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Modelling and Analysis Study Of Plastic Waste Flow Dynamics System

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Abstract

Waste has become an inextricable aspect of contemporary human existence. The classification of waste is multifaceted, encompassing solid, liquid, and gas waste, as well as organic and inorganic waste categories. Plastic waste is among the most prevalent types of waste produced by society. As indicated by the National Waste Management Information System (SIPSN), plastic waste constitutes the predominant proportion of inorganic waste. Consequently, a solution is imperative to address this issue. One approach to understanding the flow patterns of plastic waste is the population dynamics model. This model is regarded as one of the most suitable approaches for understanding plastic waste flow. The process commences with the establishment of multiple compartments implicated in the plastic waste flow, subsequently leading to the generation of a transition diagram. Subsequently, a system of differential equations is formulated based on the transition diagram of plastic waste flow. According to the model, the equilibrium point is contingent upon the plastic production factor and the plastic consumption or usage pattern. The present topic will also discuss the stability of the equilibrium point and numerical simulations for influential parameters, as well as trace them from several initial values.

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1. Introduction

Waste comes from the unused and discarded remains of living things from households, industries, agriculture, and other commercial activities. From Sistem Infromasi Pengolahan Sampah Nasional (2025) or the Ministry of Environment's National Waste Management Information System (SIPSN), plastic waste has the second highest percentage (19.74%) after food waste (39.31%). Plastic is one of the most widely used synthetic materials in everyday life because of its lightness, strength, and cheapness (Saleh *et al*., 2023) and (Anwar *et al.,* 2023). Its use has expanded in various sectors, such as food and beverage packaging, agriculture, textiles, and the electronics industry (Evode *et al.,* 2021). However, the massive use of plastics and the lack of adequate waste management systems have led to an increased accumulation of plastic waste in the environment. Further impacts arising from poor management are the pollution of land and marine ecosystems, threatening human health, and generating high carbon emissions during production and combustion.

In an effort to reduce plastic waste that pollutes the environment, the identification of plastic waste is analyzed through a life cycle approach, starting from the production process, consumption by the community, to its disposal (Heshmati, 2015). After use, most plastic waste is disposed of in the trash. In the environmental context, some types of plastic still have economic value, so they are taken by collectors or scavengers to be reused. However, the majority of plastic waste that does not have high economic value tends to be left to accumulate and pose a serious threat to the environment and human health (Huang *et al.,* 2022). In the long-term, this will pose a serious threat to the environment and human health. Therefore, an effort is needed to overcome these problems. Maximum waste treatment will have an impact on human health, a stable community economy, and most importantly, a healthier environment (Zhao *et al*., 2022).

In other disciplines this form of analyzing plastic waste flow patterns is better known as circular economy. Circular economy is an adaptation and modification of the linear economy model. Linear economics emphasizes that a product, in this case plastic, has a production-consumption-waste and disposal pattern (Robaina *et al.,* 2020). In response to this pattern, a circular economy was established to manage the waste produced. The idea of a circular economy is to optimally reuse the waste generated in a linear economy (Tambovceva *et al.,* 2021). Studies on the circular economy have been conducted by several scientists related to the circular economy and resulted in several important points. An overview of how important the circular economy is as well as the challenges and constraints faced has been conducted by Heshmati (2015). The article shows that the circular economy is able to contribute to the SDGs. Another article mentioned that a circular economy that focuses on proper waste management can improve health, economic cycles, and has a huge impact on the environment (Zhang *et al.,* 2022) and (Ikponmwosa *et al*., 2024).

The compartment model applied to the case of plastic waste streams is an approach that is often used to describe interactions between ecosystem components or commonly called population dynamics models. The approach to the plastic waste flow problem can use a compartment model adopted from the SIR or SEIR infectious disease compartment model. In the plastic waste stream analysis described earlier, each cycle or process is formed in a compartment. Furthermore, the unit transfer between components in the flow is symbolized in the form of parameters.

Several articles have discussed plastic waste flow models with a compartmental approach and produced interesting conclusions to be followed up. In the article (Busu & Busu, 2018), the plastic waste process is divided into several compartments and examined in a discrete way, as described in Figure 1.

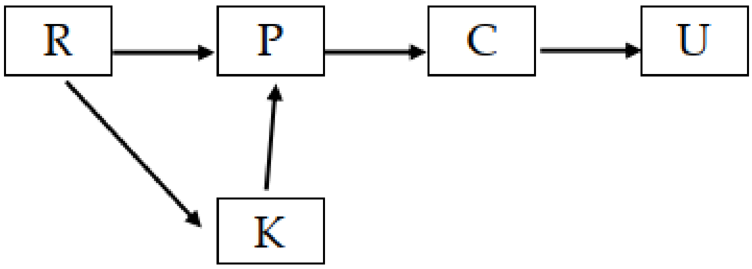

Figure 1. The conventional linear economy processes. R—natural resources, P—production, C—Consumption, U—Utility, and K—Kapital goods (Busu & Busu, 2018)

The study compared the process of the linear economy with the circular economy using the Shannon entropy process. The results of the study concluded that the circular economy process guarantees a reduced amount of waste. Another study that applies the compartment model is the compartment model and numerical simulation by Yang *et al.* (2019). The article provides an overview of the differential equation system of the circular economy and numerical simulation, as described in Figure 2. The simulation results of the article show that the maximum processing of resources can not only realize the accumulated amount of natural resources and improve the efficiency of resource use, but also can minimize the amount of waste generated from the process through the adjustment of each compartment parameter in the model.

The present article employs population dynamics models derived from the SIR and SEIR models, along with several of their analytical processes. The research commences with the segmentation of the plastic waste flow process into five distinct processes: The production, consumption, landfills, collection, and final waste management processes are critical components of the waste management hierarchy. Each transition from one compartment to another is assigned a value or weight, known as a parameter. These parameters are critical to understanding the movement of plastic waste. Subsequently, a system of differential equations is formulated, encompassing the aforementioned compartments and parameters. Subsequently, the equilibrium point of the formed equation system is determined, along with its stability properties. The final section of this discussion presents a numerical analysis, which is used to determine the relationship between each compartment and to identify the parameters that significantly influence the system.

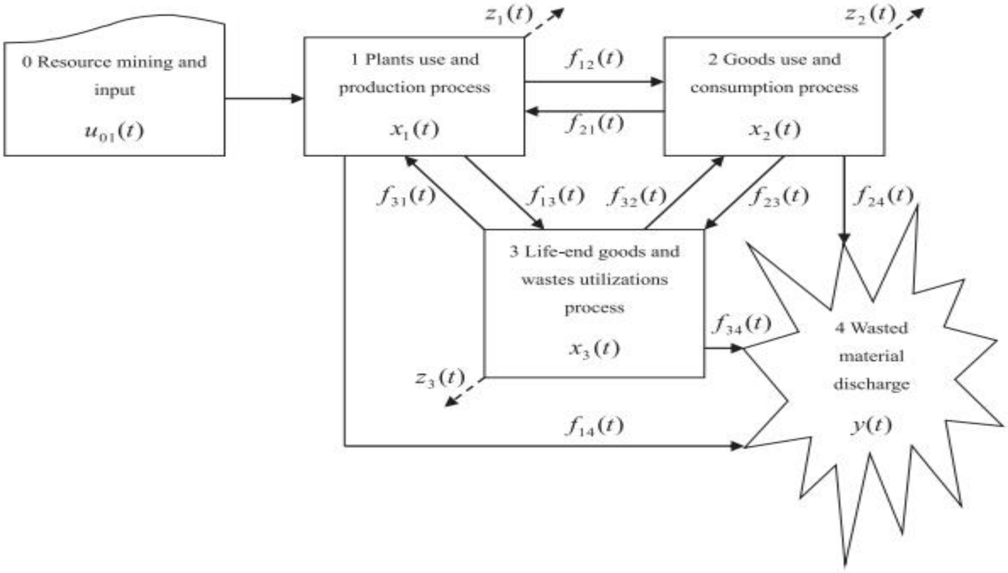

Figure 2. Compartment model structure of circulatory economy (Yang *et al*., 2019)

2. Methods

This research begins by formulating a transition diagram based on the facts obtained in real conditions. This transition diagram model consists of five compartments and five parameters.

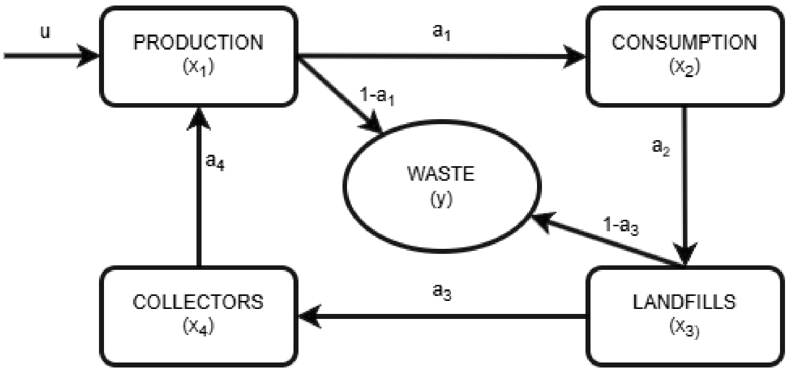

Figure 3. Transition Diagram of 5 compartment

With the following details,

Table 1. Compartment Description

Compartments	Symbols	Meaning
Production	x_1	The amount of plastics produced is influenced by time t
Consumption	x_2	The amount of plastics consumption in time t
Landfills	x_3	The amount of plastics in landfills in time t
Collectors	x_4	Number of plastics picked up by collectors in time t
Waste	y	The amount of waste plastics in time t

and

Table 2. Parameters Description

Parameter	Meaning	Source
u	Amount of resource	(Yang *et al*., 2019)
a_1	Rate of movement from production to consumption	(Busu & Busu, 2018)
a_2	Rate of movement from consumption to landfills	(Yang *et al*., 2019)
a_3	Rate of movement from landfills to collectors	(Yang *et al.,* 2019)

a_4	Rate of movement from collectors to production again	Assumed

From figure 1, a system of differential equations involving four input variables and one output variable is constructed, as follows

$$\begin{aligned} \frac{dx_1}{dt} &= u - a_1x_1 - (1-a_1)x_1 = u - x_1 \\ \frac{dx_2}{dt} &= a_1x_1 - a_2x_2 \\ \frac{dx_3}{dt} &= a_2x_2 - a_3x_3 - (1-a_3)x_3 = a_2x_2 - x_3, \\ \frac{dx_4}{dt} &= a_3x_3 - a_4x_4 \\ y &= (1-a_1)x_1 + (1-a_3)x_3 \end{aligned} \quad (1)$$

Furthermore, the system of differential equations 1) is equivalent to the form of

$$\dot{x} = Ax + B$$

with,

$$\dot{x} = \begin{bmatrix} \frac{dx_1}{dt} & \frac{dx_2}{dt} & \frac{dx_3}{dt} & \frac{dx_4}{dt} & y \end{bmatrix}^T ; A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ a_1 & -a_2 & 0 & 0 & 0 \\ 0 & a_2 & -1 & 0 & 0 \\ 0 & 0 & a_3 & -a_4 & 0 \\ 1-a_1 & 0 & 1-a_3 & 0 & 0 \end{bmatrix}; B = [u\ 0\ 0\ 0\ 0]^T$$

From the form of the coefficient matrix A, the eigenvalues corresponding to the stability of the equilibrium point (EP) of system (1) can be investigated. Next, numerical simulation of system (1) will also be investigated to find out the pattern of the solution of each variable.

3. Results and Discussion

In this session, the differential equation system that has been formulated will be analysed both analytically and numerically so that the dynamic behaviour of the modelled process can be understood and the impact of control parameters on the evolution of the system over time can be evaluated.

3.1 Analitycal Result

The system of differential equations presented by system (1) gives a system of linear differential equations. Furthermore, the equilibrium point of the system can be investigated by

$$\frac{dx_i}{dt} = 0; i = 1, \dots, 4$$

and

$$y = 0$$

Hence, we obtain equilibrium point *EP* of the system 1) is an EP column matrix as follows

$$\boldsymbol{EP} = \left[\boldsymbol{u}, \frac{\boldsymbol{a_1}}{\boldsymbol{a_2}}\boldsymbol{u}, \boldsymbol{a_1 u}, \frac{\boldsymbol{a_3 a_1}}{\boldsymbol{a_4}}\boldsymbol{u}, \boldsymbol{u}(\boldsymbol{1} - \boldsymbol{a_1 a_3})\right]^{\boldsymbol{T}}.$$

Furthermore, after determining the equilibrium point of the system 1), the stability of the equilibrium point will be investigated. The process of investigating the type of equilibrium point and the stability of the equilibrium point will use the following definitions and theorems. According to Wiggins (2000), Let *EP* $\bar{x}$ be a fixed point of $\dot{x} = f(x), x \in R^n$. Fix point $\bar{x}$ be the hyperbolic fixed point if none of the eigenvalues of A have zero real part.

From system 1), the coefficient matrix $\boldsymbol{A}$ is obtained as follows

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ a_1 & -a_2 & 0 & 0 & 0 \\ 0 & a_2 & -1 & 0 & 0 \\ 0 & 0 & a_3 & -a_4 & 0 \\ 1-a_1 & 0 & 1-a_3 & 0 & 0 \end{bmatrix}.$$

Next, we will find the eigenvalue of the coefficient matrix system with $|A - \lambda I| = 0$. The characteristic equation is obtained as follows,

$$-\lambda(a_2a_4\lambda^2 + 2a_2a_4\lambda + a_2a_4 + a_2\lambda^3 + 2a_2\lambda^2 + a_2\lambda + a_4\lambda^3 + 2a_4\lambda^2 + a_4\lambda + \lambda^4 + 2\lambda^3 + \lambda^2) = 0.$$

Hence, we obtain the eigen value

$$\lambda_{1,2} = -1; \lambda_3 = -a_2; \lambda_4 = -a_4; \lambda_5 = 0.$$

Considering the eigenvalues generated by the coefficient matrix A, the resulting eigenvalue is the eigenvalue $\lambda_i \leq 0; i = 1,\ldots 5$. Therefore, based on Wiggins (2000), the equilibrium point *EP* is a non-hyperbolic equilibrium point because it has one eigenvalue with the real part equal to 0.

Next, the stability of the *EP* equilibrium point will be investigated by theorem from Olsder (2003). According to Olsder (2003), Given the differential equation $\dot{x} = Ax + B$, with A an $n \times n$ matrix with different eigenvalues $\lambda_1, \ldots, \lambda_k$ $(k \leq n)$. The Equilibrium point *EP* is stable if $Re(\lambda_i) \leq 0$ for $i = 1, \ldots k$. As a result of the eigenvalues generated by the coefficient matrix A of system 1), the equilibrium point *EP* is a stable equilibrium point. Furthermore, the equilibrium point *EP* is a non-hyperbolic and stable equilibrium point. Since the *EP* equilibrium point is a non-hyperbolic equilibrium point, further investigation is needed to determine the behavior of the *EP* equilibrium point. Numerical simulation of system 1) can determine the behavior of the *EP* equilibrium point by approaching several tools in MATLAB or VSCode. The next subsections will present some numerical simulations for system 1) and the *EP* equilibrium point.

3.2 Numerical Analysis

The numerical simulation begins by plotting each compartment or variable against time, taking some parameter values that refer to several papers.

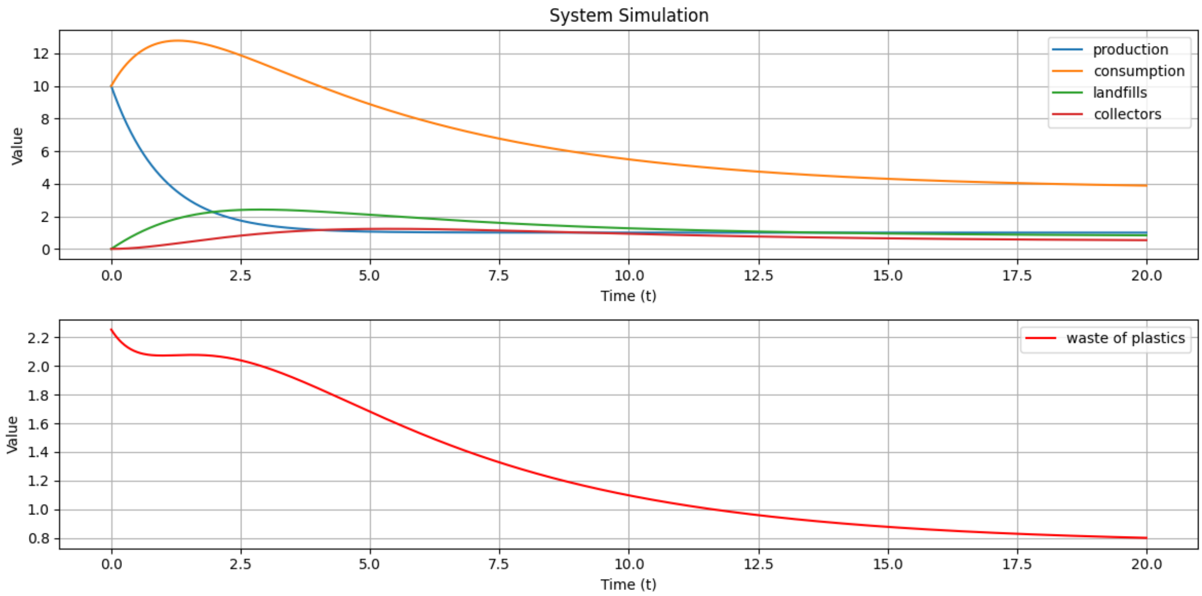

Figure 4. Plot of the system 1) (four Compartments and one Output) with parameters $u = 1$; $a_1 = 0.8$; $a_2 = 0.9$; $a_3 = 0.5$; $a_4 = 0.5$.

The interpretation of Figure 4 shows that the values of parameters $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$ are quite high. It illustrates that if the production and consumption levels have high values, the graph moves dynamically to the *EP* equilibrium point. Moreover, the simulation revealed that when the initial conditions $(\boldsymbol{x_{10}}, \boldsymbol{x_{20}}, \boldsymbol{x_{30}}, \boldsymbol{x_{40}})$ were set to (10, 10, 0, 0), there was observed dynamic movement of the four compartments in relation to system 1. The consumption compartment $(\boldsymbol{x_2})$ exhibited an increase in the short term following a decline in production activity. This suggests that the landfills compartment $(\boldsymbol{x_3})$ and the collector compartment $(\boldsymbol{x_4})$ will also exhibit a short-term increase before reaching a state of stability. Concurrently, the movement of final plastic waste has exhibited a downward trend, although over an extended time period.

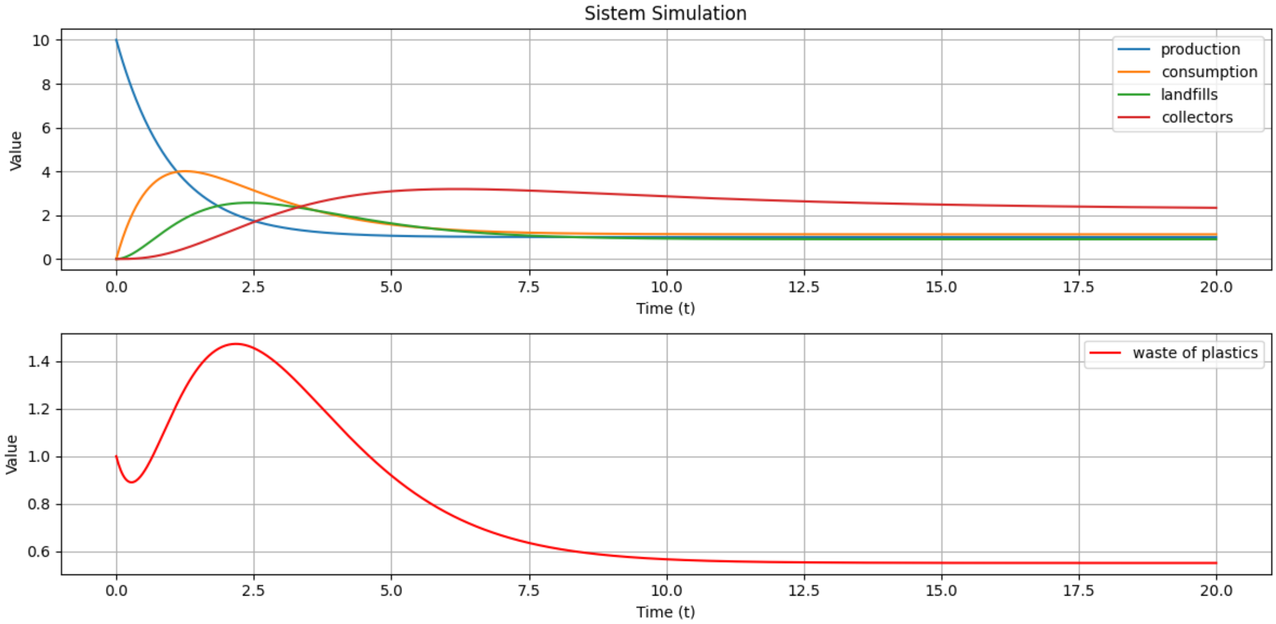

Figure 5. Plot of the system 1) with parameters $u = 1; a_1 = 0{,}8; a_2 = 0{,}8; a_3 = 0{,}5; a_4 = 0{,}3$.

In Figure 5, slight changes made to parameters a_1 and a_2 have an impact on the output variable, namely the amount of plastic waste, which increases over a certain period of time and will then decrease and go to 0. Illustratively, all variables move dynamically towards the equilibrium point. The initial value of the model is set at the starting point $(x_{10}, x_{20}, x_{30}, x_{40})$ equal to (10, 0, 0, 0). The dynamic movement of the four compartments, namely production, consumption, waste disposal, and waste collection, demonstrates a stable direction. At the beginning of the time period under consideration, the consumption, waste disposal, and waste collection compartments increase in line with the specified parameters. Consequently, the movement of the final waste compartment escalates, influenced by the production and consumption compartments.

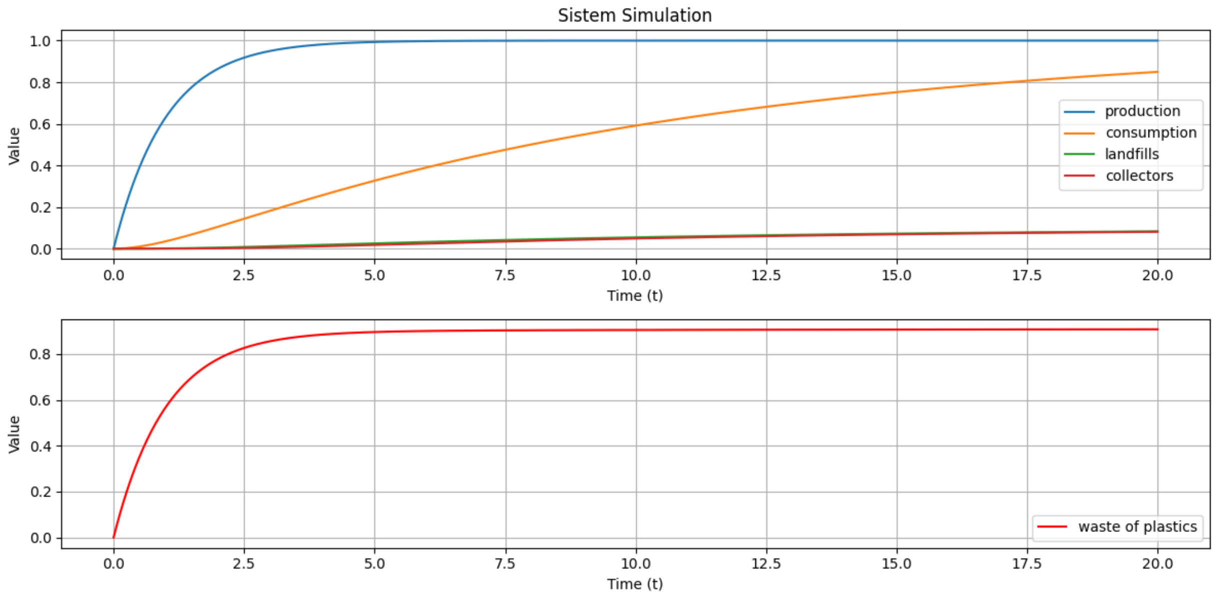

Figure 6. Plot of system 1) with parameters $u = 1, a_1 = 0{,}1. a_2 = 0{,}1, a_3 = 0{,}9, a_4 = 0{,}9$

The figure further explains that when parameters a_3 and a_4 are increased dan take initial value $(x_{10}, x_{20}, x_{30}, x_{40})$ equal to (10, 0, 0, 0), the amount of plastic waste will increase following the logistic curve. This shows that plastic waste should be managed properly and its growth can be suppressed. Next, simulations are conducted between compartments or variables to see the behavior or pattern of each compartment that is affected by each compartment.

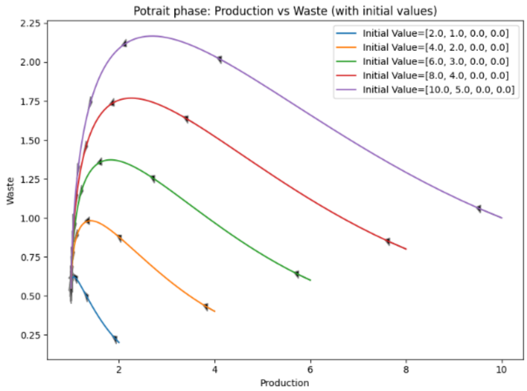

Figure 7. Plot Production vs Waste

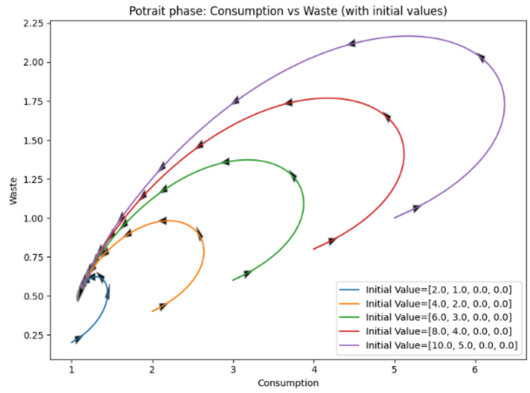

Figure 8. Plot Consumption vs Waste

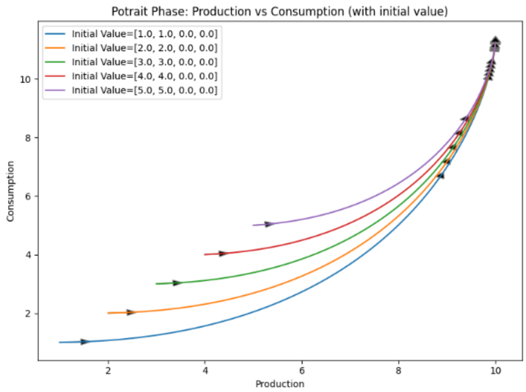

Figure 9. Plot Production vs Consumption

As shown in the figure, each compartment can be analyzed to determine its effect on the others when certain initial values are considered. Figure 7 shows the movement of solutions between the production and waste compartments. It can be seen that when production decreases, waste increases within a certain time interval. This is due to the amount of plastic consumed. Figure 8 shows the direct proportional relationship between the consumption compartment and waste consumption. When the consumption graph rises, the plastic waste graph rises simultaneously. Figure 9 shows the exponential relationship between the consumption compartment and the plastic waste compartment.

4. Conclusion

The plastic waste flow modeling described in this article adopts the SIR model used to study the spread of infectious diseases. In this model, the plastic waste flow is divided into several corresponding compartments at each stage. The model is then analyzed both analytically and numerically. Analytically, system 1) has a non-hyperbolic equilibrium point $\boldsymbol{EP} = \left[\boldsymbol{u}, \frac{\boldsymbol{a_1}}{\boldsymbol{a_2}}\boldsymbol{u}, \boldsymbol{a_1 u}, \frac{\boldsymbol{a_3 a_1}}{\boldsymbol{a_4}}\boldsymbol{u}, \boldsymbol{u(1 - a_1 a_3)}\right]^{\boldsymbol{T}}$ that is non-asymptotically stable. This system was also analysed numerically using MATLAB and Python. The numerical results showed the sensitivity of each parameter to the number of compartments involved. Changes in the initial values of each compartment and the values of each parameter greatly affected the dynamic movement of the numerical solution. This enables more comprehensive further analysis. Although the population dynamics model is suitable for describing plastic waste flow patterns, several factors have not been considered. One such factor is plastic waste recycling. This factor will also impact the model that has been developed as well as both analytical and numerical solution analyses. Various articles often mention recycling as a way to optimise plastic use or consumption.

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