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Predator-Prey Model in the Growth Phase of Rice Plants with Pest Control Using Pesticides

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Abstract

In Indonesia, Rice is the primary food crop in Indonesia, with rice being the staple food for more than half of the population. However, pest infestations at each stage of rice growth pose a serious threat that can reduce crop yields. This study developed a predator-prey mathematical model to describe the interactions between rice plants at different growth stages, such as the vegetative, reproductive, and maturation phases, and three major pests: rice stem borers, brown planthoppers, and rats. The model also incorporates Pontryagin's maximum principle for optimal control through pesticide application. The method yields state and costate equations, which are solved using the fourth-order Runge-Kutta forward-backward sweep method with the assistance of Scilab 2024 software. The objective of this research is to examine the model, analyze its stability and controllability, and conduct numerical simulations. The results indicate that the model has a stable equilibrium point, suggesting that the system is controllable. Based on the results of the analysis, the use of pesticides proved to play an important role in controlling pestpopulations and increasing the growth of rice plants in each phase of growth. Pest control also proved effective with pesticides. Rice stem borer (PBP) was controlled in 22 days, brown planthopper (WBC) in 13 days, and rats in 23 days. Overall, the use of pesticides not only helps control pest populations but also supports the balance of agricultural ecosystems and reduces damage to rice plants.

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1. Introduction

The role of the agricultural sector in Indonesia's development is unquestionable. Agriculture plays a crucial role in human life as it serves as a source of food, animal feed, and bioenergy. In addition, agriculture also contributes to encouraging the development of the agro-industry in the downstream sector and increasing the export of agricultural commodities to boost foreign exchange (Renstra Kementerian Pertanian tahun 2020-2024, 2021). Indonesia has a total of 8.087 million hectares of paddy fields. With vast and fertile agricultural land, Indonesia is recognized as an agricultural country (Badan Pusat Statistik Indonesia, 2018). This makes a large portion of Indonesia's population work as farmers, including rice farmers (Sidharta *et al*., 2021).

The growth cycle of the rice plant is divided into three main phases: 1) the vegetative phase, 2) the reproductive phase, and 3) the maturation phase (Mohapatra *et al*., 2023). During the growth phase of rice, pest attacks become a serious threat as they can disrupt normal growth, causing plants to fail to grow and develop optimally, which affects crop yields (Priya *et al*., 2024). Therefore, pest control efforts are crucial, particularly through the proper application of pesticides. However, it is essential to note that pest control must be adapted to the growth phase of the rice and the type of pests that may appear. One of the mathematical models commonly used to describe predator-prey interactions is the Lotka-Voltera model or commonly called the Predator-Prey model. In this model, prey growth follows a logistic model that slows down as it approaches environmental capacity. (Pratama & Baqi, 2019).

The author is interested in developing a redator-prey mathematical model between the growth phases of rice plants and pests. The rice growth phases, as the prey, include the vegetative, reproductive, and maturation phases, while the predators include rice stem borers (PBP), brown planthoppers (WBC), and rats. By adding optimal control to the predator-prey model, through pesticide application, this study aims to control the pest population. The optimal control theory employed is Pontryagin's Maximum Principle, and the fourth-order Runge-Kutta forward-backward sweep method is used for numerical simulation. The goal of this research is to reduce pest populations on rice plants, thereby minimizing losses and helping to increase rice production. It is also expected to provide practical solutions for maximizing the sustainability of rice farming. This research is a development of studies Triwidodo *et al.* (2020) and Syafii (2022).

2. Method

The author conducts a literature review by examining various articles, scientific journals, and other reference sources relevant to the topic of discussion. The literature sources utilized include knowledge and data related to predator-prey mathematical models, rice plants, pests, pesticide use, and other supporting theories.The formation of mathematical models begins by determining the variables and parameters to be used in the model. A mathematical model diagram, differential equations, and model assumptions are then developed.

A dynamic analysis of the predator-prey mathematical model is conducted by linearizing it to determine the equilibrium points and stability properties of the model. Subsequently, a controllability analysis is performed by constructing a controllability matrix and determining its rank. The optimal control model is solved using Pontryagin's Maximum Principal method to obtain the best control for the dynamic system, from the initial state to the final state, by maximizing the objective function. Then numerical simulations are performed using the fourth-order Runge-Kutta forward and backward sweep method, along with Scilab 2024 software. The simulation results are interpreted and analyzed, with the numerical simulation outputs presented in graphical form. Conclusions are drawn based on the literature review and the analysis of the problem-solving results from the research conducted.

3. Results and discussions

3.1. Mathematical Model

The mathematical model developed is a Lotka-Volterra predator-prey model consisting of three predators: rice stem borers (PBP), brown planthoppers (WBC), and rats; and one prey, rice plants, with growth divided into three phases: the vegetative phase, reproductive phase, and maturation phase. In this study it is assumed that if there is no interaction between prey and predators, the predators will die, and the prey population will increase (prey and predator growth follows a logistic model). Then it is also assumed that rats only attack during the reproductive and maturation phases. This is because replanting can still be done if rats attack during the vegetative phase, and the damaged seedlings can still form new saplings, resulting in minimal losses.

The variables used in the model are presented in Table 1.

Table 1. List of variables

Symbol	Description	Unit	Value
$\boldsymbol{P_1}$	population of rice plants in the vegetative phase (prey)	Clumps	$\boldsymbol{P_1 \geq 0}$
$\boldsymbol{P_2}$	population of rice plants in the reproductive phase (prey)	Clumps	$\boldsymbol{P_2 \geq 0}$
$\boldsymbol{P_3}$	population of rice plants in the mature phase (prey)	Clumps	$\boldsymbol{P_3 \geq 0}$
$\boldsymbol{B}$	population of rice stem borers	Individuals	$\boldsymbol{B \geq 0}$
$\boldsymbol{W}$	population of brown planthoppers	Individuals	$\boldsymbol{U \geq 0}$
$\boldsymbol{T}$	population of rats	Individuals	$\boldsymbol{T \geq 0}$

The parameters used in the model are presented in Table 2.

Table 2. List of parameters

Symbol	Description	Unit	Value
α	Intrinsic growth rate of rice plants in the vegetative phase naturally (prey)	Per time	$0 < \alpha < 1$
β	Intrinsic growth rate of rice plants from vegetative phase to reproductive phase | Per time	Per time	$0 < \beta < 1$
φ	Intrinsic growth rate of rice plants from reproductive phase to mature phase	Per time	$0 < \varphi < 1$
δ	Natural birth rate of rice stem borers	Per time	$0 < \delta < 1$
ω	Natural birth rate of brown planthoppers	Per time	$0 < \omega < 1$
τ	Natural birth rate of rats	Per time	$0 < \tau < 1$
ϵ	Natural death rate of rice plants in the mature phase	Per time	$0 < \epsilon < 1$
θ	Natural death rate of rice stem borers	Per time	$0 < \theta < 1$
γ	Natural death rate of brown planthoppers|	Per time	$0 < \gamma < 1$
μ	Natural death rate of rats	Per time	$0 < \mu < 1$
a_1	Rate of damage to rice plants in the vegetative phase due to rice stem borers	Per clump per time	$0 < a_1 < 1$
a_2	Rate of damage to rice plants in the reproductive phase due to rice stem borers	Per clump per time	$0 < a_2 < 1$
b_1	Rate of damage to rice plants in the vegetative phase due to brown planthoppers	Per clump per time	$0 < b_1 < 1$
b_2	Rate of damage to rice plants in the reproductive phase due to brown planthoppers	Per clump per time	$0 < b_2 < 1$
c_2	Rate of damage to rice plants in the reproductive phase due to rats	Per clump per time	$0 < c_2 < 1$
c_3	Rate of damage to rice plants in the mature phase due to rats	Per clump per time	$0 < c_2 < 1$
K	Carrying capacity or environmental support for rice plants	Clumps	$K > 0$
u_1	Rate of pesticide application on rice stem borers	Per time	$0 < u_1 \leq 1$
u_2	Rate of pesticide application on brown planthoppers	Per time	$0 < u_2 \leq 1$
u_3	Rate of pesticide application on rats	Per time	$0 < u_3 \leq 1$

The diagram of the predator-prey mathematical model in the rice plant growth phases with pests is presented in Figure 1.

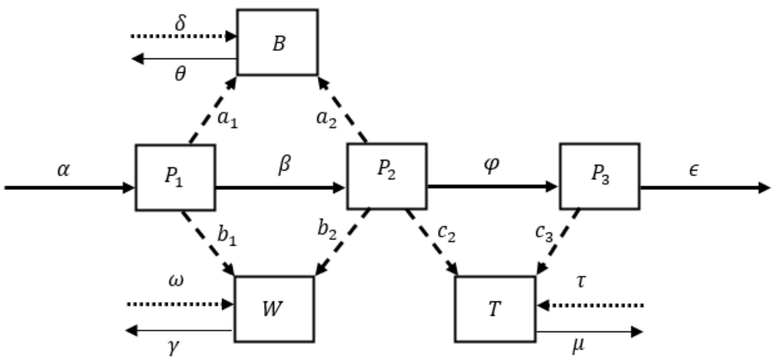

Figure 1. The diagram of the predator-prey mathematical model

The mathematical model is formulated as a system of differential equations, as follows.

$$\frac{dP_1}{dt} = \alpha P_1 \left(1 - \frac{P_1}{K}\right) - a_1 P_1 B - b_1 P_1 W - \beta P_1 \quad (1)$$

$$\frac{dP_2}{dt} = \beta P_1 - a_2 P_2 B - b_2 P_2 W - c_2 P_2 T - \varphi P_2 \quad (2)$$
$$\frac{dP_3}{dt} = \varphi P_2 - c_3 P_3 T - \epsilon P_3 \quad (3)$$
$$\frac{dB}{dt} = \delta - \theta B \quad (4)$$
$$\frac{dW}{dt} = \omega - \gamma W \quad (5)$$
$$\frac{dT}{dt} = \tau - \mu T \quad (6)$$

3.2. Equilibrium Points and Stability Analysis

From equations (1) to (6), the equilibrium points are obtained as follows: $\boldsymbol{E_0^*(P_1^*, P_2^*, P_3^*, B^*, W^*, T^*)} = \left(\boldsymbol{0, 0, 0, \frac{\delta}{\theta}, \frac{\omega}{\gamma}, \frac{\tau}{\mu}}\right)$ and $\boldsymbol{E_1^*(P_1^*, P_2^*, P_3^*, B^*, W^*, T^*)}$

$$= \left(\frac{K(\alpha\theta\gamma - a_1\delta\gamma - b_1\theta\omega - \beta\theta\gamma)}{\alpha\theta\gamma}, \frac{\beta K\mu(\alpha\theta\gamma - a_1\delta\gamma - b_1\theta\omega - \beta\theta\gamma)}{\alpha(a_2\delta\gamma\mu + b_2\omega\theta\mu + c_2\tau\theta\gamma + \varphi\theta\gamma\mu)}, \frac{\varphi\beta K\mu^2(\alpha\theta\gamma - a_1\delta\gamma - b_1\theta\omega - \beta\theta\gamma)}{\alpha(a_2c_3\tau\delta\gamma\mu + a_2\delta\gamma\mu^2\epsilon + b_2c_3\tau\omega\theta\mu + b_2\omega\theta\epsilon\mu^2 + c_2c_3\tau^2\theta\gamma + c_2\tau\theta\gamma\epsilon\mu + c_3\tau\varphi\theta\gamma\mu + \varphi\theta\gamma\epsilon\mu^2)}, \frac{\delta}{\theta}, \frac{\omega}{\gamma}, \frac{\tau}{\mu}\right)$$

The stability analysis of the model is conducted only at the equilibrium point $\boldsymbol{E_1^*}$ because at the equilibrium point $\boldsymbol{E_0^*}$, the number of rice plants is zero. The first step in determining the stability of the equilibrium point $\boldsymbol{E_i^*}$ is to evaluate the equilibrium point $\boldsymbol{E_1^*}$ in the Jacobian matrix, resulting in:

$$J_{E_1^*} = \begin{bmatrix} \ell_1 & 0 & 0 & -a_1P_1^* & -b_1P_1^* & 0 \\ \beta & \ell_2 & 0 & -a_2P_2^* & -b_2P_2^* & -c_2P_2^* \\ 0 & \varphi & \ell_3 & 0 & 0 & -c_3P_3^* \\ 0 & 0 & 0 & \ell_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ell_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ell_6 \end{bmatrix}$$

where $\ell_1 = \alpha - \frac{2\alpha P_1^*}{K} - a_1B^* - b_1W^* - \beta$, $\ell_2 = -a_2B^* - b_2W^* - c_2T^* - \varphi$, $\ell_3 = -c_3T^* - \epsilon$, $\ell_4 = -\theta$, $\ell_5 = -\gamma$, and $\ell_6 = -\mu$.
Based on the matrix $\boldsymbol{J_{E_1^*}}$, the characteristic equation can be formed using $\mathbf{det}(\boldsymbol{\lambda I - J_{E_1^*}}) = \mathbf{0}$, giving:

$$\lambda^6 + \lambda^5 L_1 + \lambda^4 L_2 + \lambda^3 L_3 + \lambda^2 L_4 + \lambda L_5 + L_6 = 0$$

where $L_1 = (-\ell_1 - \ell_2 - \ell_3 - \ell_4 - \ell_5 - \ell_6)$
$L_2 = (\ell_1\ell_2 + \ell_1\ell_3 + \ell_2\ell_3 + \ell_1\ell_6 + \ell_2\ell_6 + \ell_6\ell_3 + \ell_1\ell_5 + \ell_2\ell_5 + \ell_5\ell_3 + \ell_5\ell_6 + \ell_1\ell_4 + \ell_2\ell_4 + \ell_4\ell_3 + \ell_4\ell_6 + \ell_4\ell_5)$
$L_3 = (-\ell_1\ell_2\ell_3 - \ell_1\ell_2\ell_6 - \ell_1\ell_6\ell_3 - \ell_6\ell_2\ell_3 - \ell_1\ell_2\ell_5 - \ell_1\ell_5\ell_3 - \ell_5\ell_2\ell_3 - \ell_1\ell_5\ell_6 - \ell_2\ell_5\ell_6 - \ell_5\ell_6\ell_3 - \ell_1\ell_2\ell_4 - \ell_1\ell_4\ell_3 - \ell_4\ell_2\ell_3 - \ell_1\ell_4\ell_6 - \ell_2\ell_4\ell_6 - \ell_4\ell_6\ell_3 - \ell_1\ell_4\ell_5 - \ell_2\ell_4\ell_5 - \ell_4\ell_5\ell_3 - \ell_4\ell_5\ell_6)$
$L_4 = (\ell_1\ell_2\ell_3\ell_6 + \ell_1\ell_2\ell_3\ell_5 + \ell_1\ell_2\ell_5\ell_6 + \ell_1\ell_6\ell_3\ell_5 + \ell_6\ell_2\ell_3\ell_5 + \ell_1\ell_2\ell_3\ell_4 + \ell_1\ell_2\ell_4\ell_6 + \ell_1\ell_4\ell_3\ell_6 + \ell_4\ell_2\ell_3\ell_6 + \ell_1\ell_2\ell_4\ell_5 + \ell_1\ell_3\ell_4\ell_5 + \ell_4\ell_2\ell_3\ell_5 + \ell_1\ell_4\ell_5\ell_6 + \ell_2\ell_4\ell_5\ell_6 + \ell_3\ell_4\ell_5\ell_6)$
$L_5 = (-\ell_1\ell_2\ell_3\ell_5\ell_6 - \ell_1\ell_2\ell_3\ell_4\ell_6 - \ell_1\ell_2\ell_3\ell_4\ell_5 - \ell_1\ell_2\ell_4\ell_5\ell_6 - \ell_1\ell_3\ell_4\ell_5\ell_6 - \ell_2\ell_3\ell_4\ell_5\ell_6)$
$L_6 = (\ell_1\ell_2\ell_3\ell_4\ell_5\ell_6)$ (7)

According to the definition in (Perko, 2001): "Given a Jacobian matrix $\boldsymbol{Jf(x)}$ with eigenvalues $\boldsymbol{\lambda}$, an equilibrium point $\boldsymbol{\dot{x}}$ in a system of differential equations $\boldsymbol{\dot{x} = f(x)}$ is said to be:

1. Locally asymptotically stable if all eigenvalues of the Jacobian matrix matrix $\boldsymbol{Jf(x)}$ have negative real parts.
2. Unstable if any eigenvalue of the Jacobian matrix matrix $\boldsymbol{Jf(x)}$ has a positive real part."

The endemic equilibrium point of the predator-prey mathematical model in the growth phases of rice plants with pests will be asymptotically stable if the characteristic roots have negative real parts.
Based on the Routh-Hurwitz criterion, equation (4.20) is stable if
$\boldsymbol{L_1L_2 > L_3, m_1L_3 > L_1m_2 \Leftrightarrow L_1L_2L_3 + L_1L_5 > L_3^2 + L_1^2L_4, n_1m_2 > m_1n_2 \Leftrightarrow L_1^2L_2L_3L_4 + L_1^2L_4L_5 + L_3^2L_5 + L_1^2L_4L_5 + L_1L_2L_3L_5 + L_1^3L_2L_6 > L_1L_3^2L_4 + L_1^3L_4^2 + L_1L_5^2 + L_1^2L_2^2L_5 + L_3^2L_5 + L_1^2L_3L_6}$, and $\boldsymbol{o_1n_2 > n_1o_2 \Leftrightarrow n_1m_1m_2L_5 + n_2m_1m_3L_1 + n_1m_2m_3L_1 > n_2m_1^2L_5 + n_1m_2m_3L_1 + n_1m_1m_3L_3}$.

3.3. Controllability Analysis

The necessary and sufficient condition for a system to be controllable is that the matrix

$$M_C = \left[\bar{B}\ \overline{AB}\ \overline{A^2B}\ \overline{A^3B}\ \overline{A^4B}\ \overline{A^5B}\right]$$
$$= \begin{bmatrix} 0 & 0 & 0 & \ell_7 & \ell_8 & 0 & k_1 & k_2 & 0 & r_1 & r_2 & 0 & h_1 & h_2 & 0 & s_1 & s_2 & 0 \\ 0 & 0 & 0 & \ell_{10} & \ell_{11} & \ell_{12} & k_3 & k_4 & k_5 & r_3 & r_4 & r_5 & h_3 & h_4 & h_5 & s_3 & s_4 & s_5 \\ 0 & 0 & 0 & 0 & 0 & \ell_{14} & k_6 & k_7 & k_8 & r_6 & r_7 & r_8 & h_6 & h_7 & h_8 & s_6 & s_7 & s_8 \\ 1 & 0 & 0 & \ell_4 & 0 & 0 & k_9 & 0 & 0 & r_9 & 0 & 0 & h_9 & 0 & 0 & s_9 & 0 & 0 \\ 0 & 1 & 0 & 0 & \ell_5 & 0 & 0 & k_{10} & 0 & 0 & r_{10} & 0 & 0 & h_{10} & 0 & 0 & s_{10} & 0 \\ 0 & 0 & 1 & 0 & 0 & \ell_6 & 0 & 0 & k_{11} & 0 & 0 & r_{11} & 0 & 0 & h_{11} & 0 & 0 & s_{11} \end{bmatrix}$$

where $\ell_1 = \alpha - \frac{2\alpha P_1^*}{K} - a_1B^* - b_1W^* - \beta$, $\ell_2 = -a_2B^* - b_2W^* - c_2T^* - \varphi$, $\ell_3 = -c_3T^* - \epsilon$, $\ell_4 = -\theta$, $\ell_5 = -\gamma$, and $\ell_6 = -\mu$

From this matrix, it can be concluded that the predator-prey model in the growth phase of rice plants with pests is controllable because the rank $\boldsymbol{M_C = 6}$ and each element in the row and column is not all zero and does not contain repeated parameters.

3.4. Optimal Control Problem Solving

The optimal control for managing pest populations in the growth phase of rice plants using pesticide controls $(\boldsymbol{u_1(t), u_2(t), u_3(t)})$is considered, where: $\boldsymbol{u_1(t)}$ represents pesticide application for rice stem borers (PBP), $\boldsymbol{u_2(t)}$ represents pesticide application for brown planthoppers (WBC) $\boldsymbol{u_3(t)}$,represents pesticide application for rat pests.
The system of mathematical model equations used in this study is as follows:

$$\frac{dP_1}{dt} = \alpha P_1\left(1 - \frac{P_1}{K}\right) - a_1P_1B - b_1P_1W - \beta P_1 \quad (8)$$

$$\frac{dP_2}{dt} = \beta P_1 - a_2P_2B - b_2P_2W - c_2P_2T - \varphi P_2 \quad (9)$$

$$\frac{dP_3}{dt} = \varphi P_2 - c_3P_3T - \epsilon P_3 \quad (10)$$

$$\frac{dB}{dt} = \delta - \theta B + u_1 \quad (11)$$

$$\frac{dW}{dt} = \omega - \gamma W + u_2 \quad (12)$$

$$\frac{dT}{dt} = \tau - \mu T + u_3 \quad (13)$$

where $0 \le u_1 \le 1$, $0 \le u_2 \le 1$, and $0 \le u_3 \le 1$.
Based on the Pontryagin Maximum Principle (Putri, 2018) obtained

a. State Equation:

$$\dot{P_1} = \frac{\partial H^*}{\partial \lambda_1} = \alpha P_1\left(1 - \frac{P_1}{K}\right) - a_1P_1B - b_1P_1W - \beta P_1$$

$$\dot{P_2} = \frac{\partial H^*}{\partial \lambda_2} = \beta P_1 - a_2P_2B - b_2P_2W - c_2P_2T - \varphi P_2$$

$$\dot{P_3} = \frac{\partial H^*}{\partial \lambda_3} = \varphi P_2 - c_3P_3T - \epsilon P_3$$

$$\dot{B} = \frac{\partial H^*}{\partial \lambda_4} = \delta - \theta B + u_1$$

$$\dot{W} = \frac{\partial H^*}{\partial \lambda_5} = \omega - \gamma W + u_2$$

$$\dot{T} = \frac{\partial H^*}{\partial \lambda_6} = \tau - \mu T + u_3$$

b. Costate Equation:

$$\dot{\lambda_1} = \frac{\partial H^*}{\partial \lambda_1} = -\left(\lambda_1\left(\alpha - \frac{2\alpha P_1}{K} - a_1B - b_1W - \beta\right) + \lambda_2\beta\right)$$

$$\dot{\lambda_2} = \frac{\partial H^*}{\partial \lambda_2} = -(-\lambda_2(a_2B + b_2W + c_2T + \varphi) + \lambda_3\varphi)$$

$$\dot{\lambda_3} = \frac{\partial H^*}{\partial \lambda_3} = -(-\lambda_3(c_3T + \epsilon))$$

$$\dot{\lambda_4} = \frac{\partial H^*}{\partial \lambda_4} = -(-\lambda_1a_1P_1 - \lambda_2a_2P_2 - \lambda_4\theta)$$

$$\dot{\lambda_5} = \frac{\partial H^*}{\partial \lambda_5} = -(-\lambda_1b_1P_1 - \lambda_2b_2P_2 - \lambda_5\gamma)$$

$$\dot{\lambda_6} = \frac{\partial H^*}{\partial \lambda_6} = -(-\lambda_2c_2P_2 - \lambda_3c_3P_3 - \lambda_6\mu)$$

3.5. Numerical Simulation

Next, the fourth-order Runge-Kutta forward-backward sweep method (Hardiyanti, 2016) was carried out.
Solve the state equation using the forward sweep with the fourth-order Runge-Kutta method until obtained:

- $$P_{1,n+1} = P_{1,n} + \frac{h}{6}(q_1 + 2q_2 + 2q_3 + q_4) = P_{1,n} + \frac{h}{6}\Bigg(\left(\alpha P_{1,n}\left(1 - \frac{P_{1,n}}{K}\right) - a_1P_{1,n}B - b_1P_{1,n}W - \beta P_{1,n}\right) + 2\Bigg(\alpha\left(P_{1,n} + q_1\frac{h}{2}\right)\left(1 - \frac{P_{1,n}+q_1\frac{h}{2}}{K}\right) - a_1\left(P_{1,n} + q_1\frac{h}{2}\right)\left(B_n + x_1\frac{h}{2}\right) - b_1\left(P_{1,n} + q_1\frac{h}{2}\right)\left(W_n + y_1\frac{h}{2}\right) - \beta\left(P_{1,n} + q_1\frac{h}{2}\right)\Bigg) + 2\Bigg(\left(P_{1,n} + q_2\frac{h}{2}\right)\left(1 - \frac{P_{1,n}+q_2\frac{h}{2}}{K}\right) - a_1\left(P_{1,n} + q_2\frac{h}{2}\right)\left(B_n + x_2\frac{h}{2}\right) - b_1\left(P_{1,n} + q_2\frac{h}{2}\right)\left(W_n + y_2\frac{h}{2}\right) - \beta\left(P_{1,n} + q_2\frac{h}{2}\right)\Bigg) + \Bigg(\alpha(P_{1,n} + q_3h)\left(1 - \frac{P_{1,n}+q_3h}{K}\right) - a_1(P_{1,n} + q_3h)(B_n + x_3h) - b_1(P_{1,n} + q_3h)(W_n + y_3h) - \beta(P_{1,n} + q_3h)\Bigg)\Bigg)$$

- $$P_{2,n+1} = P_{2,n} + \frac{h}{6}(r_1 + 2r_2 + 2r_3 + r_4) = P_{2,n} + \frac{h}{6}\Bigg((\beta P_{1,n} - a_2P_{2,n}B - b_2P_{2,n}W - c_2P_{2,n}T - \varphi P_{2,n}) + 2\left(\beta\left(P_{1,n} + q_1\frac{h}{2}\right) - a_2\left(P_{2,n} + r_1\frac{h}{2}\right)\left(B_n + x_1\frac{h}{2}\right) - b_2\left(P_{2,n} + r_1\frac{h}{2}\right)\left(W_n + y_1\frac{h}{2}\right) - c_2\left(P_{2,n} + r_1\frac{h}{2}\right)\left(T_n + z_1\frac{h}{2}\right) - \varphi\left(P_{2,n} + r_1\frac{h}{2}\right)\right) + 2\left(\beta\left(P_{1,n} + q_2\frac{h}{2}\right) - a_2\left(P_{2,n} + r_2\frac{h}{2}\right)\left(B_n + x_2\frac{h}{2}\right) - b_2\left(P_{2,n} + y_2\frac{h}{2}\right)\left(W_n + z_2\frac{h}{2}\right) - c_2\left(P_{2,n} + r_2\frac{h}{2}\right)\left(T_n + z_2\frac{h}{2}\right) - \varphi\left(P_{2,n} + r_2\frac{h}{2}\right)\right) + \big(\beta(P_{1,n} + q_3h) - a_2(P_{2,n} + r_3h)(B_n + x_3h) - b_2(P_{2,n} + r_3h)(W_n + y_3h) - c_2(P_{2,n} + r_3h)(T_n + z_3h) - \varphi(P_{2,n} + r_3h)\big)\Bigg)$$

- $$P_{3,n+1} = P_{3,n} + \frac{h}{6}(v_1 + 2v_2 + 2v_3 + v_4) = P_{3,n} + \frac{h}{6}\Bigg((\varphi P_{2,n} - c_3P_{3,n}T - \epsilon P_{3,n}) + 2\left(\varphi\left(P_{2,n} + r_1\frac{h}{2}\right) - c_3\left(P_{3,n} + v_1\frac{h}{2}\right)\left(T_n + z_1\frac{h}{2}\right) - \epsilon\left(P_{3,n} + v_1\frac{h}{2}\right)\right) + 2\left(\varphi\left(P_{2,n} + r_1\frac{h}{2}\right) - c_3\left(P_{3,n} + v_2\frac{h}{2}\right)\left(T_n + z_1\frac{h}{2}\right) - \epsilon\left(P_{3,n} + v_2\frac{h}{2}\right)\right) + \big(\varphi(P_{2,n} + r_3h) - c_3(P_{3,n} + v_3h)(T_n + z_3h) - \epsilon(P_{3,n} + v_3h)\big)\Bigg)$$

- $B_{n+1} = B_n + \frac{h}{6}(x_1 + 2x_2 + 2x_3 + x_4) = B_n + \frac{h}{6}\Bigg((\delta - \theta B + u_1) + 2\left(\delta - \theta\left(B_n + x_1\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{1_j} + u_{1_{j+1}}\right)\right)\right) + 2\Bigg(\delta - \theta\left(B_n + x_1\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{1_j} + u_{1_{j+1}}\right)\right)\Bigg) + \left(\delta - \theta(B_n + x_3h) + u_{1_{j+1}}\right)\Bigg)$
- $W_{n+1} = W_n + \frac{h}{6}(y_1 + 2y_2 + 2y_3 + y_4) = W_n + \frac{h}{6}\Bigg((\omega - \gamma W + u_2) + 2\left(\omega - \gamma\left(W_n + y_1\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{2_j} + u_{2_{j+1}}\right)\right)\right) + 2\Bigg(\omega - \gamma\left(W_n + y_1\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{2_j} + u_{2_{j+1}}\right)\right)\Bigg) + \left(\omega - \gamma(W_n + y_3h) + u_{2_{j+1}}\right)\Bigg)$
- $T_{n+1} = T_n + \frac{h}{6}(z_1 + 2z_2 + 2z_3 + z_4) = T_n + \frac{h}{6}\Bigg((\tau - \mu T + u_3) + 2\left(\tau - \mu\left(T_n + z_1\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{3_j} + u_{3_{j+1}}\right)\right)\right) + 2\Bigg(\tau - \mu\left(T_n + z_2\frac{h}{2}\right) + \left(\frac{1}{2}\left(u_{3_j} + u_{3_{j+1}}\right)\right)\Bigg) + \left(\tau - \mu(T_n + z_3h) + u_{3_{j+1}}\right)\Bigg)$

Solve the costate equation using the backward sweep with the fourth-order Runge-Kutta method, until obtained:

- $\lambda_{1,n-1} = \lambda_{1,n} - \frac{h}{6}(q_1 + 2q_2 + 2q_3 + q_4) = \lambda_{1,n} - \frac{h}{6}\Bigg(\left(-\lambda_{1,n}\left(\alpha - \frac{2\alpha P_{1,n}}{K} - a_1B - b_1W - \beta\right) + \lambda_2\beta\right) + 2\Bigg(\left(-\lambda_{1,n} - \frac{h}{2}q_1\right)\Bigg(\alpha - \frac{2\alpha\left(\frac{1}{2}(P_{1,n}+P_{1,n-1})\right)}{K} - a_1\frac{1}{2}(B_n + B_{n-1}) - b_1\frac{1}{2}(W_n + W_{n-1}) - \beta\Bigg) + \left(\lambda_{2,n} - \frac{h}{2}r_1\right)\beta\Bigg) + 2\Bigg(\left(-\lambda_{1,n} - \frac{h}{2}q_2\right)\Bigg(\alpha - \frac{2\alpha\left(\frac{1}{2}(P_{1,n}+P_{1,n-1})\right)}{K} - a_1\frac{1}{2}(B_n + B_{n-1}) - b_1\frac{1}{2}(W_n + W_{n-1}) - \beta\Bigg) + \left(\lambda_{2,n} - \frac{h}{2}r_2\right)\beta\Bigg) + \Bigg((-\lambda_{1,n} - hq_3)\Bigg(\alpha - \frac{2\alpha P_{1,n-1}}{K} - a_1B_{n-1} - b_1W_{n-1} - \beta\Bigg) + (\lambda_{2,n} - hr_3)\beta\Bigg)\Bigg)$
- $\lambda_{2,n-1} = \lambda_{2,n} - \frac{h}{6}(r_1 + 2r_2 + 2r_3 + r_4) = \lambda_{2,n} - \frac{h}{6}\Bigg(\left(\lambda_{2,n}(a_2B + b_2W + c_2T + \varphi) - \lambda_{3,n}\varphi\right) + 2\left(\left(\lambda_{2,n} - \frac{h}{2}r_1\right)\left(a_2\frac{1}{2}(B_n + B_{n-1}) + b_2\frac{1}{2}(W_n + W_{n-1}) + c_2\frac{1}{2}(T_n + T_{n-1}) + \varphi\right) - \left(\lambda_{3,n} - \frac{h}{2}v_1\right)\varphi\right) + 2\left(\left(\lambda_{2,n} - \frac{h}{2}r_2\right)\left(a_2\frac{1}{2}(B_n + B_{n-1}) + b_2\frac{1}{2}(W_n + W_{n-1}) + c_2\frac{1}{2}(T_n + T_{n-1}) + \varphi\right) - \left(\lambda_{3,n} - \frac{h}{2}v_2\right)\varphi\right) + \left((\lambda_{2,n} - hr_3)(a_2B_{n-1} + b_2W_{n-1} + c_2T_{n-1} + \varphi) - (\lambda_{3,n} - hv_3)\varphi\right)\Bigg)$
- $\lambda_{3,n-1} = \lambda_{3,n} - \frac{h}{6}(v_1 + 2v_2 + 2v_3 + v_4) = \lambda_{3,n} - \frac{h}{6}\Bigg(\left(\lambda_{3,n}(c_3T + \epsilon)\right) + 2\left(\left(\lambda_{3,n} - \frac{h}{2}v_1\right)\left(c_3\frac{1}{2}(T_n + T_{n-1}) + \epsilon\right)\right) + 2\left(\left(\lambda_{3,n} - \frac{h}{2}v_2\right)\left(c_3\frac{1}{2}(T_n + T_{n-1}) + \epsilon\right)\right) + \left((\lambda_{3,n} - hv_3)(c_3T_{n-1} + \epsilon)\right)\Bigg)$
- $\lambda_{4,n-1} = \lambda_{4,n} - \frac{h}{6}(x_1 + 2x_2 + 2x_3 + x_4) = \lambda_{4,n} - \frac{h}{6}\Bigg(\left(\lambda_{1,n}a_1P_{1,n} + \lambda_{2,n}a_2P_{2,n} + \lambda_{4,n}\theta\right) + 2\left(\left(\lambda_{1,n} - \frac{h}{2}q_1\right)a_1\frac{1}{2}(P_{1,n} + P_{1,n-1}) + \left(\lambda_{2,n} - \frac{h}{2}r_1\right)a_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{4,n} - \frac{h}{2}x_1\right)\theta\right) + 2\left(\left(\lambda_{1,n} - \frac{h}{2}q_2\right)a_1\frac{1}{2}(P_{1,n} + P_{1,n-1}) + \left(\lambda_{2,n} - \frac{h}{2}r_2\right)a_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{4,n} - \frac{h}{2}x_2\right)\theta\right) + \left((\lambda_{1,n} - hq_3)a_1P_{1,n-1} + (\lambda_{2,n} - hr_3)a_2P_{2,n-1} + (\lambda_{4,n} - hx_3)\theta\right)\Bigg)$
- $\lambda_{5,n-1} = \lambda_{5,n} - \frac{h}{6}(y_1 + 2y_2 + 2y_3 + y_4) = \lambda_{5,n} - \frac{h}{6}\Bigg(\left(\lambda_{1,n}b_1P_{1,n} + \lambda_{2,n}b_2P_{2,n} + \lambda_{5,n}\gamma\right) + 2\left(\left(\lambda_{1,n} - \frac{h}{2}q_1\right)b_1\frac{1}{2}(P_{1,n} + P_{1,n-1}) + \left(\lambda_{2,n} - \frac{h}{2}r_1\right)b_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{5,n} - \frac{h}{2}y_1\right)\gamma\right) + 2\left(\left(\lambda_{1,n} - \frac{h}{2}q_2\right)b_1\frac{1}{2}(P_{1,n} + P_{1,n-1}) + \left(\lambda_{2,n} - \frac{h}{2}r_2\right)b_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{5,n} - \frac{h}{2}y_2\right)\gamma\right) + \left((\lambda_{1,n} - hq_3)b_1P_{1,n-1} + (\lambda_{2,n} - hr_3)b_2P_{2,n-1} + (\lambda_{5,n} - hy_3)\gamma\right)\Bigg)$
- $\lambda_{6,n-1} = \lambda_{6,n} - \frac{h}{6}(z_1 + 2z_2 + 2z_3 + z_4) = \lambda_{6,n} - \frac{h}{6}\Bigg(\left(\lambda_{2,n}c_2P_{2,n} + \lambda_{3,n}c_3P_{3,n} + \lambda_{6,n}\mu\right) + 2\left(\left(\lambda_{2,n} - \frac{h}{2}r_1\right)c_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{3,n} - \frac{h}{2}v_1\right)c_3\frac{1}{2}(P_{3,n} + P_{3,n-1}) + \left(\lambda_{6,n} - \frac{h}{2}z_1\right)\mu\right) + 2\left(\left(\lambda_{2,n} - \frac{h}{2}r_2\right)c_2\frac{1}{2}(P_{2,n} + P_{2,n-1}) + \left(\lambda_{3,n} - \frac{h}{2}v_2\right)c_3\frac{1}{2}(P_{3,n} + P_{3,n-1}) + \left(\lambda_{6,n} - \frac{h}{2}z_2\right)\mu\right) + \left((\lambda_{2,n} - hr_3)c_2P_{2,n-1} + (\lambda_{3,n} - hv_3)c_3P_{3,n-1} + (\lambda_{6,n} - hz_3)\mu\right)\Bigg)$

Repeat the forward and backward sweep process until the optimal solution to the control problem is obtained.

3.6. Analysis and Simulation Results

In this section, we will analyze the simulation results of the model through graphs by inputting parameter values, using Scilab 2024 software and the Runge-Kutta forward-backward sweep method. The variable and parameter values are used in the numerical simulations are presented in Table 3.

Table 3. The variable and parameter values

No	Variables and Parameters	Value	Source	No	Variables and Parameters	Value	Source
1	P_1	1050	Assumptions	16	μ	0.2963	(Padilah *et al.*, 2018)
2	P_2	980	Assumptions	17	a_1	0.002	(Padilah *et al.*, 2018)
3	P_3	910	Assumptions	18	a_2	0.002	(Padilah *et al.*, 2018)
4	B	350	Assumptions	19	b_1	0.0002	Assumptions
5	W	525	Assumptions	20	b_2	0.002	(Padilah *et al.*, 2018)
6	T	70	Assumptions	21	c_2	0.001	(Triwidodo *et al.*, 2020)
7	α	0.25	(Triwidodo *et al.*, 2020)	22	c_3	0.003	Assumptions
8	β	0.0268	(Padilah *et al.*, 2018)	23	A_1	-10	Assumptions
9	φ	0.0268	(Padilah *et al.*, 2018)	24	A_2	-8	Assumptions
10	δ	0.014	Asumsi	25	A_3	-6	Assumptions
11	ω	0.08	Asumsi	26	S_1	0.1	Assumptions
12	τ	0.0087	(Nurrohman *et al.*, 2019)	27	S_2	0.2	Assumptions
13	ϵ	0.02	Assumptions	28	S_3	0.3	Assumptions
14	θ	0.29	Assumptions	29	K	1330	Assumptions
15	γ	0.5	(Taufiq & Agustito, 2019)				

The simulation results of the predator-prey model in the growth phase of rice plants with pest control using pesticides are as follows.

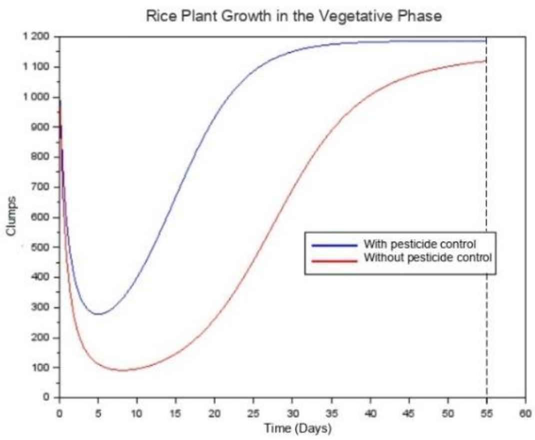

Figure 2. Growth of rice plants in the vegetative phase

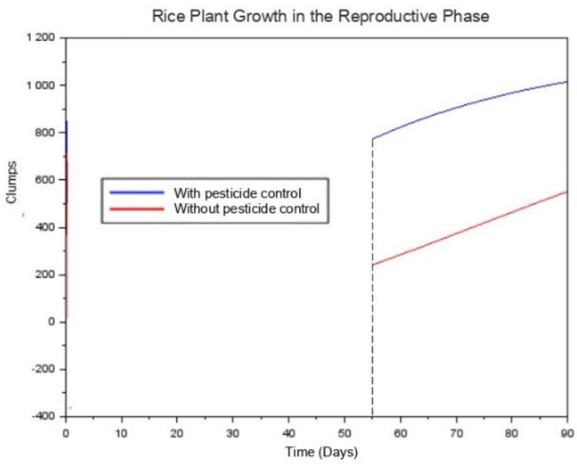

Figure 3. Growth of rice plants in the reproductive phase

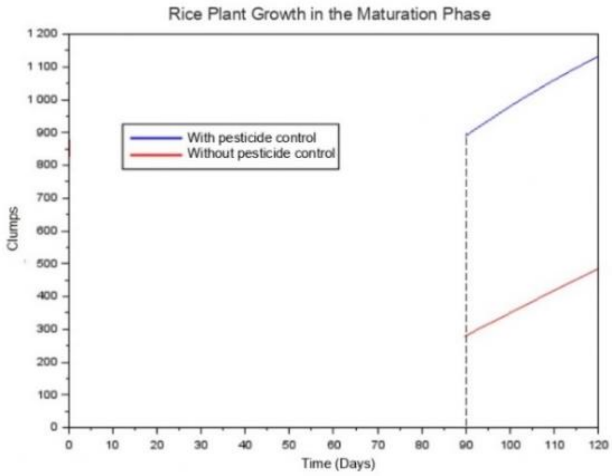

Figure 4. Growth of rice plants in the maturation phase

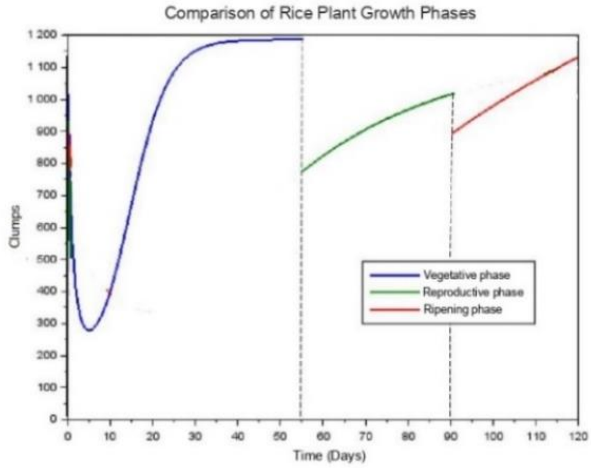

Figure 5. Growth of rice plants in the maturation phase

Figure 2 shows the growth of rice in the vegetative phase infested with PBP and WBC, both without and with pesticide control. Without control, the population increased from 1050 clumps ($\boldsymbol{t = 0}$) to 1110 clumps ($\boldsymbol{t = 55}$), indicating limited growth due to pest attacks. With pesticide control, the population increased more significantly, from 1050 clumps to 1180 clumps, demonstrating the effectiveness of pesticides in supporting optimal growth. The application of pesticides not only increased the rice population in the vegetative phase but also supported maximum production yields and reduced losses due to pests. Figure 3 shows the growth of rice in the reproductive phase attacked by PBP, WBC, and rats, both without and with pesticide control. Without control, the population only increased from 200 clumps ($\boldsymbol{t = 56}$) to 500 clumps ($\boldsymbol{t = 90}$) due to high pest infestation. With pesticide control, the population increased more significantly, from 780 clumps to 1020 clumps, indicating that the pesticide was effective in controlling pests and maintaining rice grain development. The population difference of 520 clumps indicates the success of pesticides in reducing losses due to pest attacks in the reproductive phase.

Figure 4 shows the growth of rice in the maturation phase under rat infestation, both without and with pesticide control. Without control, the population increased from 280 clumps ($\boldsymbol{t = 90}$) to 490 clumps ($\boldsymbol{t = 120}$), but rat infestation inhibited seed growth and reduced rice grains. With pesticide control, the population increased more significantly, from 890 clumps to 1140 clumps, indicating the effectiveness of pesticides in controlling pests and supporting optimal seed maturation. The population difference of 650 clumps confirms the importance of pesticides in reducing pest losses and ensuring maximum yields. Figure 5 shows a comparison of rice growth in the vegetative, reproductive, and maturation phases with pesticide control. In the vegetative phase (0-55 HST), the population reached 1180 clumps, reflecting effective control against PBP and WBC pests. In the reproductive phase (56-90 HST), the population decreased from 1180 to 780 clumps due to attacks from PBP, WBC, and rats. However, with pesticide control, the population increased to 1020 clumps, supporting optimal grain formation. In the ripening phase (91-120 HST), rat attacks decreased the population from 1020 to 890 clumps, but pesticide control increased the population to 1140 clumps, confirming the success of pesticides in reducing losses due to pests.

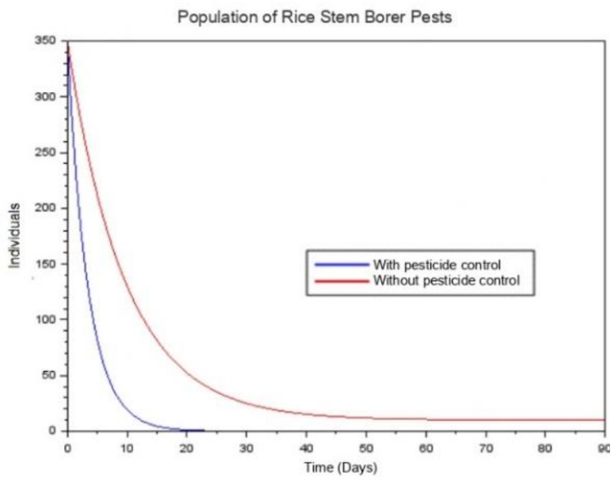

Figure 6. Population of rice stem borer pests

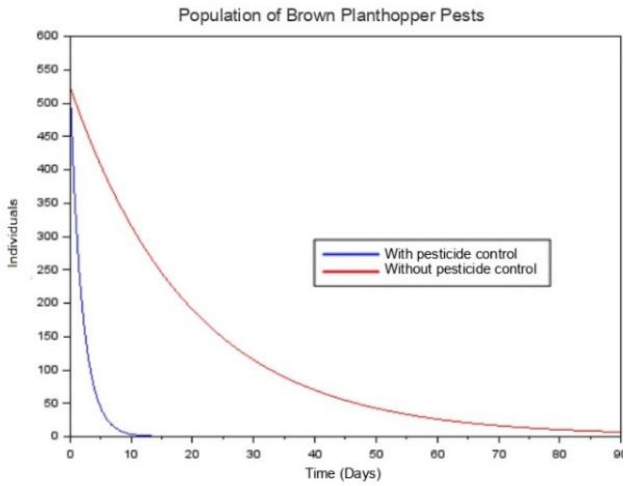

Figure 7. Population of brown planthopper (WBC)

Figure 6 shows the population of rice stem borers (PBP) without and with pesticide control. Without control, PBP continues to attack rice plants due to its rapid life cycle and aggressive nature. With pesticide control, PBP was eradicated in 22 days, utilizing its life cycle, which lasts only 35-60 days. This demonstrates the effectiveness of pesticides in reducing pest populations and accelerating control. Figure 7 the population of brown planthopper (WBC) without and with pesticide control. WBC attacks during the vegetative and reproductive phases. Without control, the population remained high due to its fast breeding ability. With pesticide control, the WBC was eradicated in 13 days, due to its short life cycle (25-30 days). Pesticides effectively inhibit the development of eggs, nymphs, and adults, making the WBC population easier to control.

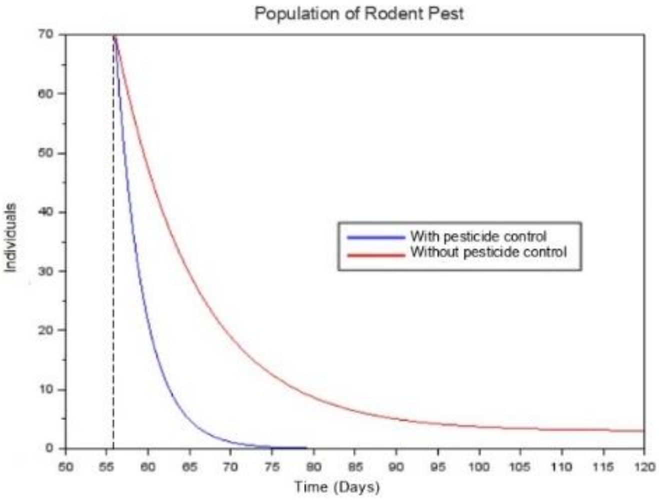

Figure 8. Population of rodent pest

Figure 8 the rat population without and with pesticide control. Rats attack during the reproductive and maturation phases. Without control, rats continue to attack due to their longlife cycle and rapid reproduction. With pesticide control, rats were eradicated in 23 days. Pesticides affect the reproduction of rats, causing birth defects, decreased endurance, and direct mortality, resulting in a significant decrease in the population.

4. Conclusion

Based on the results of the analysis, the use of pesticides proved to play an important role in controlling pestpopulations and increasing the growth of rice plants in each phase of growth. In the vegetative phase, the rice population increased by 60 clumps without pesticide control, and 130 clumps with pesticide control. In the reproductive phase, without pesticide control, the rice population decreased drastically to 860 clumps, while with pesticide control, although there was a decrease of 400 clumps, it still showed better results. In the maturation phase, without pesticide control, the rice population decreased by 270 clumps, while with pesticide control the decrease was only 130 clumps, still showing better results.

Pest control also proved effective with pesticides. Rice stem borer (PBP) was controlled in 22 days, brown planthopper (WBC) in 13 days, and rats in 23 days. Overall, the use of pesticides not only helps control pest populations but also supports the balance of agricultural ecosystems and reduces damage to rice plants.

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