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PSNR and SSIM Performance Analysis of Schur Decomposition for Imperceptible Steganography

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Abstract.
Purpose: This research examines how well Schur decomposition-based steganography can hide data in digital images without being noticed, while also keeping the image quality high and keeping the hidden information safe.
Methods: The study starts by choosing regular test images (Lena, Plane, Peppers, Cameraman, Baboon) to use for hiding messages in. The Schur decomposition is used to hide information within images in a subtle way. To test how well the new method works, we added Gaussian noise and Salt & Pepper noise after embedding. The quality of the image is determined by looking at the Peak Signal to Noise Ratio (PSNR) and Structural Similarity Index (SSIM) metrics.
Result: The research shows that Schur decomposition results in very good SSIM values (greater than 0.92) and high PSNR scores (as high as 90.27 dB) for various image sizes (64x64, 128x128, 256x256). This means that the quality of the images is not greatly reduced even after steganography is applied.
Novelty: This research introduces a unique use of Schur decomposition for hiding data in images without affecting their quality. The study highlights how this method can securely hide information in digital media, which could be really useful for improving steganography techniques in the future. Future studies should concentrate on making improvements to Schur decomposition-based steganography, especially for bigger images. One possibility is to create adaptive methods that can change how images are hidden based on their content. This could make it harder for others to detect and analyze hidden information in the images.

Keywords: Imperceptible steganography, PSNR, SSIM, Schur decomposition.
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INTRODUCTION

Digital Image Processing is a field of study where a tangible image is generated by applying the process, and then analyzed to create information which can be easily understood by humans [1]–[4]. Digital image processing is aimed at manipulating images for improving viewing quality, making the objects more conspicuous to human eyes and extracting needed features of these [5], [6]. Contemporary, the speedy-looking development of image processing is an area of immense interest to research in robotics and computing [7], [8]. The image quality can be evaluated especially for Image processing to ensure how much effectively an original information is retained from the given transformation and compression techniques.

Various metrics are employed to evaluate the quality of images, including Mean Squared Error (MSE), Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index Measure (SSIM), and additional metrics for assessing image quality [9]. PSNR and SSIM are common metrics in image quality evaluation, particularly in steganography for assessing imperceptibility. PSNR and SSIM are widely-used measurements for evaluating the quality of images, especially in the field of steganography where they are used to gauge how well imperceptible hidden data is. PSNR is a commonly used metric that has been proven effective in measuring digital image quality, while SSIM is a newer metric that takes into account luminance, contrast, and structure to better model human visual perception [9], [10]. However, obtaining high-quality images relies on numerous important elements that have the ability to impact image clarity, including resolution, color depth, brightness, contrast, distortion, noise, artifacts, and disturbances [11], [12]. Noise, which can reduce the quality of images, must be removed to improve information retrieval. It

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is typically the result of technical disruptions or inadequate lighting conditions [13], [14]. There are different types of noise, such as Speckle Noise, Gaussian Noise, and Salt and Pepper Noise [15]. In order to attain the best possible image quality and enhance the results of image processing assessments, it is crucial to eliminate any noise that may have a negative impact on the quality of the image. Sound can lower the PSNR and SSIM metrics and corrupt crucial details within the image. One way to address this issue is by using algorithms that are able to reduce noise [16].

Many researchers study noise reduction to improve image quality and achieve better evaluation results. Liu et al. [17] proposed a blind color digital image steganography algorithm that uses affine transformation for steganography encryption and Schur decomposition for block matrices in the color channels. In experiments, the method employed was proven to be effective, showing excellent camouflage abilities, top-notch security measures, and impressive durability.

In the study by Haris et al. [18], An approach to noise reduction in digital images involved utilizing deep backprojection and down projection layers in a denoising technique. According to the experimental findings, this method has the potential to enhance quality by as much as 6 dB when compared to alternative techniques using the SIDO dataset. Fauziah et al. [[19] utilized the Gaussian Blur technique with OpenCV and Python to improve the resolution of the image. The findings of the study suggested that this technique effectively decreased noise and improved the clarity of image details.

Su et al. [20] investigated the safeguarding of copyrights for color images within extensive multimedia datasets by employing a steganography algorithm based on Schur decomposition. They chose image blocks that had minimal visual distortion to create steganographies that are less detectable than previous techniques. Awasthi et al. [21] Dual steganography techniques were employed on DICOM images in this study, utilizing Schur decomposition, lifting wavelet transformation, and firefly optimization. They showed strong resistance to different forms of attacks and achieved positive evaluation outcomes based on metrics such as PSNR, NCC, and SSIM.

Li et al. [19] utilizing Schur decomposition and contourlet transformation without sub-sampling to safeguard copyrights. They improved security by encrypting steganographic data prior to insertion and implementing synchronization methods to defend against geometric threats. The experimental results indicated that this scheme outperformed comparable methods in terms of imperceptibility, robustness, and capacity to carry data. It is anticipated that this method will improve steganography abilities by effectively concealing messages while also maintaining high PSNR and SSIM values.

According to a previous study, the goal of this research is to incorporate Gaussian attacks and salt and pepper noise into image histograms as a means of enhancing image security. This process involves obscuring the image through the Schur decomposition method, which has been shown to be successful in minimizing the signs of digital attacks. The findings reveal that incorporating these different attack variations greatly impacts the integrity of the image, as evidenced by the SSIM value showing how closely the modified image retains its original structure, along with the PSNR indicating the extent to which image quality declines as a result of obfuscation.

METHODS

The noise embedding procedure employed in the proposed image steganography technique merges Gaussian noise with Salt & Pepper noise. Once the ceramic noise image is generated, picture decomposition is carried out to ensure that the noise is imperceptible to the observer. This procedure ensures the preservation of the original image's visual quality while concealing steganography. SSIM and PSNR are the primary metrics utilized for assessing the results of noise embedding. The assessment evaluates the effectiveness of noise embedding in image steganography by comparing the fidelity and degree of detail of the steganographed image to the original image. Figure 1 displays the operational procedure proposed.

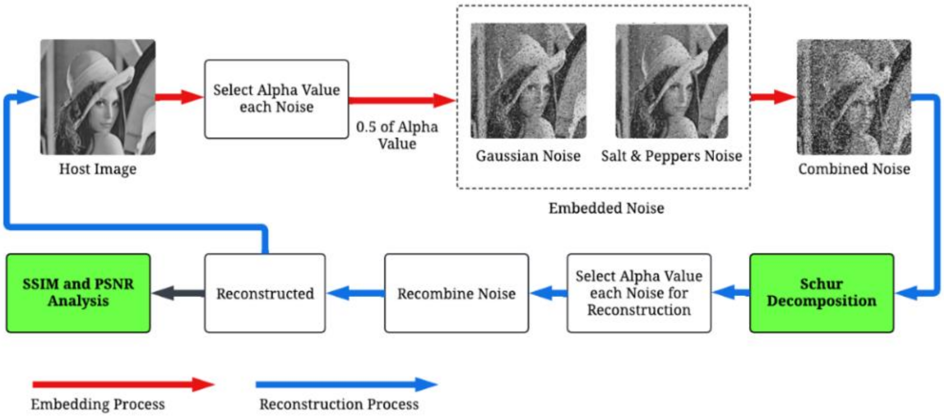


Figure 1. Proposed scheme

Variant attack noise

Noise attacks can have a negative effect on the quality of digital photographs. Different forms of noise interference, such as Gaussian, salt and pepper, speckle, and other variations [20], [22], [23]. Various types of noise exhibit unique characteristics and have varied effects on photographs, such as pixel intensity fluctuations and recognizable distortion. Caution is required when developing solutions for picture protection and restoration due to the potential for combined attacks involving multiple types of interference, which can result in complex and difficult-to-manage effects. As suggested by equations (1) and (2), this study proposes the integration of Gaussian noise and Salt & Pepper noise.

1) Gaussian Noise: Gaussian noise is generally described as random pixel values added to a digital image from a Gaussian distribution with zero mean and specified variance.

$$f\ (x, y) = g\ (x, y) + N\ (0, \sigma^2) \tag{1}$$

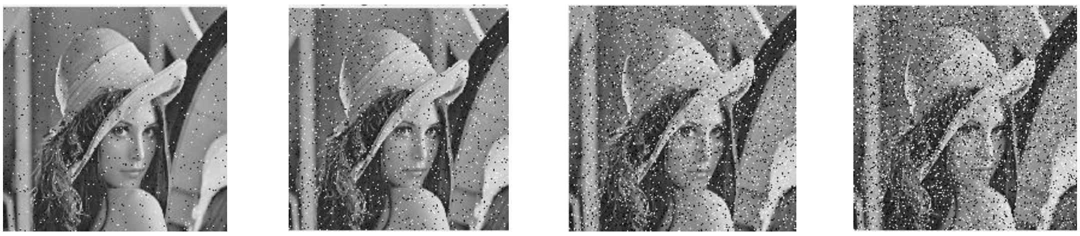
Where, $f\ (x, y)$ represents embedded gaussian noise, $g\ (x, y)$ represents original/host image, and $N\ (0, \sigma^2)$ represents adding gaussian noise to the host image with intensity $\mu = 0$ and varians σ^2.

2) Salt-and-Pepper Noise: Salt-and-Pepper noise causes random black and white dots to appear in an image. This is usually described as adding noise to certain pixels in the image with a certain probability.

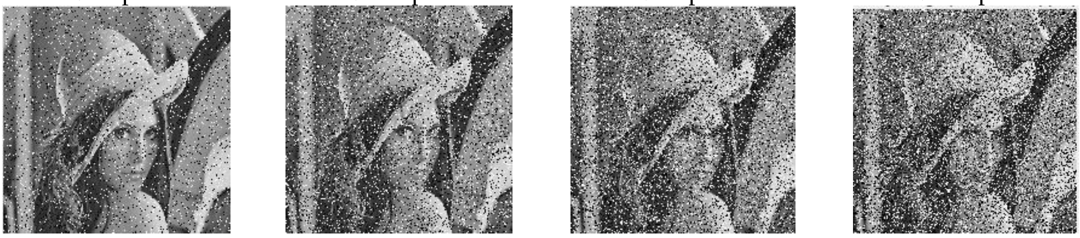
$$\begin{cases} salt & if\ u \leq P1 \\ g\ (x, y) & if\ P1 < u\ \leq P1 + P0 \\ peppers & if\ P1 + P0 < u\ \leq 1 \end{cases} \tag{2}$$

where $P1$ and $P0$ are the probabilities of occurrence of salt and pepper noise, and u is a random value between 0 and 1.

The difference between Gaussian noise and Salt and Pepper noise can be seen in Figure 2. In this figure, the image is measured based on the addition of noise with various alpha values. Gaussian noise tends to spread evenly across the image, giving a haze-like or grainy effect, while Salt and Pepper noise produces white and black dots that are randomly scattered across the image, giving a speckled effect.



(a) Salt&Papper noise with 0.1 alpha value | (b) Salt&Papper noise with 0.25 alpha value | (c) Salt&Papper noise with 0.5 alpha value | (d) Salt&Papper noise with 0.75 alpha value



(e) Gaussian noise with 0.1 alpha value | (f) Gaussian noise with 0.25 alpha value | (g) Gaussian noise with 0.5 alpha value | (h) Gaussian noise with 0.75 alpha value

Figure 2. The different between gaussian and salt & papper noise

Schur decomposition

The proposed technique for enhancing picture clarity and ensuring private communication is the Schur decomposition [24], [25]. By streamlining the image matrix, the Schur decomposition makes it easier to manipulate and analyze the image structure in greater depth [21]. Schur decomposition can be improve an image's ability to distinguish between its essential elements, resulting in less distortion from noise or hidden message insertions and sharper images [26], [27]. The Schur decomposition equation is the source of equation (3) in matrix A.

$$A = QTQ^* \tag{3}$$

where T is an upper triangular Schur matrix, and Q is an orthogonal matrix. The decomposition and reconstruction procedure using Schur decomposition is demonstrated in Algorithm 1. This approach demonstrates the key steps in breaking down a square matrix A into an upper triangular matrix T and an orthogonal matrix Q, and how to recreate the image using the decomposition results. The first algorithm describes the steganography technique known as Schur decomposition and shows how to reconstitute an embedded image while maintaining its visual quality.

1st Algorithm: Schur Decomposition

```
function [Q,T] = schur_decomposition (A)
    Q = I_n
    R = A
    for k = 1 to max_iterations do
        [Q_k, R_k] = qr(R)
        R = R_k * Q_k
        Q = Q * Q_k
    end for
    T = R
    return Q,T
end function
```

The starting point for steganography image embedding is breaking down a square matrix A into two matrices: an orthogonal matrix Q and an upper triangular matrix T, as shown in eq (3). In steganography, this breakdown can be used on either an image's pixel intensity matrix or its transformed domain representation. The information that needs to be concealed is inserted into the upper triangular matrix T, with little impact on the original image's quality. After the modification, the altered matrix T is combined again with matrix Q to create the steganographic image. The secret to this method is choosing elements in T to embed carefully, so that information can be hidden effectively without compromising the image's appearance. This technique improves the confidentiality and undetectability of concealed information, making it a reliable option for image steganography uses.

SSIM and PSNR evaluation

The image quality [9] is considered better when the PSNR value is higher, as this indicates less distortion in the image. Alternatively, SSIM quantifies the structural likeness of two images by analyzing variations in brightness, contrast, and overall composition. The SSIM value falls within the range of -1 to 1, with a higher value suggesting a stronger resemblance between the two images according to reference [8]. Combining these two techniques can enhance the thoroughness of image quality assessment, resulting in a more precise evaluation of visual sharpness and structural accuracy during image processing. Equations (4) and (5) display the formulas for calculating PSNR and SSIM.

$$PSNR = 10 \cdot Log_{10}\left(\frac{Max^2}{MSE}\right) \tag{4}$$

$$SSIM\ (x, y) = \frac{(\mu x2 + \mu y2 + c1)(\sigma x2 + \sigma y2 + c2)}{(2\mu x\mu y + c1)(2\sigma xy + c2)} \tag{5}$$

PSNR is determined by multiplying ten by the square of the maximum pixel value (MAX) and dividing this product by the mean squared error (MSE) of pixel intensity between the original and processed images. SSIM, on the other hand, analyzes the variance and covariance of both images to quantify their structural similarity. To enhance the robustness of the algorithm and prevent division by zero, a constant factor is incorporated.

RESULTS AND DISCUSSIONS

The research starts by setting up the host images that will be used for the steganography process. The study uses host images including Lena, Plane, Peppers, Cameraman, and Baboon, as depicted in Figure 3. These images were chosen because of their diverse visual characteristics, allowing for a thorough assessment of the effectiveness and strength of the steganography technique being tested. Every image undergoes a process involving Schur decomposition, reconstruction steps, and the introduction of Gaussian noise and Salt and Pepper noise to evaluate how well the embedded steganography works and how difficult it is to detect.

(a) Lena (b) Plane (c) Peppers (d) Cameraman (e) Baboon

Figure 3. Sample host images

In the following steps, this research included noise from Gaussian and Salt & Pepper sources as recommended. This noise is used to evaluate the steganography's resistance to external disruptions. Figure 4 illustrates the impact of adding noise to the original image. Both types of noise are used to evaluate the effectiveness of steganography produced by the Schur decomposition method under various conditions.

(a) Lena (b) Plane (c) Peppers (d) Cameraman (e) Baboon

Figure 4. Embedded variant noise

Table 1 presents the SSIM and PSNR measurements for each original image after noise is added to it. This table compares the image quality from steganography methods not using Schur decomposition, under the influence of Gaussian and Salt & Pepper noise. Various image dimensions of 64x64, 128x128, and 256x256 were used in the testing phase.

Table 1. Results of SSIM and PSNR without schur decomposition

Sample Hoost	SSIM			PSNR (decibels)		
	64 x 64	128 x 128	256 x 256	64 x 64	128 x 128	256 x 256
Lena	0.32	0.26	0.17	19.18 dB	17.88 dB	16.28 dB
Plane	0.42	0.33	0.30	20.30 dB	17.21 dB	15.98 dB
Peppers	0.44	0.34	0.29	20.11 dB	17.33 dB	16.25 dB
Cameraman	0.38	0.30	0.26	21.77 dB	17.41 dB	16.88 dB
Baboon	0.42	0.32	0.30	22.22 dB	18.59 dB	17.15 dB

The information in Table 1 makes it abundantly evident that the presence of noise significantly lowers the quality of the SSIM and PSNR measurements. To solve this issue, the steganography method's resistance and invisibility were improved by applying the Schur decomposition methodology. Table 2 presents the enhanced outcomes obtained with the use of Schur decomposition, exhibiting notable increases in both SSIM and PSNR values under the same noise condition.

Table 2. Results of SSIM and PSNR with schur decomposition

Sample Hoost	SSIM			PSNR (decibels)		
	64 x 64	128 x 128	256 x 256	64 x 64	128 x 128	256 x 256
Lena	0.99	0.94	0.92	88.11 dB	80 dB	72.42 dB
Plane	0.98	0.94	0.92	88.70 dB	79.07 dB	71.32 dB
Peppers	0.99	0.94	0.92	88.68 dB	79.32 dB	72.57 dB
Cameraman	0.97	0.94	0.91	89.37 dB	79.91 dB	72.08 dB
Baboon	0.98	0.94	0.92	90.27 dB	81.19 dB	74.12 dB

Figure 5 shows the pictures that were built using the Schur decomposition. It is evident that the embedded pictures closely resemble the original photos, even if steganography was used to embed them. This demonstrates how messages may be successfully hidden in photos using Schur decomposition without affecting the way they look.

(a) Lena (b) Plane (c) Peppers (d) Cameraman (e) Baboon

Figure 5. Reconstructed host images (256 x256)

CONCLUSION

Schur decomposition-based imperceptible steganography has demonstrated high PSNR and SSIM performance in a variety of sample sizes. Experimental results have shown excellent watermarking capacity lower bound by only the methods based on Schur decomposition at many different cover image resolutions, PSNR and SSIM. The achieved SSIM values for 64x64 and bigger images are consistently close to the theoretical maximum of just under (0.99), resulting in nearly perfect structural similarity with respect to this metric between predicted images from our model and corresponding original ones. In parallel, the PSNR values are very high (between 88.11 dB and 90.27 db) representing good image quality with almost no visible noise in the images either side of our threshold at gray-tone level =20). Similarly in the super-resolution from 8x to 4x, though SSIM and PSNR ware reduced a little during bulid larger images which are (128*128) or (256*256). However, it is a good performance of SSIM higher than 0.91 and PSNRs are in the optimal range from us 72.08 dB to ca 81.19dB. This study shows that Schur decomposition successfully preserves excellent visual quality and strong steganography performance at various image resolutions, making it ideal for discreet steganography needs in situations where maintaining the integrity of the original image is essential, like secure communication and digital watermarking. Future studies should concentrate on making specific improvements to steganography using Schur decomposition, especially for images of increased size. One possible approach is to create adaptive methods that can change the embedding process in real-time according to the image content. This could enhance both invisibility and resilience against different steganalysis techniques. Moreover, investigating the incorporation of machine learning algorithms into enhancing the embedding parameters may result in more efficient steganography techniques, ultimately achieving improved detection rates. Evaluating how well this method works with various image formats and noise conditions is important, as is adapting it for video and other multimedia content, which poses additional challenges in maintaining invisibility and strength.

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