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GeoGebra-supported Learning Kit for Teaching Area of Trapezoid Based on TPMK Framework

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Abstract

The use of technology in education has expanded fast in the twenty-first century. The capacity to use technology in teaching materials has become a crucial talent for mathematics teachers, and it is a key topic in the current study. This is why, as a new teacher, you must be able to create ICT-based learning materials. This study aims to create an interactive mathematical learning media tool kit for teaching the area of trapezoid based on the TPMK framework and the GeoGebra dynamic mathematics program. This study was research and development (R&D) conducted with the Plomp development model. The designing stage is divided into two steps. In the first step, we created a model for developing a GeoGebra-supported teaching kit based on the TPMK framework originated from Shulman's (1986) stated in our theoretical foundation. In the second step, a concrete example of a task circumstance was created to demonstrate the model. This result shows that using GeoGebra dynamic mathematics software with a TPMK framework can help teachers explain abstract mathematics concepts to make them easier for students to understand.

1. Introduction

Most educators throughout the world are no longer unfamiliar with the use of information and communication technology (ICT) in education. It is a powerful tool for teachers to innovate teaching methods towards active learning - student-centred instead of "content-oriented" or teacher-centred teaching. ICT can help teachers and students succeed in most subjects' teaching and learning processes. Typically, in math, authors Niyibizi and Mutarutinya (2024) argue that math is an important subject of comprehensive education, as it provides students with essential problem-solving and critical thinking skills. However, this is a dry, difficult-to-understand, abstract subject for many S. According to author Dang Trung Van (2023), ICT applied in the teaching and learning process helps students' learning to be lively, attractive, easy to understand and teacher has the conditions to teach differentiated, individualized to enhance the positivity, activeness and creativity of each student; create favourable conditions for teacher's teaching and student's exploration and self-study suitable for the needs, interests, abilities, interests of each student. In addition, author Do Thi Hai (2016) also argues that the application of ICT is an essential requirement for the renewal of teaching methods and that teachers must be interested in teaching all subjects, including math. However, along with the advantages, there are also challenges that the application of ICT must face. Therefore, the author Salehi and Salehi (2012) argued that to integrate ICT into the curriculum, teacher training institutions need appropriate and adequate support for teachers.

On the other hand, the teacher needs to be aware of the situations and changes in the classroom. Therefore, ICT can effectively teach and learn, leading to improved educational programs. Based on the objective requirements of the renewed education program, the teacher needs to be equipped with full knowledge, expertise, and skills in active teaching methods with the support of ICT. According to Mishra and Koehler (2006), effective teaching with technology involves integrating technological, pedagogical,

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and content knowledge. This integration creates a unique technological pedagogical content knowledge (TPACK) expertise, which emerges when teachers design lessons incorporating technology (Koehler et al., 2007). In designing these lessons, teachers utilize their technological, pedagogical, and content knowledge, along with the intersecting areas of technological pedagogical knowledge, technological content knowledge, and pedagogical content knowledge, to develop technology-enhanced instructional strategies that address students' learning challenges. These lesson strategies, created through the design process, represent teachers' TPACK (Cox and Graham, 2009). Applying Mishra and Koehler's concept of TPACK to mathematics teaching, this specialized expertise can be referred to as technological pedagogical mathematics knowledge (TPMK).

1.1 Dynamic mathematic software GeoGebra

The National Council of Teachers of Mathematics (NCTM) specifies six key principles for effective mathematics instruction: equity, curriculum, teaching methods, learning methods, assessment methods, and technology assistance. Technology support is particularly significant in shaping mathematics teaching and learning, enhancing instructional efficacy and improving student learning outcomes. Niyibizi and Mutarutinya (2024) suggest that contemporary students are more motivated to engage in mathematics learning when it is delivered through interactive techniques that incorporate information and communication technology applications. Boo and Leong (2016) acknowledge the availability of numerous advanced technological tools for teaching mathematics, including Geometer's Sketchpad, Cabri Geometry, Cinderella, and GeoGebra. Researchers and educators have widely recognized GeoGebra as the most optimal and comprehensive tool. Developed by Austrian professor Markus Hohenwarter at Salzburg University, GeoGebra was initiated in 2001 and has since undergone continuous development. The software runs on all operating systems, is Java-based, and is available for free download from https://www.geogebra.org. GeoGebra is open-source and multilingual, available in approximately 63 languages, including Vietnamese. Its user-friendly interface and intuitive toolboxes facilitate easy interaction with the software, allowing users to dynamically define and manipulate points, vectors, line segments, lines, conic sections, and functions. Additionally, users can directly enter equations and coordinates. Consequently, GeoGebra can handle variables, vectors, and points, find derivatives and integrals of functions, and execute commands such as "Solve" or "Extrema."

The dynamic mathematics software GeoGebra was created for use in elementary school through university education (Phan Trong Hai, 2023). The software integrates a dynamic geometry environment, algebraic expression manipulation, calculus, and a spreadsheet within a coordinate plane, thus bridging the gap between the mathematical fields of geometry, algebra, calculus, and computation. Le Viet Minh Triet (2021) asserts that with GeoGebra's support, teachers can empower students to enhance their thinking processes during problem-solving. Students have ample opportunities to engage with scientific methods such as data collection through experimentation, data analysis, conjecture, verification of conjecture, generalization, and problem extension. Consequently, students learn mathematics and methods to explore mathematical concepts in a teaching environment. Overall, GeoGebra is easy to use, has a user-friendly interface and numerous prominent features, and is particularly notable for being free of charge. It has the potential to become widely adopted and replace commercial software such as Geometry Cabri and Geometer's Sketchpad and can be readily utilized for web applications (such as GeoGebra Applets) without copyright concerns.

Authors Tran Trung et al. (2012) employed GeoGebra as a visual tool in teaching to help students predict, prove, illustrate, limit, and expand transformation geometry problems in 11th-grade geometry. Phan Trong Hai (2013) utilized GeoGebra to support the exploration of theorems in high school, yielding many positive results. Le Viet Minh Triet (2021) highlighted the benefits of using GeoGebra in teaching 10th-grade exploratory geometry, asserting that its proper use can positively impact the teaching process, contribute innovative teaching methods, and make student learning more active and autonomous. Vu Thi Phuong (2021) employed GeoGebra to reinforce geometric theory, helping students grasp and internalize newly learned concepts. Hang Duy Thanh (2022) used GeoGebra to assist in teaching the solution of locus problems through exploratory methods, increasing student engagement, simplifying the process of finding the locus and instilling confidence in students regarding their problem-solving abilities. Pham Huyen Trang et al. (2023) presented views on the competency in using mathematical tools, manifestations, and the process of teaching to enhance this competency (GeoGebra software) through teaching the representation of the solution region of a system of linear inequalities in two variables, along with illustrative examples. This approach helps students identify the advantages of using GeoGebra in solving mathematical problems compared to conventional methods. Notably, at the elementary level, research by Tran Hoa Hiep and Nguyen Tan Tai (2022a, 2022b) focused on using GeoGebra to design dynamic geometric products for teaching the areas of triangles and trapezoids in 5th-grade mathematics. Similarly, Tran Hoa Hiep and Tran Long Quang (2022) designed dynamic geometric products for teaching rhombuses in 4th grade. These studies affirm that this approach fosters student interest and passion for learning mathematics while developing mathematical thinking.

1.2 TPACK and TPMK

The TPACK framework originated from Shulman's (1986) notion of pedagogical content knowledge. Shulman posited that teachers' specialized instructional expertise is rooted in their ability to integrate pedagogical and content knowledge to develop lesson strategies tailored to diverse student profiles. With the advent of technology, Mishra and Koehler (2006) expanded Shulman's concept by incorporating technological knowledge, resulting in the TPACK framework.

Mishra and Koehler (2006) propose that TPACK comprises three primary knowledge domains: technological knowledge (TK—teachers' understanding of technology tools), pedagogical knowledge (PK—teachers' understanding of instructional methodologies), and content knowledge (CK—teachers' understanding of subject matter). These domains are conceptualized as overlapping circles, producing intermediary knowledge forms from their intersections. These intermediary forms are defined as technological pedagogical knowledge (TPK—teachers' understanding of using technology tools to enhance pedagogical strategies), technological content knowledge (TCK—teachers' understanding of using technology tools to support content representation), and pedagogical content knowledge (PCK—teachers' understanding of teaching without technology). TPACK encapsulates the intersection of TK, PK, and CK, which also encompasses the overlap of TPK, TCK, and PCK and can be exemplified through technology-integrated lessons created by teachers (Cox & Graham, 2009).

The seven constructs of TPACK have gained widespread acceptance in both teacher and mathematics education. However, Cox and Graham (2009) and Graham (2011) advise caution in fully endorsing the framework, as its construct validity and predictive utility require further refinement. Angeli and Valanides (2009) contend that teachers' knowledge of technology integration is transformative, given that the outcomes from the dynamic interplay among technology, pedagogy, content, learners, and context cannot be merely summed up. Similarly, Guerrero (2010) asserts that TPMK does not necessarily have a one-to-one correspondence with Mishra and Koehler's (2006) seven-component TPACK model; instead, the intersection among technology, pedagogy, and content can serve as a basis for considering TPMK. From a teacher development perspective, Niess et al. (2009) argue that TPACK for mathematics is an extension of teachers' PCK. Drawing on the varying conceptions of PCK for mathematics (Depaepe et al., 2013), Niess et al. utilize the works of Grossman and Lynn (1990) and Borko and Putnam (1996). These authors propose four themes for transitioning teachers from PCK to TPACK: developing beliefs about technology use in mathematics, understanding how students engage with mathematics through technology, knowledge of technology-based mathematics curricula and resources, and knowledge of technology-based instructional strategies for mathematics.

They hypothesized that teachers can leverage seven types of knowledge to develop technology-integrated lesson strategies. Besides the three primary knowledge domains of technological, pedagogical, and content knowledge, four additional types of knowledge can emerge from their intersections.

Adapting from Mishra and Koehler's (2006) visualization of TPACK as three intersecting knowledge circles, the constructs describing teachers' TPMK can be represented as follows (see Figure 1). These seven constructs are defined as follows:

1. Mathematical knowledge (MK) – Teachers' mathematics content knowledge.
2. Technological knowledge (TK) – Teachers understand various technologies.
3. Pedagogical knowledge (PK) – Teachers understand instructional processes or methods.
4. Technological mathematical knowledge (TMK) – Teachers' knowledge of technological tools for representing mathematical knowledge or using technology to represent mathematical knowledge. Mishra and Koehler's (2006) framework depicts this construct as technological content knowledge (TCK).

5. Technological pedagogical knowledge (TPK) – Teachers' understanding of using technology to implement various teaching methods.
6. Mathematical knowledge for teaching (MKT) – As defined by Silverman and Thompson (2008), this represents teachers' pedagogical content knowledge (PCK) for teaching mathematics.
7. TPMK – Teachers' understanding of using technology to implement teaching methods for mathematics.

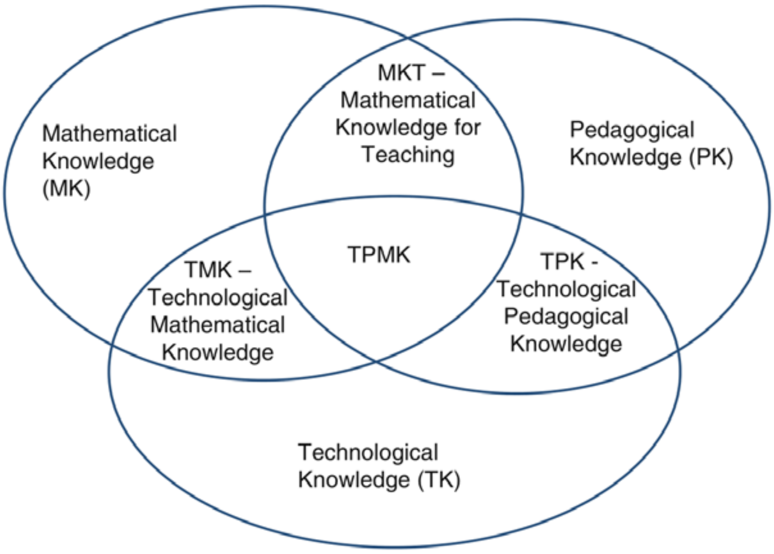

Figure 1. The TPMK framework

This paper presents a tool-kit design process and an example of teaching math in primary school based on the TPMK framework.

2. Methods

This type of research was the development of research, known as research and development (R&D). The development of Geogebra-supported learning tool in this study was developed by adapting Plomp's model development from 1997 (Plomp & Nieveen, 2014). This method is used to produce research products that are developed scientifically. The research stages consist of (1) investigation, (2) designing, (3) realization/construction, (4) testing, evaluation, and revision, (5) implementation. But in this paper, we only focus on the first two stages, which are described:

(1) *Investigation:* In this phase, the theories regarding GeoGebra-based learning as a reference in developing teaching materials and establishment of basic competencies that must be achieved are identified and studied.

(2*) Designing:* After the preliminary investigation phase has been carried out, next is the design phase. In this phase, the principal thing is to design the instrument teaching materials, and also to determine the validity and effectiveness of the teaching materials. The designing is divided into two stages. In the first phase, we developed a teaching toolkit supported by GeoGebra, based on the key concepts from our theoretical framework. This phase is detailed in Section 3.1. In the second phase, we designed a practical task example to illustrate how the model works, which is thoroughly described in Section 3.2.

3. Results & Discussions

3.1 Developing a tool-kit for supporting mathematics teaching with dynamic mathematics software GeoGebra based on the TPMK model

Based on the components of the TPMK model, we propose a process for building an interactive dynamic model to support teachers and students in the teaching and learning of a specific mathematical concept.

Step 1. Content analysis of knowledge to be taught

- Select the mathematical knowledge to be taught using GeoGebra software.

- Study and analyze the mathematical knowledge taught in the curriculum, textbooks, and teacher's guides to accurately understand the content, including its mathematical nature and any pedagogical constraints.
- Analyze the difficulties and common errors students encounter with the mathematical knowledge to be taught.

Step 2. Analyzing and selecting pedagogical ideas for organizing teaching activities

Analyze the current teaching situation, including the difficulties and obstacles teachers and students face in teaching and learning mathematical knowledge based on personal experience or through surveys and classroom observations.

Based on pedagogical knowledge of learning theories such as cognitive theory and constructivism, the teacher analyzes and selects teaching methods and formulates pedagogical ideas to build a teaching sequence using GeoGebra. These pedagogical ideas may also arise from the teacher's needs, such as dynamically simulating static illustrations in textbooks or demonstrating the variation of a quantity or object according to one or more other quantities/objects.

Identify the main output requirements needed for the digital teaching material.

Step 3. Analyzing and selecting GeoGebra functions

Analyze the main output requirements of the digital teaching material and study the functions and potential of the tools provided by GeoGebra. Then, select the appropriate GeoGebra functions to implement the teaching ideas from step 2.

Step 4. Building the interactive dynamic model (tool kit)

Use GeoGebra to develop the digital teaching material.

Step 5. Describing how to use and apply in teaching

Describe the functions, operations, and usage of the product.

Provide suggestions for applying the product in teaching.

Step 6. Operation, testing, and adjustment if necessary

Operate, test, and make adjustments to the product as needed.

3.2 An illustration

To demonstrate how the model could be utilized, we introduce a concrete example. This case focuses on the following lesson “Area of a Trapezoid” which was mentioned in Grade 5th Geometry textbook (Hoan, 2012a, 2012b).

Step 1. Analysis of the mathematical knowledge to be taught (MK)

Objectives: Present the formation of the rule and formula for the area of a trapezoid;

Calculate the area of a trapezoid; Solve simple problems related to the area of a trapezoid.

The teacher's guide directs students to explore the knowledge of the "Area of a Trapezoid" as follows: "Use the prepared trapezoidal cardboard piece → identify the midpoint of one of the legs, draw a line as shown → cut the cardboard into two pieces and reassemble it into a triangle → compare the area of the trapezoid with the area of the assembled triangle → write the calculation for finding the area of the triangle → write the calculation for finding the area of the trapezoid." (Hoan, 2012a)

Step 2. Analysis and selection of pedagogical ideas for organizing teaching activities (PMK)

Pedagogical Idea: Use GeoGebra to design an interactive dynamic product to support the exploration of new knowledge – the rule for calculating the area of a trapezoid from the perspective of constructivist theory.

Main output requirements: The limitation of static images following the procedure needs to be addressed by dynamically simulating the illustrations, allowing learners to interact with them.

Step 3. Analysis and selection of GeoGebra functions

The functional tools are selected and designed according to the following steps:

"Construct the trapezoid and display its area measurement → identify the midpoint of one of the legs, draw a line through this midpoint and a vertex of the trapezoid → simulate cutting the trapezoid into two pieces along this line using a scissors tool → use the rotation tool to simulate cutting and reassembling the pieces into a triangle → display the area measurement of the newly formed triangle."

Step 4. Building the interactive dynamic model

Use GeoGebra to build the digital teaching material:

1. Hide the grid and coordinate axes.

2. Create a segment AB, then use the unique line tool to draw a line d parallel to segment AB. Select two points C and D on the line d.
3. Use the polygon tool to draw trapezoid ABCD.
4. Use the point tool to find the midpoint M of segment BC.
5. Use the polygon tool to draw AMCD and ABM polygons.
6. Create a slider. Use the rotation tool combined with the command Neu() to rotate polygon ABM around point M. The command is as follows:
   Rotato (polygon (A, B, M), (a > 0, 180° + (1 – a) * 180°, 0°), M)
7. Hide the unnecessary names and objects, format the colours and appearance conditions of the remaining objects, and add images and text to make the product more user-friendly and accessible.

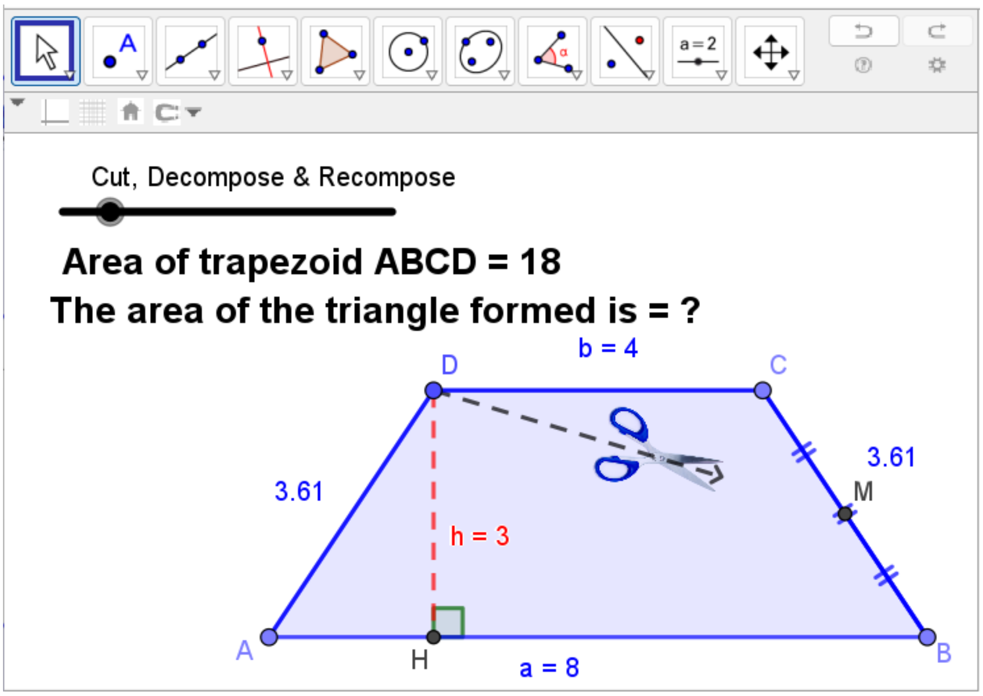

Figure 2. Decompose the trapezoid

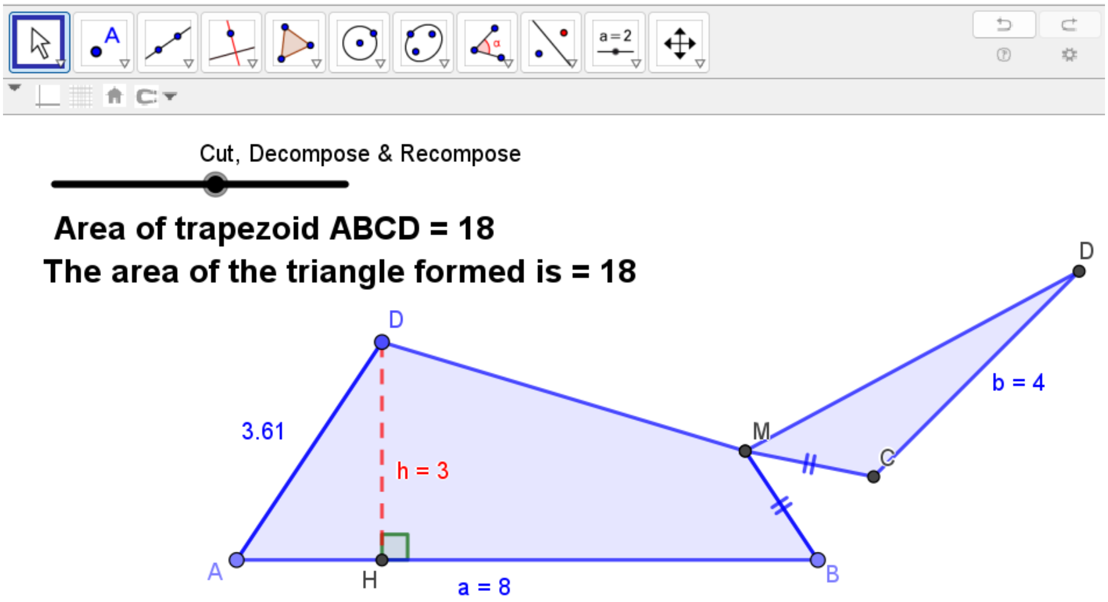

Figure 3. Decompose the trapezoid (cont.)

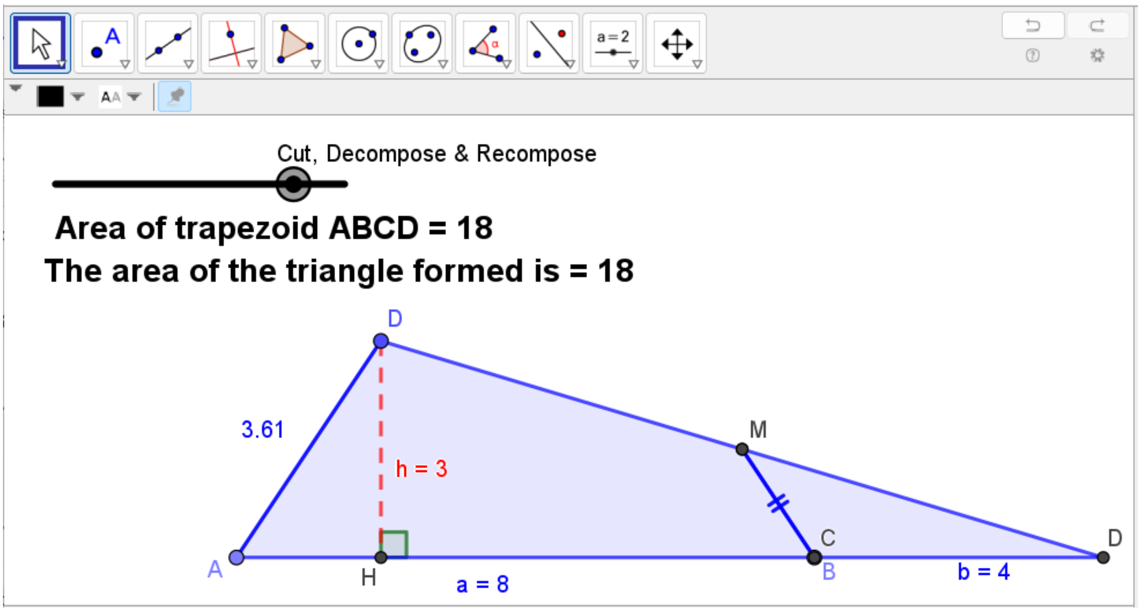

Figure 4. Recompose to create a triangle

Step 5. Describing how to use and apply in teaching

Describe the functions, operations, and usage of the GeoGebra tool-kit (Figure 2-4).

Provide suggestions for applying the product in teaching: The GeoGebra tool-kit not only creates an environment for students to engage in hands-on practice and experiential learning, but it also provides feedback that helps students explore and discover new knowledge. Therefore, teachers can use this tool-kit to facilitate students' exploration of the rule for calculating the area of a trapezoid through active teaching methods such as inquiry-based learning, the 5E instructional model, and Kolb's experiential learning cycle.

4. Conclusion

Dynamic mathematics software GeoGebra offers numerous possibilities to help students develop an intuitive understanding and effectively visualize mathematical processes. Utilizing GeoGebra's tools allows students to explore a broader range of function types and make connections between symbolic, visual, and numeric representations. This software enables students to learn new techniques and model and evaluate challenging, engaging, and real-world situations. Additionally, using GeoGebra within a TPMK framework assists teachers in explaining abstract mathematical concepts, making them more accessible for students to comprehend. Based on research findings, it is recommended that other mathematical topics also incorporate learning media such as GeoGebra to aid teachers in explaining fundamental concepts.

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