research not only focuses on mechanical aspects but also considers factors such as mass distribution, as seen in the study of the "bridge pendulum crane with distributed mass." This underscores the importance of a deeper understanding of how mass distribution affects system dynamics, which is crucial in designing more efficient and stable systems.

Additionally, the integration of drone technology with pendulum systems, as seen in the study "quadrotor carrying a double pendulum," opens new opportunities in the development of more complex transportation and load control applications.

Further analysis shows that most research uses mathematical models, with the Equation of Motion approach being the most widely applied method. This approach is widely used because it directly derives the dynamic equations from the forces acting on the system and is easy to implement in numerical simulations and control algorithms. The Equation of Motion is considered more effective, especially in the context of crane and robotic systems, where analyzing external forces and load dynamics are critical aspects for maintaining system stability and performance.

Thus, the use of appropriate mathematical models is crucial for understanding the dynamic behavior of double pendulum systems and designing effective solutions to various challenges in practical applications. These findings contribute significantly to the further development of mathematical models and their application in various disciplines, including robotics and control systems.

Conclusion

Based on the results of the research and analysis, it can be concluded that the soil pH in the agricultural land of Gemuhblanten Village for maize plants is 7.75 which is included in the "slightly alkaline" criteria. Meanwhile, in chili plants, soil pH of 7.08 was obtained, included in the "neutral" category. The pH value of the soil in chili plants is still within the range of soil pH requirements for chili growth. Meanwhile, the soil pH value for maize plants was quite high, exceeding the range of good soil pH for maize plants.

Soil quality is one of the important factors that must be considered in plant cultivation, especially if the land is used for cultivating different plants. One way to know the quality of the soil is by measuring the pH or acidity level of the soil. Each plant requires a different soil pH according to the criteria that have been set in order to grow well. Through this research, recommendations or suggestions can be given, such as measuring and adjusting soil pH so that the resulting agricultural products are of good quality.

References

Alhassan, A. B., Shehu, M. A., & Muhammad, B. B. (2023). Sliding Mode Observer-based Super-twisting Controller for Under-actuated Bridge Cranes Subject to Double-pendulum Effect. IEEE Access, PP, 1. https://doi.org/10.1109/ACCESS.2023.3315321

Amer, A., Zhang, W., Amer, T. S., & Li, H. (2025). Nonlinear vibration analysis of a 3DOF double pendulum system near resonance. *Alexandria Engineering Journal*, 113(November 2024), 262–286. https://doi.org/10.1016/j.aej.2024.11.018

Baker, G. L., & Gollub, J. P. (1996). Chaotic Dynamics (2nd ed., p. 42). https://doi.org/10.1017/CBO9781139170864

Chai, L. I. N., Guo, Q., Liu, H., & Ding, M. (2021). Linear Active Disturbance Rejection Control for Double-Pendulum Overhead Cranes. XX, 1–14. https://doi.org/10.1109/ACCESS.2021.3070048

Chen, F., Hou, Y. F., Wu, H. Y., & Wang, H. W. (2012). Research on dynamic characteristics of disc brake. *Advanced Materials Research*, 383–390(3), 2845–2849. https://doi.org/10.4028/www.scientific.net/AMR.383-390.2845

Cunha Jr, A., Felix, J.L.P. & Balthazar, J.M. (2015). On the nonlinear dynamics of an inverted double pendulum over a vehicle suspension subject to random excitations. Proceedings of the 23rd ABCM International Congress of Mechanical Engineering, December. https://doi.org/10.20906/cps/cob-2015-1694

de Menezes Neto, J. L., Araujo, G. C., Rothen, Y. P., & Vidal, C. (2022). Parametric Stability of a Double Pendulum With Variable Length and With Its Center of Mass in an Elliptic Orbit. *Journal of Geometric Mechanics*, 14(3), 381–408. https://doi.org/10.3934/jgm.2021031

Estevez, J., Lopez-Guede, J. M., Garate, G., & Graña, M. (2021). A hybrid control approach for the swing free transportation of a double pendulum with a quadrotor. *Applied Sciences (Switzerland)*, 11(12). https://doi.org/10.3390/app11125487

Fan, L., Zhang, A., Pan, G., Du, Y., & Qiu, J. (2023). Swing-up and fixed-time stabilization control of underactuated cart-double pendulum system. IET Control Theory and Applications, 17(6), 662–671. https://doi.org/10.1049/cth2.12386

Jaafar, H. I., Mohamed, Z., Ahmad, M. A., Wahab, N. A., Ramli, L., & Shaheed, M. H. (2021). Control of an underactuated double-pendulum overhead crane using improved model reference command shaping: Design, simulation and experiment. Mechanical Systems and Signal Processing, 151. https://doi.org/10.1016/j.ymssp.2020.107358

Kurcbart, S., Oliveira, E., & Amaral, G. (2009). Chaotic double pendulum construction proposals. May 2014.

Li, F., Bai, X., Su, Z., Tang, S., Wang, Z., Li, F., & Yu, H. (2023). Development of a Control System for Double-Pendulum Active Spray Boom Suspension Based on PSO and Fuzzy PID. Agriculture (Switzerland), 13(9). https://doi.org/10.3390/agriculture13091660

Lizuain, I., Tobalina, A., Rodriguez-Prieto, A., & Muga, J. G. (2020). Invariant-based inverse engineering for fast and robust load transport in a double pendulum bridge crane. *Entropy*, 22(3). https://doi.org/10.3390/e22030350

Maraslidis, G. S., Kottas, T. L., Tsipouras, M. G., & Fragulis, G. F. (2022). Design of a Fuzzy Logic Controller for the Double Pendulum Inverted on a Cart. *Information (Switzerland)*, 13(8). https://doi.org/10.3390/info13080379

Nenuwe, N. O. (2019). Application of Lagrange equations to 2D double spring-pendulum in generalized coordinates. *Ruhuna Journal of Science*, 10(2), 120. https://doi.org/10.4038/rjs.v10i2.78

Orsini, V. (2021). A Robust Offline Precomputed Optimal Feedforward Control Action for the Real Time Feedback/Feedforward Control of Double Pendulum Gantry Cranes. IEEE Access, 9, 158273–158286. https://doi.org/10.1109/ACCESS.2021.3131032

Ouyang, H., Xu, X., & Zhang, G. (2021). Boom motion trajectory generation approach for load sway rejection in rotary cranes considering double-pendulum effect. *Measurement and Control (United Kingdom)*, 54(5–6), 924–934. https://doi.org/10.1177/0020294020944964

Qi, J., Ping, Y., Wang, M., & Wu, C. (2022). Online trajectory planning method for double-pendulum quadrotor transportation systems. *Electronics (Switzerland)*, 11(1), 1–14. https://doi.org/10.3390/electronics11010050

Qiang, H., Sun, Y., Lyu, J., & Dong, D. (2021). Anti-Sway and Positioning Adaptive Control of a Double-Pendulum Effect Crane System With Neural Network Compensation. 8(April), 1–9. https://doi.org/10.3389/frobt.2021.639734

Shi, H., Yao, F., Yuan, Z., Tong, S., Tang, Y., & Han, G. (2022). Research on nonlinear coupled tracking controller for double pendulum gantry cranes with load hoisting/lowering. *Nonlinear Dynamics*, 108(1), 223–238. https://doi.org/10.1007/s11071-021-07185-6

Smirnov, A., & Smolnikov, B. (2021). Collinear control of oscillation modes of spatial double pendulum with variable gain. Cybernetics and Physics, 10(2), 88–96. https://doi.org/10.35470/2226-4116-2021-10-2-88-96

Steven H. Strogatz. (2018). Nonlinier With Applications to Physics, Dynamics Biology, Chemistry, and Engineering And Chaos.

Strogatz, S. H. (2018). Nonlinier With Applications to Physics, Dynamics Biology, Chemistry, and Engineering And Chaos.

Vrabel, R. (2021). Design of the State Feedback-Based Feed-Forward Controller Asymptotically Stabilizing the Overhead Crane at the Desired End Position. *Journal of Applied Nonlinear Dynamics*, 10(2), 339–350. https://doi.org/10.5890/JAND.2021.06.012

Wang, J., Liu, K., Wang, S., & Sun, Y. (2024). Dynamic analysis and experiment of underactuated double-pendulum anti-swing device for ship-mounted cranes. *Journal of Huazhong University of Science and Technology (Natural Science Edition)*, 52(1), 72–77. https://doi.org/10.13245/j.hust.240553

Wang, T., Tan, N., Zhang, X., Li, G., Su, S., Zhou, J., Qiu, J., Wu, Z., Zhai, Y., Donida Labati, R., Piuri, V., & Scotti, F. (2021). A Time-Varying Sliding Mode Control Method for Distributed-Mass Double Pendulum Bridge Crane with Variable Parameters. IEEE Access, 9, 75981–75992. https://doi.org/10.1109/ACCESS.2021.3079303

Williams, H. (2023). A compound double pendulum with friction. Forces in Mechanics, 10(October 2021). https://doi.org/10.1016/j.finmec.2022.100164

Wojewoda, J. (2023). Kinovea analysis of high-speed video recording to determine kinematics of double pendulum in the long time scale. *Bulletin of the Polish Academy of Sciences: Technical Sciences*, 71(6), 1–10. https://doi.org/10.24425/bpasts.2023.147918

Wu, Q., Sun, N., & Wang, X. (2022). Equivalent Rope Length-Based Trajectory Planning for Double Pendulum Bridge Cranes with Distributed Mass Payloads. *Actuators*, 11(1), 1–14. https://doi.org/10.3390/act11010025

Yu, H., Lei, X., Song, Z., Wang, J., Wei, X., & Yu, B. (2018). Functional brain connectivity in Alzheimer's disease: An EEG study based on permutation disalignment index. *Physica A: Statistical Mechanics and Its Applications*, 506, 1093–1103. https://doi.org/10.1016/j.physa.2018.05.009

Zhang, M., Zhang, Y., Ji, B., Ma, C., & Cheng, X. (2020). Modeling and energy-based sway reduction control for tower crane systems with double-pendulum and spherical-pendulum effects. *Measurement and Control (United Kingdom)*, 53(1–2), 141–150. https://doi.org/10.1177/0020294019877492

Zhang, W., Chen, H., Chen, H., & Liu, W. (2021). A Time Optimal Trajectory Planning Method for Double-Pendulum Crane Systems with Obstacle Avoidance. IEEE Access, 9, 13022–13030. https://doi.org/10.1109/ACCESS.2021.3050258