

Mathematical modeling of voter participation in Indonesia's 2024 Presidential Election with consideration of unregistered voters in the final voter list

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Abstract

Voter participation is a key indicator of the legitimacy of an election, yet turnout in Indonesia's 2024 presidential election was 81.78%, and some eligible citizens abstain or are excluded from the final voter list. This study builds a compartmental model of voter participation in the 2024 election with four compartments: potential voters, registered voters, unregistered voters, and abstainers. A compartmental model was chosen because abstention spreads through contact between voters and abstainers, and because it yields thresholds that can be derived analytically. The analysis determined the abstaining-free and abstaining equilibrium points, the basic reproduction number R_0 by the next-generation matrix method, and local stability by eigenvalues and the Routh–Hurwitz criterion; numerical simulations in Maple 18 varied the rates of registration (β), registration failure (δ), re-registration (θ), abstention of unregistered voters (α), and abstention of registered voters (γ). Abstention disappears when $R_0 < 1$ and persists at a unique positive equilibrium when $R_0 > 1$. Sensitivity indices show that raising β and θ lowers R_0 , whereas δ , α , and γ raise it, and the time needed to eliminate abstention is long even when $R_0 < 1$. Better registration and easier re-registration of unregistered voters are therefore recommended as policy priorities.

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1. Introduction

Voter participation is a key indicator of the legitimacy of an election, and in Indonesia it remains a concern. The General Elections Commission (KPU) reported a turnout of 81.78% in the 2024 presidential election, slightly below the 81.97% of 2019, among 204,422,181 registered voters; the two figures are not computed on exactly the same basis, because the 2024 figure also counts voters on the special voter list (Bisnis.com, 2024; Subarkah, 2024). Citizens who do not vote, commonly called *golput* or abstainers, include those who choose not to vote and those who are prevented by administrative problems.

One concrete administrative cause is the accuracy of the voter list. Updating the list faces duplicate entries, deceased voters, changes of residence, and eligible residents who are not recorded (Sabdo, 2023; Adli Wa et al., 2023). Eligible citizens who are in neither the permanent voter list (DPT) nor the additional list (DPTb) may still vote on election day through the special voter list (DPK) by showing an electronic ID card at the polling station of their address, but this requires that they know the procedure (Huda, 2024). Eligible citizens who are not listed and do not use this mechanism are unregistered voters, who may be registered later or may abstain. The concrete problem studied here is how the number of abstainers evolves when such unregistered voters are considered explicitly, and which of the registration-related rates decide whether abstention persists. The urgency is that these registration problems recur in every election cycle, so that registration policy for the coming elections needs quantitative evidence on which interventions matter most.

Mathematical modeling has been used to study voter behavior and to compare policy options. Compartmental models borrowed from epidemiology treat abstention as a behavior that spreads through contact between voters and abstainers, as in the models of Balatif et al. (2020), Putri et al. (2021), and Putri & Widyasari (2024); they allow equilibrium points, a basic reproduction number as a threshold, and stability to be derived analytically, which is why this approach was used. Recent studies of behavioral dynamics support this view: Charalambous (2024) showed that social norms can shape collective decisions in a network, and Albi et al. (2025) showed that social influence, especially from well-connected individuals, shifts the distribution of behavior in a population.

Existing models of participation divide the population into potential voters, registered voters, and abstainers (Yubelian & Megawati, 2023), study awareness campaigns and optimal control (Balatif et al., 2020; Putri et al., 2021), or compare participation in Medan in the 2019 and 2024 elections (Putri & Widyasari, 2024). To the authors' knowledge, none treats eligible citizens who are missing from the final voter list as a separate group. The novelty of this study is the new compartment of unregistered voters, who are eligible but not listed and who can be registered later (rate θ) or can abstain (rate α), together with an explicit basic reproduction number that shows how registration failure (δ) and re-registration (θ) enter the threshold.

This study aims to analyze the dynamics of voter participation with unregistered voters in the 2024 presidential election. The research questions are: (1) what are the equilibrium points of the model and its basic reproduction number; (2) under what conditions are the equilibrium points locally asymptotically stable; and (3) how do the registration-related rates affect abstention in the numerical simulations, and what do the results imply for registration policy?

2. Method

There are stages that must be carried out so that the research objectives can be achieved, as follows.

(1) Literature Review

A review of previous studies and relevant literature was conducted to understand the theoretical background of mathematical modeling, especially that related to voter participation and abstention dynamics. Sources included books, scientific journals, and official election reports.

(2) Model Construction

In this stage, the mathematical model was constructed to represent the dynamics of voter participation.

- Model Assumptions: several assumptions were made to define the model framework, including a constant total population, fixed recruitment and removal rates, and consistent transition rates between voters compartments.
- Model Formulation: based on the assumptions, a system of differential equations was constructed. The population was divided into four compartments, potential voters (P), registered voters (R), unregistered voters (U), and abstainers (A).

(3) Mathematical Analysis

Analytical techniques were applied to study the qualitative behavior of the model. The following steps were carried out:

- Determination of the equilibrium points,
- Construction of the Jacobian matrix to linearize the system around the equilibrium points,
- Calculation of eigenvalues from the Jacobian matrix,
- Stability analysis of each equilibrium point using the eigenvalues.

The basic reproduction number was derived with the next-generation matrix method, taking A as the only compartment that carries abstention. Local stability of the abstaining-free equilibrium was determined from the eigenvalues of the Jacobian matrix, and that of the abstaining equilibrium from the Routh–Hurwitz criterion applied to the cubic characteristic polynomial. The influence of each parameter p on R_0 was measured with the normalized forward sensitivity index $(\partial R_0 / \partial p)(p/R_0)$, which gives the percentage change of R_0 for a one percent change of p .

(4) Numerical Simulation

Numerical simulations were performed using Maple 18 software to verify the analytical results. These simulations aimed to observe the dynamic behavior of each subpopulation over time and to confirm the conditions under which abstention behavior persists or disappears.

(5) Conclusion Drawing

The final stage involved drawing conclusions based on the analytical and simulation results. These conclusions evaluated whether the research objectives were achieved and provided interpretations of the model in the context of real world voter participation dynamics.

3. Results and Discussion

3.1. Mathematical Model

This study presents a compartmental mathematical model to describe the dynamics of voter participation in Indonesia's 2024 Presidential Election. The population is divided into four compartments based on their status in the voting process: potential voters (P), registered voters (R), unregistered voters (U), and abstainers (A). Here, $P(t)$ represents individuals who are eligible to vote but have not yet registered or abstained, $R(t)$ denotes voters who have successfully registered, $U(t)$ includes eligible individuals who are not listed in the official registry, and $A(t)$ accounts for those who ultimately choose not to vote, either voluntarily or due to technical barriers. The model uses the following variables and parameters in Table 1.

Table 1. List of parameters.

Parameter	Description	Unit
Λ	Rate of which individuals become eligible to vote.	$\frac{\text{individuals}}{\text{year}}$
μ	Rate of which individuals leave the voting population (e.g., due to death or ineligibility).	$\frac{\text{individuals}}{\text{year}}$
β	Rate of transition from potential voters (P) to registered voters (R).	$\frac{\text{individuals}}{\text{year}}$
δ	Rate of transition from potential voters (P) to unregistered voters (U).	$\frac{\text{individuals}}{\text{year}}$
γ	Rate of transition from registered voters (R) to abstainers (A).	$\frac{\text{individuals}}{\text{year}}$
θ	Rate of transition from unregistered voters (U) to registered voters (R).	$\frac{\text{individuals}}{\text{year}}$
α	Rate of transition from unregistered voters (U) to abstainers (A).	$\frac{\text{individuals}}{\text{year}}$

The flow diagram below illustrates the interactions among the four compartments.

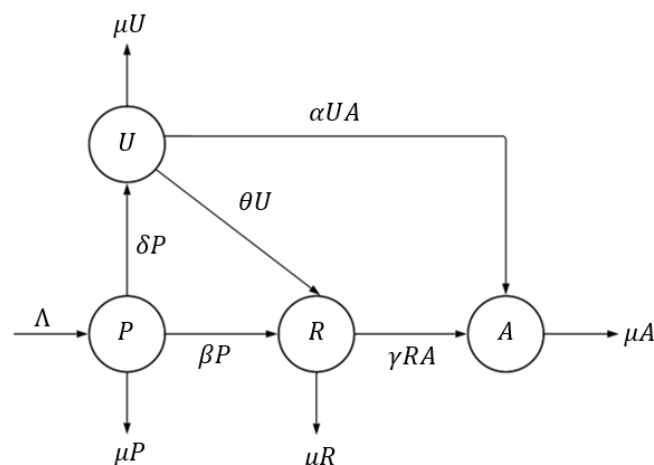


Figure 1. Transfer diagram of PRUA mathematical model.

Based on the transition structure shown in the diagram, the system of differential equations is formulated as follows,

$$\frac{dP(t)}{dt} = \Lambda - \beta P - \delta P - \mu P, \quad (1)$$

$$\frac{dR(t)}{dt} = \beta P + \theta U - \gamma RA - \mu R, \quad (2)$$

$$\frac{dU(t)}{dt} = \delta P - \theta U - \alpha UA - \mu U, \quad (3)$$

$$\frac{dA(t)}{dt} = \gamma RA + \alpha UA - \mu A. \quad (4)$$

3.2. Mathematical Model Analysis

3.2.1. Equilibrium Points

Based on Equations (1) – (4) the equilibrium point of the system can be determined by setting the right-hand side of each equation to zero. The system of equations becomes,

$$\Lambda - \beta P - \delta P - \mu P = 0 \quad (5)$$

$$\beta P + \theta U - \gamma RA - \mu R = 0 \quad (6)$$

$$\delta P - \theta U - \alpha UA - \mu U = 0 \quad (7)$$

$$\gamma RA + \alpha UA - \mu A = 0 \quad (8)$$

Solving this system leads to a second-degree polynomial in A , given by,

$$a_0 A^2 + a_1 A + a_2 = 0 \quad (9)$$

where the coefficients are defined as,

$$a_0 = -\alpha\beta\gamma\mu - \alpha\delta\gamma\mu - \alpha\gamma\mu^2$$

$$a_1 = \Lambda\alpha\beta\gamma + \Lambda\alpha\delta\gamma - \alpha\beta\mu^2 - \alpha\delta\mu^2 - \alpha\mu^3 - \beta\gamma\mu^2 - \beta\gamma\mu\theta - \delta\gamma\mu^2 - \delta\gamma\mu\theta - \gamma\mu^3 - \gamma\mu^2\theta$$

$$a_2 = \Lambda\alpha\delta\mu + \Lambda\beta\gamma\mu + \Lambda\beta\gamma\theta + \Lambda\delta\gamma\theta - \beta\mu^3 - \beta\mu^2\theta - \delta\mu^3 - \delta\mu^2\theta - \mu^4 - \mu^3\theta$$

At an equilibrium with $A > 0$, Equations (5) to (7) give $U = \delta P / (\theta + \alpha A + \mu)$ and $R = (\beta P + \theta U) / (\gamma A + \mu)$ with $P = \Lambda / (\beta + \delta + \mu)$, and Equation (8) requires $\gamma R + \alpha U = \mu$. The left-hand side is a strictly decreasing function of A , and its value at $A = 0$ is μR_0 with R_0 given in Equation (13). Hence Equation (9) has a positive root if and only if $R_0 > 1$, and the root is then unique, so that an abstaining equilibrium exists if and only if $R_0 > 1$. This agrees with Descartes' rule of signs, because $a_0 < 0$ and $a_2 = \mu^2(\beta + \delta + \mu)(\theta + \mu)(R_0 - 1)$.

3.2.1.1. Abstaining-free Equilibrium Point (E_0)

The abstaining-free equilibrium represents the condition in which no abstainers exist in the population. This equilibrium is achieved when $A = 0$, resulting in the following equilibrium point,

$$E_0 = (P^*, R^*, U^*, A^*) = \left(\frac{\Lambda}{\beta + \delta + \mu}, \frac{1}{\mu} \left[\frac{\beta\Lambda(\theta + \mu) + \theta\delta\Lambda}{(\beta + \delta + \mu)(\theta + \mu)} \right], \frac{\delta\Lambda}{(\beta + \delta + \mu)(\theta + \mu)}, 0 \right) \quad (10)$$

3.2.1.2. Endemic Equilibrium Point (E_1)

The endemic equilibrium corresponds to a condition in which abstention continues to exist within the population. This equilibrium occurs when all compartments contain a non-zero number of individuals, such that $P^{**} \neq 0, R^{**} \neq 0, U^{**} \neq 0$, and $A^{**} \neq 0$. The equilibrium point is given by the following expressions,

$$P^{**} = \frac{\Lambda}{\beta + \delta + \mu}$$

$$R^{**} = \frac{\alpha\beta\Lambda + \theta\mu(\beta + \delta + \mu)}{(\beta + \delta + \mu)(\theta\gamma + \gamma A^{**}\alpha + \mu\alpha)}$$

$$U^{**} = \frac{1}{\alpha} \left[\mu - \frac{\gamma(\alpha\beta\Lambda + \theta\mu(\beta + \delta + \mu))}{(\beta + \delta + \mu)(\theta\gamma + \gamma A^{**}\alpha + \mu\alpha)} \right]$$

Here, A^{**} is a positive root of the quadratic polynomial

$$a_0 A^2 + a_1 A + a_2 = 0 \quad (11)$$

3.2.2. Basic Reproduction Number

The basic reproduction number R_0 is the average number of new abstainers produced by one abstainer in a population without abstainers. It is derived with the next-generation matrix method, in which A is the only compartment carrying abstention. Starting from the abstainer equation,

$$\frac{dA}{dt} = \gamma R A + \alpha U A - \mu A$$

the rate at which new abstainers appear at the abstaining-free equilibrium is $F = \gamma R^* + \alpha U^*$, and the rate at which abstainers are removed is $V = \mu$, so that $R_0 = FV^{-1}$. Equivalently, the derivative of the right-hand side with respect to A at $A = 0$ is

$$\frac{\partial(dA/dt)}{\partial A} = \gamma R + \alpha U - \mu \quad (12)$$

By substituting the abstainer-free equilibrium into the equation, we derive the following threshold condition,

$$R_0 = \frac{\Lambda(\gamma[\beta(\theta + \mu) + \theta\delta] + \mu\alpha\delta)}{\mu^2(\beta + \delta + \mu)(\theta + \mu)} \quad (13)$$

3.2.3. Stability Analysis of the Equilibrium Point

The stability of an equilibrium point can be assessed by analyzing the eigenvalues of the Jacobian matrix, which is obtained by linearizing the system described in Equations (1) – (4) around the equilibrium point. The Jacobian matrix evaluated at a general equilibrium point is given by,

$$J^*(E) = \begin{pmatrix} -(\beta + \delta + \mu) & 0 & 0 & 0 \\ \beta & -\gamma A - \mu & \theta & -\gamma R \\ \delta & 0 & -\theta - \alpha A - \mu & -\alpha U \\ 0 & \gamma A & \alpha A & \gamma R + \alpha U - \mu \end{pmatrix} \quad (14)$$

3.2.3.1. Stability of the Abstaining-free Equilibrium Point (E_0)

The stability of the abstaining equilibrium is analyzed by evaluating the Jacobian matrix in Equation (14) at the point $E_0 = (P^*, R^*, U^*, A^*)$. Substituting the abstaining-free equilibrium into the Jacobian yields the following matrix.

$$J^*(E_0) = \begin{pmatrix} \lambda + (\beta + \delta + \mu) & 0 & 0 & 0 \\ -\beta & \lambda + \mu & -\theta & \gamma R \\ -\delta & 0 & \lambda + \theta + \mu & \alpha U \\ 0 & 0 & 0 & \lambda - (\gamma R^* + \alpha U^* - \mu) \end{pmatrix} \quad (15)$$

The corresponding characteristic equation is given by,

$$(\lambda + (\beta + \delta + \mu))(\lambda + \mu)(\lambda + \theta + \mu)(\lambda - (\gamma R^* + \alpha U^* - \mu)) = 0 \quad (16)$$

The eigenvalues of the Jacobian matrix are,

$$\lambda_1 = -(\beta + \delta + \mu)$$

$$\lambda_2 = -\mu$$

$$\lambda_3 = -(\theta + \mu)$$

$$\lambda_4 = \gamma R^* + \alpha U^* - \mu, \text{ under the condition that } R_0 = \frac{\Lambda(\gamma[\beta(\theta + \mu) + \theta\delta] + \mu\alpha\delta)}{\mu^2(\beta + \delta + \mu)(\theta + \mu)} < 1$$

Since all eigenvalues have negative real parts, it follows that the abstainer-free equilibrium point is locally asymptotically stable.

3.2.3.2. Stability of the Abstaining Equilibrium Point (E_1)

The stability of the abstaining equilibrium point is analyzed by substituting the Jacobian matrix in Equation (14) with the abstaining equilibrium. The Jacobian matrix evaluated at is given by,

$$J^*(E_1) = \begin{pmatrix} -(\beta + \delta + \mu) & 0 & 0 & 0 \\ \beta & -\gamma A - \mu & \theta & -\gamma R \\ \delta & 0 & -\theta - \alpha A - \mu & -\alpha U \\ 0 & \gamma A & \alpha A & \gamma R + \alpha U - \mu \end{pmatrix} \quad (17)$$

From the matrix, one eigenvalue can be directly obtained as $\lambda_1 = -(\beta + \delta + \mu)$. The remaining eigenvalues are determined from the characteristic equation,

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0 \quad (18)$$

At E_1 , Equation (8) with $A \neq 0$ gives $\gamma R^{**} + \alpha U^{**} = \mu$, so the last diagonal entry of the Jacobian matrix is zero, and the coefficients are

$$b_1 = \gamma A^{**} + \alpha A^{**} + \theta + 2\mu,$$

$$b_2 = (\gamma A^{**} + \mu)(\theta + \alpha A^{**} + \mu) + \gamma^2 R^{**} A^{**} + \alpha^2 U^{**} A^{**},$$

$$b_3 = (\gamma A^{**} + \mu)\alpha^2 U^{**} A^{**} + \theta\alpha\gamma U^{**} A^{**} + \gamma^2 R^{**} A^{**}(\theta + \alpha A^{**} + \mu).$$

According to the Routh-Hurwitz criterion, all roots of the characteristic equation have negative real parts if the following conditions are satisfied $b_1, b_2, b_3 > 0$ dan $b_1b_2 - b_3 > 0$. From this explanation, several point can be concluded, namely:

- All terms of b_1, b_2 , and b_3 are positive, so $b_1, b_2, b_3 > 0$.
- Furthermore, $b_1b_2 - b_3 = b_1(\gamma A^{**} + \mu)(\theta + \alpha A^{**} + \mu) + \gamma^2 R^{**} A^{**}(\gamma A^{**} + \mu) + \alpha U^{**} A^{**}[\alpha(\alpha A^{**} + \theta + \mu) - \theta\gamma]$. Since $\alpha U^{**} = \mu - \gamma R^{**} \leq \mu$, the last term is not less than $-\theta\gamma\mu A^{**}$, which is smaller than the first term, because $b_1(\gamma A^{**} + \mu)(\theta + \alpha A^{**} + \mu) \geq 2\mu \cdot \gamma A^{**} \cdot \theta$. Hence $b_1b_2 - b_3 > 0$.

By the Routh-Hurwitz criterion, E_1 is therefore locally asymptotically stable whenever it exists, that is, whenever $R_0 > 1$.

3.3. Numerical Simulation

Numerical simulations were conducted to examine the dynamics of voter participation, considering unregistered individuals. The initial conditions used were the number of potential voters was set as $P(0) = 2.06$, registered voters as $R(0) = 2.04$, unregistered voters as $U(0) = 0.02$, and abstainers as $A(0) = 0.4$. The recruitment rate was $\Lambda = 0.017$, and under the assumption of a constant total population, the removal rate was set equal to $\mu = 0.017$.

[Authors: the initial values add up to 4.52, whereas $\Lambda/\mu = 1$ is the total population to which the model converges, so the total decreases during the simulation and the abstainers first increase even for $R_0 < 1$. Please give the initial values as proportions that add up to Λ/μ , state the unit of time, and state the units of the parameters (they are rates per unit of time, not individuals per year).]

To observe the system dynamics under both conditions $R_0 < 1$ and $R_0 > 1$, five key parameters were varied based on specific assumptions. These parameters are β (rate of transition from potential voters to registered voters), δ (rate of transition from potential voters to unregistered voters), θ (rate of re-registration from unregistered to registered voters), α (rate of transition from unregistered voters to abstainers), and γ (rate of abstention among registered voters). Sensitivity index using Maple 18 indicates that increasing β and θ plays a crucial role in reducing the basic reproduction number R_0 , thereby suppressing the spread of abstention. In contrast, higher values of δ, α , and γ tend to increase R_0 , which may accelerate and sustain abstainer growth. The combinations of parameter variations used in the simulations are summarized in Table 2.

Table 2. Parameters value variations of $\beta, \delta, \theta, \alpha$, and γ .

Sensitivity index		Parameters value variation			R_0	Equilibrium Point (P, R, U, A)
		β	δ	θ		
β	-0.29498469	0.4	0.1	0.06	0.97548145	(0.03288201, 0.92441407, 0.042703911, 0)
		0.8	0.1	0.06	0.77320588	(0.01853871, 0.95738503, 0.02407625, 0)
		0.97	0.1	0.06	0.73231811	(0.01563937, 0.96404974, 0.02031087, 0)
δ	+0.31352341	0.8	0.05	0.06	0.64500666	(0.01960784, 0.96765979, 0.01273236, 0)

θ	-0.27303623	0.8	0.1	0.06	0.77320588	(0.01853871, 0.95738503, 0.02407625, 0)
		0.8	0.2	0.06	0.99178743	(0.01671583, 0.93986642, 0.04341774, 0)
		0.8	0.1	0.03	0.94613905	(0.01853871, 0.94201721, 0.03944407, 0)
		0.8	0.1	0.06	0.77320588	(0.01853871, 0.95738503, 0.02407625, 0)
		0.8	0.1	0.09	0.69724458	(0.01853871, 0.96413538, 0.01732590, 0)
		θ	α	γ		
θ	-0.27303623	0.003	0.2	0.0087	1.54535249	(0.01853871, 0.84152669, 0.04839358, 0.09154100)
		0.01	0.2	0.0087	1.27492557	(0.01853871, 0.87240213, 0.04705050, 0.06200864)
		0.024	0.2	0.0087	1.01109444	(0.01853871, 0.93329772, 0.04440154, 0.00376201)
α	$+0.36633193$	0.01	0.2	0.0087	1.27492557	(0.01853871, 0.87240213, 0.04705050, 0.06200864)
		0.01	0.4	0.0087	2.08271264	(0.01853871, 0.83430218, 0.02435392, 0.12280517)
		0.01	0.8	0.0087	3.69828677	(0.01853871, 0.81564518, 0.01237985, 0.15343623)
γ	$+0.63366806$	0.01	0.2	0.004	1.02256339	(0.01853871, 0.91087950, 0.06678240, 0.00379937)
		0.01	0.2	0.0087	1.27492557	(0.01853871, 0.87240213, 0.04705050, 0.06200864)
		0.01	0.2	0.01	1.344727883	(0.01853871, 0.85484972, 0.04225751, 0.08435404)

The numerical simulation graphs of the mathematical model for the dynamics of voter participation in Indonesia's 2024 Presidential Election, with consideration of unregistered voters in the final voter list, are presented in Figures 2 to 6. Each figure illustrates the behavior of four subpopulations under different parameter variations.

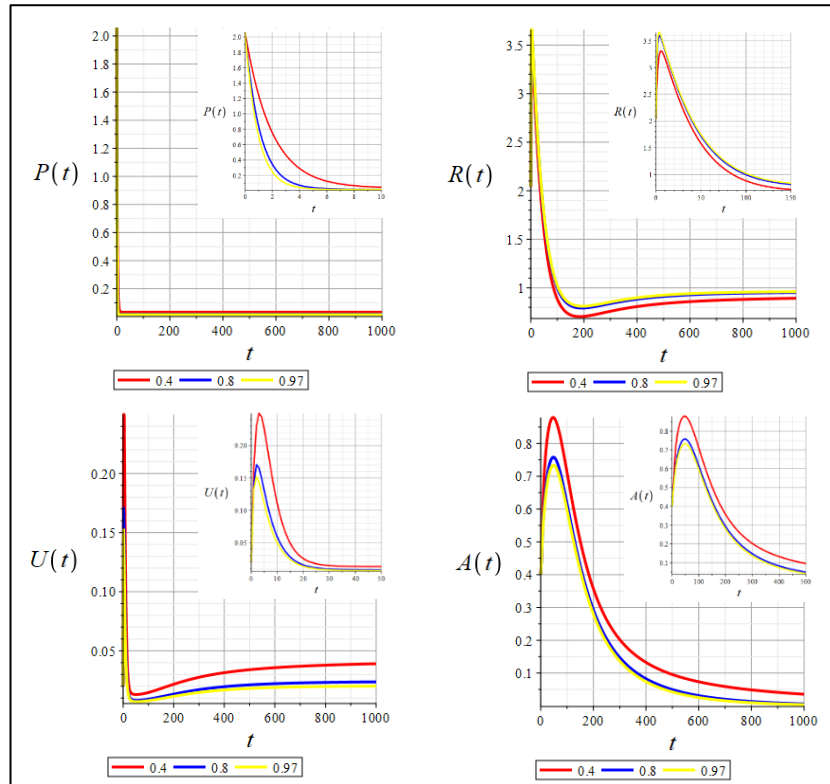


Figure 2. Graph of varying parameter β on the four subpopulations over time, each subplot represents the population dynamics of potential voters ($P(t)$), registered voters ($R(t)$), unregistered voters ($U(t)$), and abstainers ($A(t)$) for three different values of β .

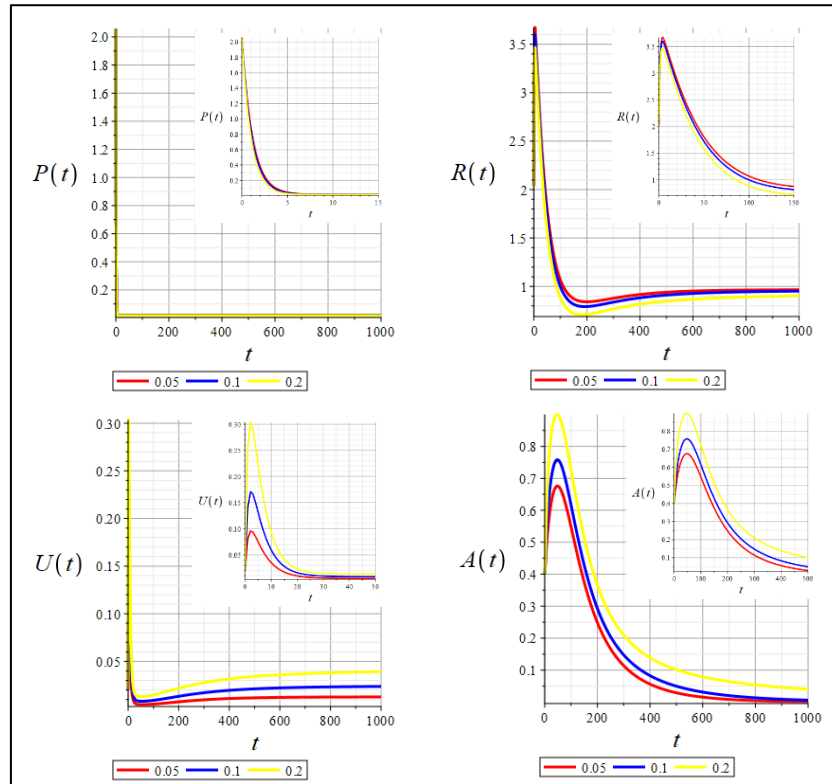


Figure 3. Graph of varying parameter δ on the four subpopulations over time, each subplot represents the population dynamics of potential voters ($P(t)$), registered voters ($R(t)$), unregistered voters ($U(t)$), and abstainers ($A(t)$) for three different values of δ .

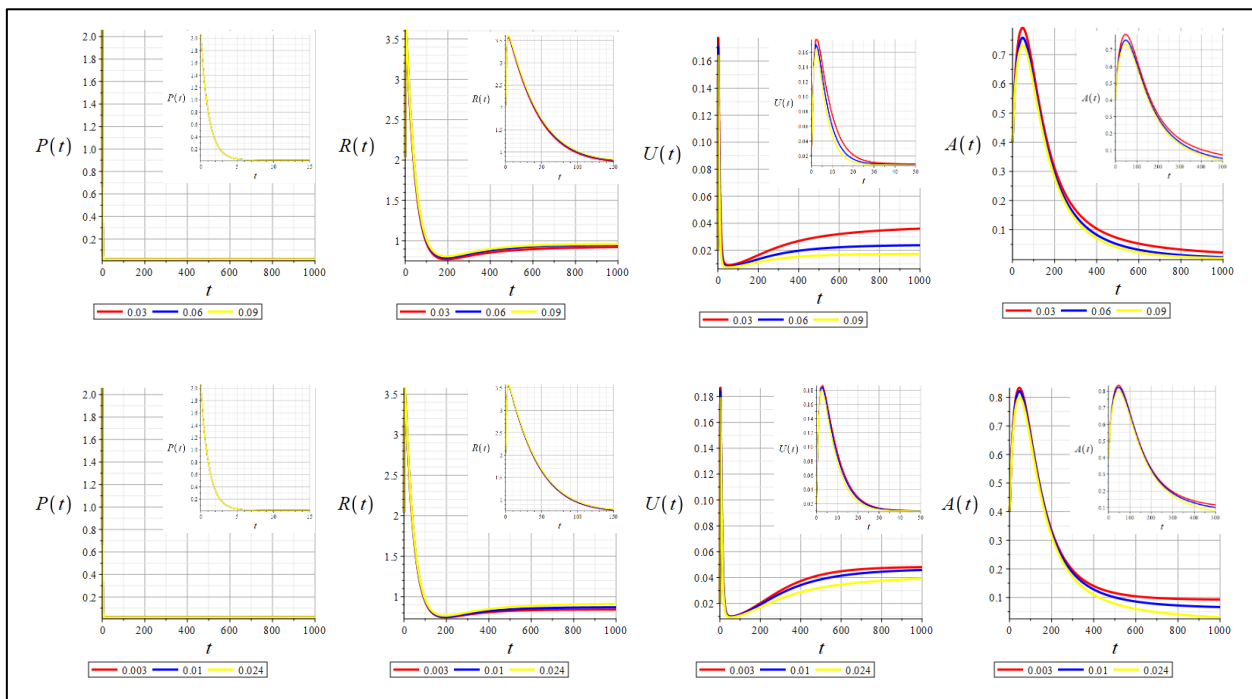


Figure 4. Graph of varying parameter θ on the four subpopulations over time, each subplot represents the population dynamics of potential voters ($P(t)$), registered voters ($R(t)$), unregistered voters ($U(t)$), and abstainers ($A(t)$) for six different values of θ .

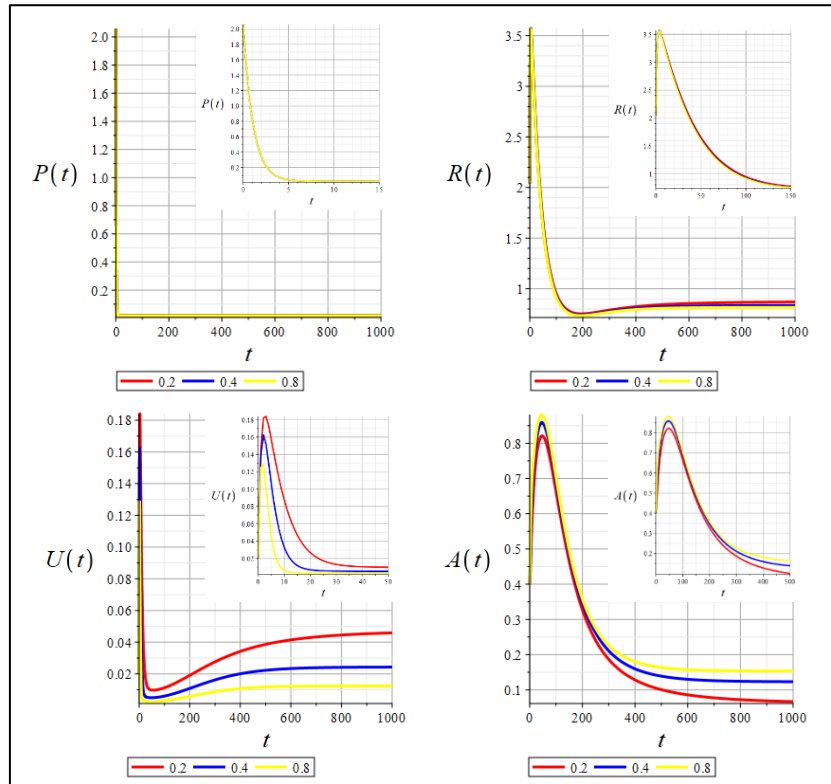


Figure 5. Graph of varying parameter α on the four subpopulations over time, each subplot represents the population dynamics of potential voters ($P(t)$), registered voters ($R(t)$), unregistered voters ($U(t)$), and abstainers ($A(t)$) for three different values of α .

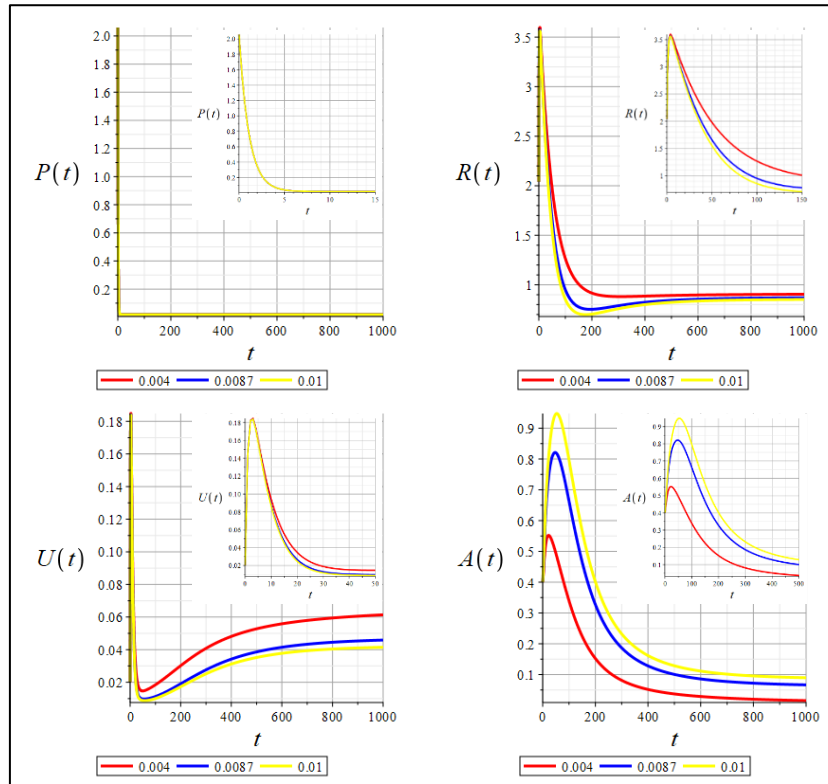


Figure 6. Graph of varying parameter γ on the four subpopulations over time, each subplot represents the population dynamics of potential voters ($P(t)$), registered voters ($R(t)$), unregistered voters ($U(t)$), and abstainers ($A(t)$) for three different values of γ .

Figures 2 to 6 illustrate the simulation outcomes for all subpopulations under varying values of the parameters β , δ , θ , α , and γ . The results highlight the following dynamics:

- (1) In Figure 2, the parameter β represents the rate of potential voters successfully register. Increasing β raises the number of registered voters and reduces abstainers, indicating that effective initial registration plays a crucial role in enhancing voter participation.
- (2) In Figure 3, the parameter δ describes the rate of registration failure among potential voters. Higher values of δ lead to an increase in unregistered and abstaining voters, showing that inefficiencies in the registration process contribute significantly to abstention.
- (3) In Figure 4, the parameter θ indicates the re-registration rate of unregistered voters. In both abstaining-free and abstaining-present conditions, higher values of θ increase the number of registered voters and reduce abstention, highlighting the importance of accessible re-registration mechanisms.
- (4) In Figure 5, the parameter α represents the rate of unregistered voters become abstainers. An increase in α leads to a higher abstainer population, indicating that unregistered individuals are more likely to disengage.
- (5) In Figure 6, the parameter γ reflects the rate of abstention among registered voters. As γ increases, abstainers become more dominant in the population, indicating the strong influence of voter disengagement and dissatisfaction on electoral participation.

Overall, the numerical simulations demonstrate how variations in key parameters affect the dynamics of voter participation. Increasing the values of β and θ generally leads to a reduction in abstention by improving registration outcomes and reintegration efforts. In contrast, higher values of δ , α , and γ tend to sustain or increase the abstainer population, reflecting the negative impact of registration failure and social disengagement. These findings highlight the importance of both administrative effectiveness and motivational factors in promoting long-term electoral participation.

The simulations and Table 2 can be read through the threshold R_0 . All values of R_0 in Table 2 agree with Equation (13), with $\alpha = 0.2$ and $\gamma = 0.0087$ in the first block and $\beta = 0.8$ and $\delta = 0.1$ in the second block. In the first block $R_0 < 1$, the equilibrium has no abstainers, and 1.3% to 4.3% of the population remains unregistered, which shows that registration failure leaves a residual group of unregistered voters even when abstention disappears. In the second block $R_0 > 1$, and abstainers persist at 0.4% to 15% of the population. With the other parameters of the second block, abstention is eliminated

if the re-registration rate θ exceeds about 0.0249, and with the parameters of the first block it would reappear if the registration failure rate δ rose above about 0.204.

The sensitivity indices show which rates matter most. At the baseline of the first block ($R_0 = 0.77$) they are -0.295 for β , +0.314 for δ , -0.273 for θ , +0.366 for α , and +0.634 for γ , so that the abstention of registered voters (γ) is the most influential rate. At the baseline of the second block ($R_0 = 1.27$), where re-registration is weak, the indices are -0.522, +0.541, -0.224, +0.634, and +0.366, and the registration rates β and δ and the abstention of unregistered voters (α) become the most influential. The ranking therefore depends on the baseline, and registration matters most when re-registration is weak.

A structural feature explains why elimination is slow. Abstainers leave the model only through the removal rate μ , since the equation of A gives $dA/dt \geq -\mu A$, so that $A(t) \geq A(0)e^{-\mu t}$. With $\mu = 0.017$ the abstainer population needs at least 41 time units to halve, even when $R_0 < 1$; this agrees with the observation that elimination takes a long time under ideal conditions.

3.4. Discussion

The results show that the persistence of abstention is controlled by a single threshold, R_0 , which depends on all five rates. Abstention can be eliminated only if the registration of potential voters (β) and the re-registration of unregistered voters (θ) are high enough relative to registration failure (δ) and to the abstention of unregistered (α) and registered (γ) voters. Because the abstainers have no way back into participation in this model, the effect of any policy that lowers R_0 below one is slow.

The threshold behavior agrees with earlier models of abstention, in which the abstaining-free equilibrium is stable when $R_0 < 1$ and the abstaining equilibrium when $R_0 > 1$ (Balatif et al., 2020; Putri et al., 2021; Putri & Widyasari, 2024), and the finding that higher registration lowers abstention agrees with Balatif et al. (2020). The model extends the three-compartment model of Yubelian and Megawati (2023) by the unregistered compartment, and the extension changes the threshold: R_0 contains δ , θ , and α , so that the exclusion of eligible citizens from the list and the possibility of re-registering them become explicit determinants of abstention. Because these models differ in structure and parameters, the comparison is limited to the qualitative behavior.

The novelty of the findings is that registration failure and re-registration enter the threshold quantitatively, that a residual group of unregistered voters remains even at the abstaining-free equilibrium, and that the influence of each rate depends on how effective re-registration is. For the election administrators this implies the following recommendations: (1) reduce registration failure (δ) by improving the accuracy of the voter list through continuous data updating and matching of population data (Adli Wa et al., 2023; Sabdo, 2023); (2) raise re-registration (θ) by making the special voter list and the additional voter list easy to use and well known, because eligible citizens outside the permanent list can still vote with an electronic ID card (Huda, 2024); (3) raise initial registration (β), especially for new voters; and (4) combine these measures with communication that reduces abstention among registered and unregistered voters (γ and α).

Several limitations should be noted. The parameter values are illustrative and were not estimated from data of the KPU; the analysis proves local stability only; the total population is assumed constant; the model has no flow of abstainers back into participation, which makes the elimination of abstention very slow; and the simulation setup needs the clarifications indicated above. Future research should estimate the parameters from the permanent, additional, and special voter lists, add a re-engagement rate from abstainers to registered voters, examine global stability, and include the time gap between the announcement of the voter list and election day.

4. Conclusion

The three research questions are answered as follows. First, the model with unregistered voters has an abstaining-free equilibrium E_0 , a unique abstaining equilibrium E_1 that exists if and only if $R_0 > 1$, and the basic reproduction number $R_0 = \Lambda(\gamma[\beta(\theta+\mu)+\theta\delta] + \mu\alpha\delta) / (\mu^2(\beta+\delta+\mu)(\theta+\mu))$. Second, E_0 is locally asymptotically stable when $R_0 < 1$, and E_1 is locally asymptotically stable whenever it exists, that is, when $R_0 > 1$. Third, the simulations and sensitivity indices show that higher registration (β) and re-registration (θ) lower R_0 and reduce abstention, whereas higher registration failure (δ) and higher abstention of unregistered (α) and registered (γ) voters raise it, and that abstention disappears only slowly even when $R_0 < 1$.

These results imply that registration policy should give priority to improving the accuracy of the voter list, to easy and well-known re-registration of eligible citizens who are not listed, and to initial registration of new voters, together with measures that reduce abstention among registered voters.

Future extensions of the model may add a return flow from abstainers to registered voters, because in the present model abstainers leave only through the removal rate. The inclusion of public figures or social media influencers, particularly those with significant appeal among younger voters, could improve the model's ability to capture real voting behavior. A comparison between the official voter list and demographic data from the national statistics agency may help address ongoing discrepancies that affect electoral accuracy. It is also important to account for deaths among eligible voters occurring between the finalization of the voter list and election day, which may require more responsive data validation involving the smallest administrative units such as neighborhood associations. Further studies should evaluate the model's applicability to elections without incumbent candidates in order to test its consistency under different political contexts. Considering the time gap between the announcement of the voter list and the election day could also help explain the persistence of unregistered individuals due to demographic and administrative changes. Lastly, the model could be extended to incorporate revoting events that result from disputes or technical issues during the election process.

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