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Hybrid Quantum Representation and Hilbert Scrambling for Robust Image Watermarking

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Abstract.
Purpose: This work aims to apply Quantum Hilbert Scrambling to enhance the security and integrity of image watermarking without affecting visual quality degradation. Further conception of the surveyed methods could result in a very good solution to conventional methods of watermarking in solving some problems of digital image security and integrity with new concepts of quantum computing.
Methods: The paper reviews Quantum Hilbert Scrambling, whose computational complexity is $O\ (n \cdot 2^{2n})$. The process involves encoding the image into a quantum state, permuting qubits by the Hilbert curve, and embedding a watermark using quantum gates.
Result: The quantitative performance evaluation metrics, like Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index (SSIM), have shown high Peak Signal to Noise Ratio (PSNR) values from 56.13 dB to 57.87 dB and Structural Similarity Index (SSIM) from 0.9985 to 0.9990, correspondingly. This justifies the fact that the quality degradation is very slight and the fine details of the structure are well maintained.
Novelty: The proposed method uniquely integrates quantum computing with traditional watermarking steps for a secure and effective approach in digital watermarking. Further development should focus on improving the quantum circuit regarding computation efficiency, extending the applicability of the method to a wide range of images, and various situations in watermarking, and finding hybrid approaches by combining quantum and classical approaches towards better performance and scalability.

Keywords: Hilbert scrambling, Image watermarking, Quantum hilbert scrambling, Performance measurement
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INTRODUCTION

Data in this digital world has grown as the most important strategic asset applied for decision-making and attached value for innovation capability enhancement to organizations [1], [2]. The helpful utilization of digital technology undoubtedly brings the effectiveness and practicality of the social activities further. On the other hand, without appropriate care in usage, it will bring adverse implications on human life. Ever-advancing Internet technology integrates and unifies all forms of digital information whereby even photos or images, which are essentially private and sensitive media, can easily be intercepted by unauthorized users [3], [4]. Besides that, the digital image itself has the inherent nature that makes it easily tampered with, manipulated, and copied illegitimately, thus rendering the detection of change or ownership practically impossible with today's technology [5]. Authentication processes and copyright of images try to trace and prevent unauthorized duplication, piracy, and their illegal manipulation and distribution which takes owners' rights away from them [6].

The one method used for digital data security include steganography [7], [8]. Steganography is the methodology of disguising any secret information in a medium in such a way that its detection is not possible, hence finds essential applications in various arenas of secure communications using digital images [9]. Watermarking, a sub-domain of steganography, is considered to be the process of embedding information into media without perceptibly changing the medium [10]. Digital Image Watermarking provides security and privacy for digital images, protecting their copyrights, hence essential in the development of multimedia applications [11], [12], [13].

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Many researchers worked on image watermarking, and their research has contributed much to security and robustness. In the technique proposed by Ma's [14], the encryption has been performed using quantum key distribution combined with pixel scrambling to overcome the shortcomings of classical image encryption algorithms. It has pointed out that the evaluation of quantum circuit performance should be done with security. The experimental results showed high NPCR, uniformly distributed encrypted histograms, low pixel correlation, and high information entropy. The work of Chen et al. [15] proposed a quantum image encryption scheme based on a binary image key and qubit rotation for counterattack against the attacks from both the spatial and frequency domains. The simulation results indicate that this approach can effectively alter image position and color information toward more security, tested with the standard deviation of quantum sequences.

Sharma et al. [16] developed a steganographic method of social network images using graphical signal processing based on quantum scrambling and graphical wavelet transform. The experiment demonstrated the improved pixel correlation without loss of visual quality of stego images along with secret image extraction. Liu et al. [17] proposed a novel quantum image encryption scheme which introduced independent bit-plane permutation into quantum Baker maps to enhance the security and efficiency of the scheme. The experimental results show that the proposed scheme performs well regarding statistical features and key sensitivity with a large capacity of concealing information with imperceptible image quality.

Jiang et al. [18] presented a quantum watermarking scheme based on quantum Hilbert mixing with Moiré fringe steganography on NEQR model is competent. The simulations showed that the watermarked and carrier images had a high degree of visual similarities and higher time complexities compared to other schemes. Kumar et al. [19] proposed an effective SVD and quantum Hilbert image mixing-based watermarking scheme that would enhance both the features of robustness and security in image authentication. Its strength has also been supported with strong experimental results relating to different image quality parameters. That said, it will also go a long way in enhancing the capacity of steganography to hide messages with high levels of security in images.

The work relies on relevant research in related work for integrating Quantum Hilbert Scrambling and Quantum Key Distribution during image watermarking, which is essential in improving image safety. Their technique lies in enhancing information security within the images in digital form using specific properties of quantum mechanics. In fact, the fundamental technique achieved partial realization of improving security with efficacy. These are integrated into this work as methods with the best results in improving security not only in the application of the watermarking process but also in ensuring that the hidden information in images is well-guarded against intrusion or unwarranted disclosures. It is predictable that such development will definitely go a long way in the enhancement of digital data security, especially in the aspects that involve image processing.

METHODS

The image watermarking scheme proposed here begins by initializing both the host and the watermark image. The initial representations of both of these images are quantum images due to efficiency and security in processing. Embedding is a process that involves embedding a quantum-represented watermark image onto a quantum-represented host image and integrated into one embedded quantum image; it is processed by Hilbert scrambling for enhanced security. Then, the wavelet transform will process the denoised shuffled image to get a watermarked image. Performance measurement is done for performance, robustness, and quality of watermarking. The host image retrieves itself from the watermarked image during extraction in such a way that the integrity and authenticity of the watermark are guaranteed. Figure 1 presents the flow of the proposed method.

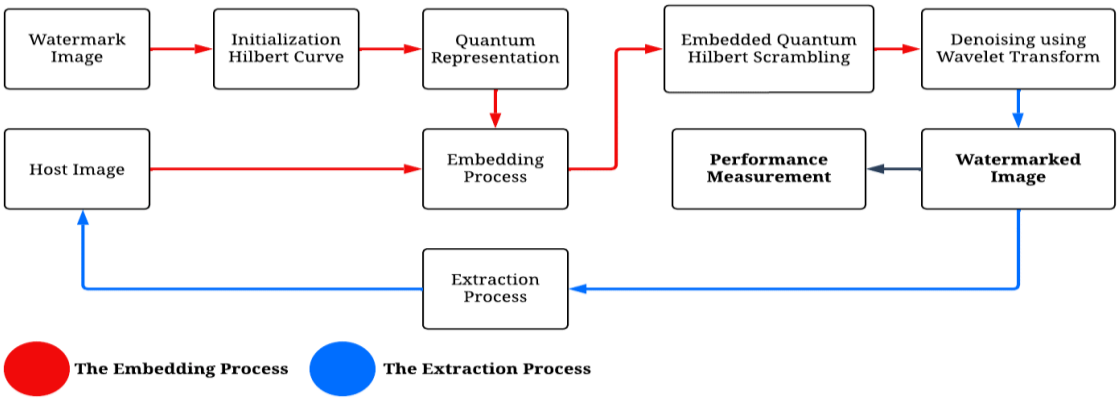


Figure 1. Flow of proposed scheme

Hilbert scrambling

Hilbert scrambling is the most well-known method of rearranging the position of pixels in a watermark image, but actually it is a space-filling curve mapping two-dimensional image coordinates to a one-dimensional sequence in a very complicated but orderly pattern [20], [21]. During the embedding process, the watermark image's pixel positions are rearranged using the Hilbert curve to enhance security by making the watermark difficult to detect. The order n of the Hilbert curve determines the granularity of the scrambling pattern: for $n = 3$, the curve divides the image into 8x8 blocks with 64 unique positions; for $n = 4$, the image is divided into 16x16 blocks with 256 positions; and for $n = 5$, the image is divided into 32x32 blocks with 1024 positions [22]. Each step in order n increases the degree of complexity and resolution in the scrambling process, making the watermark image even more hidden and resistant to various types of attack. That way, every pixel of the watermark image will be assigned a new place according to the Hilbert sequence until a scrambled but still structured image is obtained. After that, it will be ready for embedding into the host image.

For the order of the Hilbert curve n, it will subdivide the given space into $2^n \times 2^n$ blocks. The key idea is to map the 2D coordinates (x, y) into a 1D index using a recursive approach. the 1D index $H(x, y)$ can be computed using eq (1).

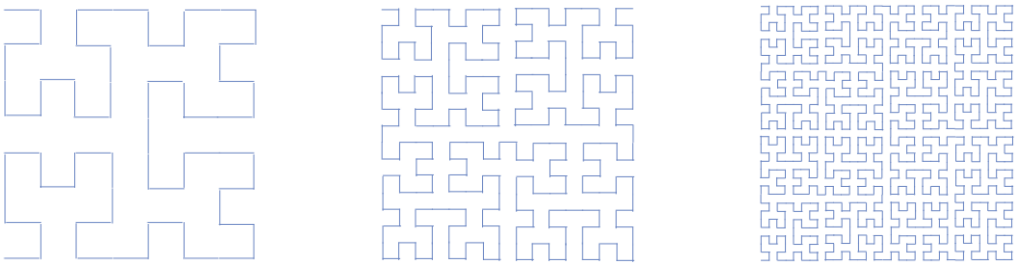
$$H(x, y) = Order - (n) + Quadrant - (x, y, n) \tag{1}$$

where $Order - (n)$ is the order-dependent base index, and $Quadrant - (x, y, n)$ is a function that maps the coordinates (x, y) to a specific quadrant of the Hilbert curve of order n. The function $Quadrant - (x, y, n)$ determines which of the four quadrants of the Hilbert curve the point (x, y) falls into. This function is defined in eq (2).

$$Quadrant - (x, y, n) = \begin{cases} 0\ if\ (x\ and\ y\ are\ both\ in\ the\ first\ quadrant) \\ 1\ if\ (x\ is\ in\ the\ second\ quadrant) \\ 2\ if\ (x\ is\ in\ the\ third\ quadrant) \\ 3\ if\ (x\ is\ in\ the\ fourth\ quadrant) \end{cases} \tag{2}$$

Eq (1) shows how to turn 2D coordinates (x, y) into a single index $H(x, y)$. The number n in the Hilbert curve determines how detailed the scrambling will be. The function $H(x, y)$ has two parts: Order-(n) determines the base index for the order, and Quadrant$-(x, y, n)$ indicates which quadrant of the Hilbert curve the pixel (x, y) is in. Quadrant mapping (shown in eq (2)) tells abour which quadrant something is in: 0 for the top right, 1 for the top left, 2 for the bottom left, and 3 for the bottom right [23]. This method scrambles the image effectively by moving each pixel to a different position using the Hilbert curve. This increases the security of the watermark image before it is added to the host image.

In the Hilbert curve visualization shown in figure 2, the order n of the Hilbert curve dictates the complexity and resolution of the curve's pattern.



(a) hilbert curve with n=3 (a) hilbert curve with n=4 (c) hilbert curve with n=5

Figure 2. Representation of hilbert curve

Quantum circuit representation

Quantum circuit essentially embeds the physics of quantum into Hilbert scrambling for the purpose of image watermarking [24], [25]. Such methodology transforms the image into its quantum state, encoding information at each pixel into qubits with the help of quantum gates and circuits [20]. It then applies the Hilbert scrambling process in this quantum platform to reorder the qubits along the Hilbert curve such that it considers generating a complex and secure pattern for the watermark. Quantum circuits allow, in this respect, subtler manipulations of the qubits by making use of quantum superposition and entanglement to effectively embed the watermark.

The quantum state of an image is represented by a vector in a Hilbert space. For an image with N pixels, where each pixel is encoded into n qubits, the state vector $|\psi\rangle$ can be seen in eq (3).

$$|\psi\rangle = \sum_{i=0}^{2^{n-1}} ci\,|i\rangle \tag{3}$$

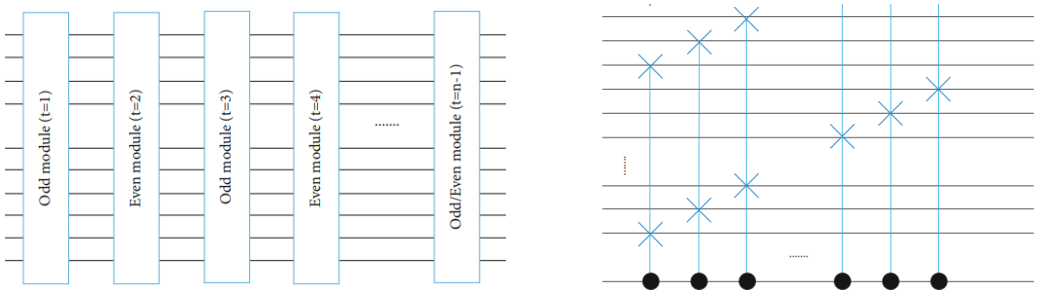
where $|i\rangle$ are the basis states, and ci are complex coefficients such eq (4):

$$|\psi\rangle = \sum_{i=0}^{2^{n-1}} ci\,|^2 = 1 \tag{4}$$

The process of embedding the watermark carries information about the watermark onto the quantum state. This is done using unitary operations that alter the quantum state of the image. This embedding operation may be represented through a unitary transformation U_{embed} If $|\psi original\rangle$ is the original quantum state of the image and $|\psi\ embedded\rangle$ is the state after embedding, the transformation is given as in eq (5).

$$|\psi embedded\rangle = U_{embed}\ \ |\psi original\rangle \tag{5}$$

where U_{embed} is a unitary matrix defined by the embedding algorithm. Based on embedded method, The quantum circuit can be seen in Figure 3.



(a) Quantum circuit of hilbert scrambling (b) Embedding quantum circuit

Figure 3. Quantum circuit

Figure 3 (a) illustrates the quantum circuit for Quantum Hilbert Scrambling and figure 3 (b) illustrates the quantum circuit of the embedding system, where the circuit is structured into alternating odd and even modules based on the recursion level t. Specifically, the circuit starts with the odd module at $t = 1$, followed by the even module at $t = 2$. This pattern continues with odd modules at $t = 3$ and even modules at $t = 4$. The process is iteratively applied, alternating between odd and even modules, until reaching the final level, denoted as $t = n - 1$, where the module type (odd or even) is determined based on the recursion depth. Based on figure 3 (a), the quantum circuit alternates between odd and even modules. For odd and even modules at recursion level t, the quantum operation can be seen in eq (6) and eq (7). And the overall quantum Hilbert scrambling process can be seen in eq (8). The final scrambled quantum state $| \psi n - 1\rangle$ at the last recursion level $t = n - 1$ is obtained by applying the alternating module operations across all levels are represented in eq (8).

$$| \psi t, odd\rangle = U_{odd}(t) \ | \psi_{t-1}\rangle \tag{6}$$

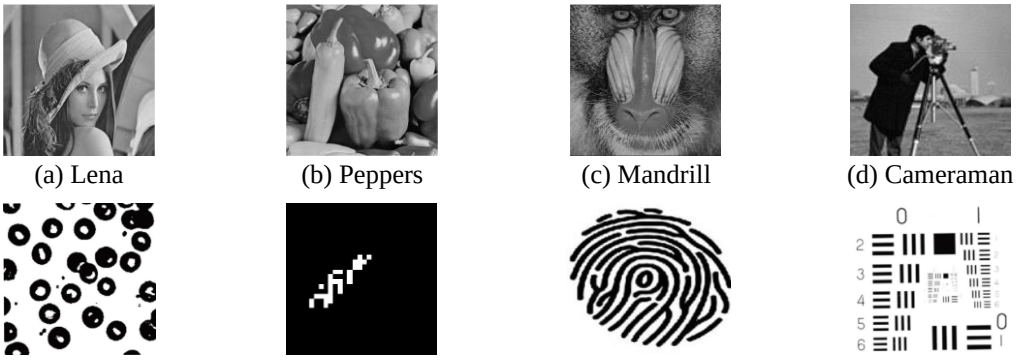
$$| \psi t, odd\rangle = U_{even}(t) \ | \psi_{t-1}\rangle \tag{7}$$

$$| \psi_{n-1}\rangle = \prod_{t-1}^{n-1} U_{module} \ (t) \ | \psi \ 0\rangle \tag{8}$$

Where $|\psi_{t-1}\rangle$ is the state from the previous recursion level. And $U_{module}\ (t)$ denotes the module operation (either odd or even) at each recursion level. For instance, quantum divide-and-conquer techniques have been used to solve traditional problems like reachability faster by applying recursive principles. This means breaking down quantum operations into different levels, just like how equations describe applying odd and even unitary operations at different steps [26], [27].

RESULTS AND DISCUSSIONS

This work presents experiments conducted for estimating the efficiency of the approach of image watermarking proposed in this paper. Before any testing or performance evaluation, the initiated software environment was executed on MATLAB 2020a on hardware with an Intel Core i9 11th Generation Processor, Iris Graphics Card. It was used in a quantum circuit with a complexity of $O\ (n \cdot 2^{2n})$. The proposed watermarking scheme has been tested on four host images, namely Lena, Peppers, Mandrill, and Cameraman, and performance is considered for four different watermarking scenarios. Initial host and watermark images are given in Figure 4, where panels (a) - (d) display the host images and panels (e) - (h) show the watermark images.

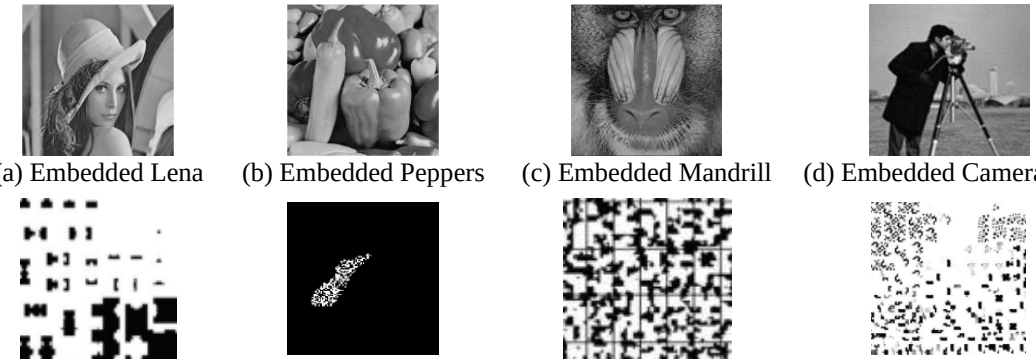


(a) Lena (b) Peppers (c) Mandrill (d) Cameraman

(e) (f) (g) (h)

Figure 4. Host and watermark image

Following the initialization and selection of host and watermark images, the embedding phase is carried out according to the proposed method. The embedding process involves incorporating the watermark into the host image based on the specified approach. The results obtained with the embedding of the watermark are shown in Figure 5. Images of watermarked images embedded by wavelet reconstruction are given in panels (a) - (d); images having watermarking achieved by Quantum Hilbert Scrambling are shown in panels (e) - (h).



(a) Embedded Lena (b) Embedded Peppers (c) Embedded Mandrill (d) Embedded Cameraman

(e) (f) (g) (h)

Figure 5. Embedded and scrambled results (a) – (d) represents watermarked image, (e) – (h) represents quantum scrambled of watermark image

The quality of the watermarked images is evaluated based on PSNR and SSIM measurements. For PSNR, the calculation is performed using the equation provided in Eq (9). Following the PSNR assessment, the Structural Similarity Index (SSIM) is computed to further evaluate image quality.

$$PSNR = 10 \log 10 \left(\frac{\max \_pixel\_value^2}{MSE} \right) \quad (9)$$

$$SSIM = \frac{(2\mu_x\mu_y + C1)(2\sigma_{xy} + C2)}{(\mu_x^2 + \mu_y^2 + C1)(\sigma_x^2 + \sigma_y^2 + C2)} \quad (10)$$

Following the PSNR [28] measurement, the quality of the watermark images is also analyzed using the Structural Similarity Index (SSIM). SSIM [28] measures the similarity between the original and watermarked images based on the structural information, luminance, and contrast.

Figure 5 shows the watermarked images, the results of the quality measurement for these watermarked images based on Figures 5(a)–(d) are shown in the table below. The table gives, with a detailed analysis, the performances concerning different metrics about the quality assessment of different watermarked images.

Table 1. PSNR measurement

Host/Watermark	Lena	Peppers	Mandrill	Cameraman
Watermark (e)	57.34 dB	54.37 dB	57.10 dB	57.87 dB
Watermark (f)	56.81 dB	55.25 dB	57.18 dB	58.03 dB
Watermark (g)	56.29 dB	54.90 dB	57.77 dB	57.33 dB
Watermark (h)	56.13 dB	55.98 dB	57.51 dB	57.40 dB

Table 2. SSIM measurement

Host/Watermark	Lena	Peppers	Mandrill	Cameraman
Watermark (e)	0.9987	0.9990	0.9989	0.9986
Watermark (f)	0.9986	0.9989	0.9989	0.9985
Watermark (g)	0.9987	0.9988	0.9989	0.9986
Watermark (h)	0.9987	0.9988	0.9988	0.9987

CONCLUSION

The evaluation of the proposed image watermarking method using Quantum Hilbert Scrambling, with a complexity of $O(n \cdot 2^{2n})$, demonstrates its efficacy in preserving image quality while embedding watermark information. Based on Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index (SSIM) metrics, it presents the best performance for different host images. The PSNR values obtained from various obtained watermarks are within the range of 56.13 to 57.87 dB, which demonstrates that no severe degradation has occurred within the image quality. The SSIM falls in the range 0.9985 to 0.9990, indicating very good structural and perceptual similarity between cover and marked images. It proves that the Quantum Hilbert Scrambling does not tamper much with the integrity of the image while embedding the watermark and thus may be considered for secure and high-quality applications of watermarking. For future

research, several avenues warrant exploration, that's one is further computational efficiency in the quantum circuit may raise the level to mitigate the complexity or even render it practical. Embedding Quantum Hilbert Scrambling into other state-of-the-art quantum image processing techniques is promising in better performance and robustness.

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