UJME 13 (2) 2024: 171-184
UNNES JOURNAL OF MATHEMATICS EDUCATION
https://journal.unnes.ac.id/journals/ujme
ISSN: 2252-6927 (print); 2460-5840 (online)

Is Learning Trajectory Necessary for Mathematics Junior High School Students' Understanding Ability?

Getut Pramesti[a,*], Mahanani Surjatiningsih[b], Bela Talita Yuli Nastiti[a]

[a] *Mathematics Education Department, Universitas Sebelas Maret, Jl. Ir Sutami 36A, Surakarta and 57126, Indonesia*
[b] *SMP Negeri 10, Jl. Kartini 12, Surakarta and 57131, Indonesia*

* *E-mail address: getutpramesti@staff.uns.ac.id*

ARTICLE INFO

Article history:
Received 10 August 2024
Received in revised form 22 August 2024
Accepted 23 August 2024

Keywords:
Learning Trajectory; Concept; Understanding; Mathematics

Abstract

A learning trajectory (LT) is needed to help teachers prepare materials, methods, and strategies to achieve learning objectives, activities, and student expectations. Hence, the purpose of this study is to determine the necessity of the LT by providing evidence of the significance of learning trajectory in improving mathematics junior high school students' conceptual understanding abilities. This quantitative research used descriptive analysis and inference by using univariate and multivariate statistical analysis. The population in this study were eighth-grade students of Public Junior high school in Surakarta Indonesia. We chose 32 students as a sample subjected to the LT class, whereas 31 were subjected to the non-LT class. We analyze the mathematics concept student's understanding ability and its components. With a confidence level of 95%, the results showed that the mathematics concept of junior high school student's understanding ability in LT classes was better than students' understanding of concepts in non-LT classes. The unique findings of this research are that students' understanding abilities in restating a concept and classifying objects according to certain properties of the concept are the same for both LT and non-LT classes. In non-LT classes, students' ability to develop necessary or sufficient conditions for a concept is better than in LT classes. Meanwhile, the understanding abilities of LT classes that are better than those in non-LT classes include giving examples and non-examples of a concept, presenting concepts in the form of mathematical representations, using and utilizing and selecting certain operating procedures, and applying concepts or algorithms to solutions.

1. Introduction

Learning difficulties experienced by students require the important role of a teacher to be responsible for overcoming these problems (Munirah, 2018). According to Isnawan and Wicaksono (2018), learning design is a design to meet learning objectives that have been set by considering students' needs. Assessment design, teaching materials, strategies, and learning objectives are all included in the learning design. Apart from that, Surya (2018) states that teachers' learning designs must consider students' needs when using student-centered learning methods, therefore teachers must consider students' learning trajectories (LT) when creating learning designs. Learning trajectories are believed to have the potential as a tool for reform by supporting "more focused standards, better-designed curricula, better assessments, and ultimately more effective instruction and improved student learning" (Corcoran, Mosher, & Rogat, 2009). Also, a mathematics learning trajectory supports teachers in creating models of students' thinking and in restructuring teachers' understandings of mathematics and students' reasoning.

Learning designs based on students' learning trajectories and the selection of the conceptual understanding procedures learning model can be a solution to the learning difficulties of junior high school students who have low conceptual understanding abilities. To find out the essence of LT in learning, we

To cite this article:
Pramesti, G., Surjatiningsih, M., & Nastiti, B. T. Y. (2024). Is Learning Trajectory Necessary for Mathematics Junior High School Students' Understanding Ability? *Unnes Journal of Mathematics Education*, *13*(2), 171-184. https://doi.org/10.15294/ujme.v13i2.11587

compared the significance of students' conceptual understanding abilities in LT and non-LT classes. Next, to find out which parts of students' conceptual understanding abilities in the LT class have improved, we analyzed the components of conceptual understanding abilities which consist of 7 components of the indicators. From this analysis, it will be known in what ways and to what extent teachers use a learning trajectory to better understand student mathematical concepts.

Along with the development of the need for learning trajectories, research on learning trajectories has been conducted. Confrey et al. (2020) uses measures of learning trajectories to support learner-centered instruction, Weber and Lockwood consider how Simon's original conceptualization of a hypothetical learning trajectory might be extended by attending to ways of thinking and ways of understanding. Wilson et al (2013) report findings from two studies examining teachers' uses of a learning trajectory. Learning trajectory is a path that describes the prerequisite knowledge that students have and each step from one point to the next, as well as the methods and thinking processes or levels of student thinking in learning (Simon in Mutaqin, 2017). Teachers should use actual learning trajectories, also known as real learning trajectories to connect student-centered learning with students' understanding and critical thinking (Fauziyah, 2023). The three main components of HLT are learning objectives, learning activities, and learning process assumptions that are predictive of students' thought processes (Hendrik et al., 2020). Hence, in this study, we will provide the essence of the learning trajectory for junior high school conceptual understanding ability students'.

Based on the background of the study, we state the purposes of the study as follows:

- Analyze the effect of the LT and non-LT on junior high school students' conceptual understanding development
- Analyze the effect of each component of students' conceptual understanding development in LT and non-LT class

Thus, for the above purposes of the study, we will provide evidence of the analysis of both. The following section contains of methods, results, and conclusion of the study.

2. Methods

This research was quantitative in the form of descriptive and inferential analysis of the conceptual understanding ability of junior high school students. The subjects were the students of Public Junior High School 10 Surakarta, namely grade VIII students in the 2023/2024 academic year from November 2023 to May 2024. Descriptive analysis includes visualization and numerical analysis of research variables, while inferential analysis includes initial inferences in the form of univariate statistical analysis of conceptual understanding ability using t-test statistics. Further inferential analysis uses multivariate statistics on the components of conceptual understanding ability. The research variables include independent variables in the form of classes, namely LT and non-LT classes, with a nominal scale. While the dependent variable was students' conceptual understanding ability denoted by y, and 7 components of conceptual understanding ability, expressed by y_1-y_7. In the LT Class, the Conceptual Understanding Procedures (CUPs) learning model was used to prepare devices and develop learning trajectories (LT) on the Pythagorean Theorem material. While the non-LT class used a conventional learning model.

The LT design in the CUPs learning model was developed by Bela (2024). A Learning trajectory is a series of activities that students go through to understand a concept to achieve predetermined learning objectives. The learning trajectory obtained from the HLT design has been tested. In this study, Hypothetical Learning Trajectory (HLT) was developed in the CUPs model of the Pythagorean Theorem material at Public Junior High School 10 Surakarta. HLT has three components consisting of learning objectives, learning activities, and alleged learning processes. In this study, we conducted a significance test on students' conceptual understanding abilities both as a whole and in each of its components. The LT developed by Bela (2024) can be seen in Figure 1.

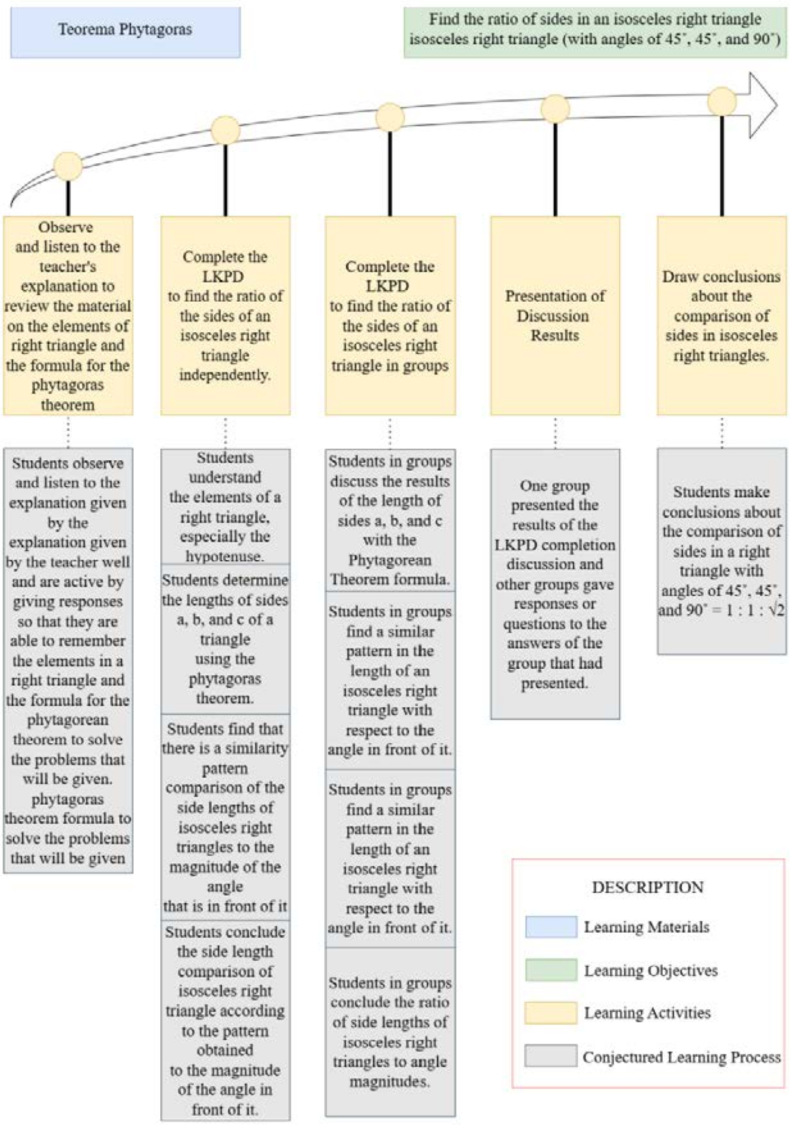

Figure 1. Learning Trajectory on the Pythagorean Theorem material.

The learning trajectory in the Conceptual Understanding Procedures (CUPs) learning model on the Pythagorean Theorem material consists of learning objectives formulated to find the ratio of sides in an isosceles right triangle (with angles 45˚, 45˚, 90˚). Learning activities are formulated into five learning activities, namely (1) observing and listening to the teacher's explanation reviewing the material of the elements in a right triangle and the Pythagorean Theorem formula, (2) completing student worksheet (LKPD) to find the ratio of sides in an isosceles right triangle independently (3) completing the LKPD to find the ratio of sides in an isosceles right triangle in groups (4) presenting the results of the discussion (5) making conclusions about the ratio of sides in an isosceles right triangle.

2.1 Participants

The population of the study is the VIII-grade students in Public Junior High School 10, Surakarta Indonesia. We did random cluster sampling with 31 students in the LT class and 32 in the non-LT class.

2.2 Data Collection and Analysis

Conceptual understanding ability is measured by a test with an interval data scale. In the initial inference analysis, the influence of LT on the overall concept understanding ability will be analyzed. In this initial inference analysis, the concept of understanding ability is denoted by y. The measurements were done in both LT and non-LT classes. The data design in the initial inference analysis can be seen in Table 1.

Table 1. The data design for y

LT	**Non-LT**
$\mathbf{y_1}$	$\mathbf{y_2}$
y_{11}	y_{21}
y_{12}	y_{22}
⋮	⋮
y_{1n}	y_{2n}

The first inference analysis was continued with an analysis of the influence of 7 components of conceptual understanding using multivariate multi-group statistical analysis. Adapting the conceptual understanding indicators from Kartika (2008) and Kharis et al. (2021), this study uses the following conceptual understanding indicators.

1) Restating a concept,
2) Classifying objects according to certain properties according to the concept,
3) Giving examples and non-examples of a concept,
4) Presenting a concept in the form of mathematical representation,
5) Developing necessary or sufficient conditions of a concept,
6) Using and utilizing and selecting certain procedures or operations,
7) Applying concepts or algorithms to problem-solving,

with each indicator 1-7 denoted by the symbol y_1-y_7. If in the initial inference analysis, y is measured from the total y_1 to y_7, then in the second inference analysis, the significance is analyzed per component y_i, i = 1,…7. The purpose of this further analysis is to determine the effect of LT on each component of the concept understanding ability of indicators y_1-y_7. The data design is as in Table 2.

Table 2. The data design for y_1-y_7

LT			**Non-LT**		
$\mathbf{y_1}$	**…**	$\mathbf{y_7}$	$\mathbf{y_1}$	**…**	$\mathbf{y_7}$
y_{11}	…	y_{71}	y_{11}	…	y_{71}
y_{12}	…	y_{71}	y_{12}	…	y_{72}
⋮	⋮	⋮	⋮	⋮	⋮
y_{1n}	…	y_{7n}	y_{1n}	…	y_{7n}

The data design in Table 1 and Table 2 can be analyzed by parametric inferential if the data follows the parametric assumptions. If the assumptions are not satisfied, then the analysis is done using non-parametric statistical analysis. Further, all data in this study were processed using IBM SPSS Statistics 26 software.

3. Results & Discussions

The LT class in CUPs learning was assigned to 31 students and the non-LT class was imposed on 32 students. This discussion begins with a descriptive analysis of y and y_1-y_7, followed by an initial inferential analysis (y) and continuation (y_1-y_7).

3.1. Descriptive analysis of students' conceptual understanding ability (y) in the LT class

The mean of conceptual understanding ability of students in LT and non-LT classes can be seen in Table 3. It appears that the conceptual understanding ability of students in LT classes is higher than in non-LT, with the dispersion of LT being smaller than the non-LT. However, to determine the significance of students' conceptual understanding ability in both LT and non-LT classes, an inference analysis needs to be conducted in sub-section 3.2.

Table 3. Estimates of y on LT and Non-LT

	LT	**Non-LT**
Mean	80.313	62.484
Std. Error	1.184	1.203

Visually, students' conceptual understanding ability (y) can be depicted in Figure 2. From Figure 2, it can be seen that the boxplot of LT class is higher with one observation identified as an outlier data with ID 30. ID 30 is detected as a student with the lowest conceptual understanding ability in the LT class. However, students with ID 30 are still at the average conceptual understanding ability of non-LT class students. This finding indicates that the use of LT in learning has a relatively different impact from non-LT classes. To determine the significance of this impact, an initial inferential analysis will be carried out on y, namely in section 3.2.

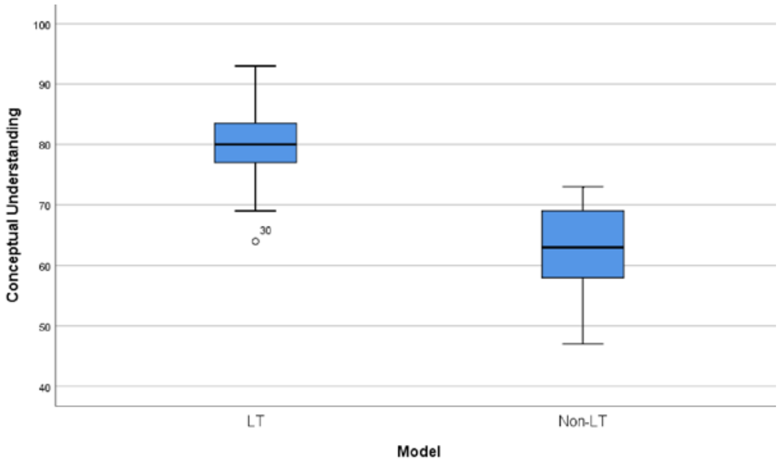

Figure 2. Plot of y on LT and non-LT.

3.2. Inferential analysis of students' conceptual understanding abilities (y) in LT and non-LT classes

Before conducting an inferential analysis of y using parametric statistical analysis, it is necessary to conduct prerequisite tests in the form of a homogeneity of variance test and a normality test.

Table 4. Levene's test of equality of error variances of y

	Value
Levene Statistics	3.679
Sig.	0.060

Based on Table 4, because of Sig. = 0.060 > 0.05 then we can conclude that the equality of error variance can be assumed satisfied.

Table 5. Tests of Normality of y. The significance values are in the parenthesis

	Kolmogorov- Smirnov	**Shapiro-Wilk**
y		
LT	1.114 (0.200)	0.970 (0.500)
Non-LT	0.136 (0.155)	0.946 (0.120)

From Table 5, because all values, both from the Kolmogorov-Smirnov and Shapiro-Wilk test statistics > 0.05, then the assumption of data normality in the LT and non-LT classes is said to be fulfilled. From the results of the analysis of Table 4 and Table 5, the parametric assumptions are met, then the main analysis can be carried out using parametric statistical analysis.

Table 6. Tests of significance of y

	F	Sig.
LT & Non-LT	111.645	0.000

From Table 6, because Sig. = 0.000 < 0.05, it can be said that students' conceptual understanding ability is significantly different in LT and non-LT classes. Concerning the analysis of Table 3 and Figure 2, the average conceptual understanding ability of students in LT classes is better than in non-LT classes. Although in LT class there are 30 ID students with very low conceptual understanding ability, these students' conceptual understanding ability is still better than in non-LT classes. To find out which components of conceptual understanding ability are significant in LT and non-LT classes, further analysis is carried out in subsections 3.3 and 3.4.

3.3. Descriptive analysis of students' conceptual understanding abilities (y_1-y_7) in LT and non-LT classes

A descriptive analysis of the mean of test scores per component of conceptual understanding ability in LT and non-LT classes appears in Table 7.

Table 7. Estimated marginal means of y_1-y_7 and its standard error on LT and non-LT

Variable response		Mean	Std. Error
y_1		7.59	0.39
	LT	8.28	0.38
	Non-LT	6.90	0.69
y_2		10.00	0.00
	LT	10.00	0.00
	Non-LT	10.00	0.00
y_3		18.33	0.24
	LT	18.31	0.47
	Non-LT	8.35	0.09
y_4		10.85	0.28
	LT	12.06	0.39
	Non-LT	9.64	0.40
y_5		10.33	0.22
	LT	9.78	0.29
	Non-LT	10.87	0.32
y_6		9.95	0.29
	LT	11.44	0.36
	Non-LT	8.45	0.46
y_7		9.49	0.36
	LT	10.44	0.57
	Non-LT	8.55	0.42

From Table 7, it appears that the mean of y_2 is 10, both in the LT and non-LT classes. Therefore, this produces a standard deviation of 0, indicating that the average ability of students' conceptual understanding in the component of classifying objects according to certain properties according to the concept; is the same in both classes. In other words, in this ability, learning trajectory has no significant effect. Overall, apart from y_2, the mean of each component of students' conceptual understanding ability in the LT class tends to be higher than in the non-LT class. To find out the significance of this difference, we need to conduct an inference analysis of $y = (y_1, y_3, ..., y_7)$, by first conducting a prerequisite test, namely the normality test and the homogeneity of variance test. Without y_2, the pre-analysis test of homogeneity of variance is obtained, namely as in Table 8 and Table 9.

Table 8. Box’s test of equality of covariance matrices y_1-y_7

	Value
Box’s M	114.793
F	4.888
df1	21
df2	13655.167
Sig.	0.000

Based on Table 8, Sig. = 0.000, and because Sig < α = 0.05 then the assumption of the equality of variance-covariance using Box's test is violated. In other words, the assumption of homogeneity of variance-covariance is not satisfied in this analysis, and results in us not being able to analyze the data using parametric statistical analysis. If using the homogeneity of variance test with Levene based on the mean, the results will be obtained as in Table 9.

Table 9. Levene’s test of equality of covariance matrices y_1-y_7, excluding y_2.

	Levene	**Sig.**
y_1	18.332	0.000
y_3	49.013	0.000
y_4	0.184	0.669
y_5	0.082	0.775
y_6	0.414	0.522
y_7	2.689	0.106

In the univariate test, using the Levene test, it was deduced that:

- The assumption of homogeneity of variance was not satisfied in the response variables y_1 and y_3
- The assumption of homogeneity of variance was satisfied in the response variables y_4, y_5, y_6, and y_7.

For the prerequisite test in the form of a normality test, Table 10 was obtained. The significance values are in the parenthesis

Table 10. Tests of Normality of y_1, y_3-y_7

	Kolmogorov- Smirnov	**Shapiro-Wilk**
y_1	0.348 (0.000)	0.723 (0.000)
y_3	0.287 (0.000)	0.747 (0.000)
y_4	0.184 (0.000)	0.892 (0.000)
y_5	0.253 (0.000)	0.991 (0.000)
y_6	0.155 (0.000)	0.943 (0.005)
y_7	0.165 (0.000)	0.895 (0.000)

Based on Table 10, using both Kolmogorov-Smirnov and Shapiro-Wilk test statistics, because the significance values < 0.05 then all y_i variables do not meet the normality assumption. Thus, it can be analyzed from Table 8-10, that the data cannot be continued with parametric statistical analysis. Further, we do inference analysis using non-parametric statistics. In the analysis using non-parametric statistics, the center measure of the data is not based on the mean but is based on rank.

Table 11. The mean rank of y_1-y_7

	LT	Non-LT
y_1	34.05	29.89
y_2	32.00	32.00
y_3	47.50	16.00
y_4	40.70	23.02
y_5	25.86	38.34
y_6	42.06	21.61
y_7	40.25	23.48

Similar to the results of the descriptive analysis of Table 3, in Table 7, the rank mean of y_2 is the same in both LT and non-LT classes. This means that the ability of y_2 in both classes is not numerically different. The highest rank is in y_3 in the LT class, conversely, the lowest average rank is in y_3 in the non-LT class. To determine the significance of the difference in rank y_1-y_7, further inferential analysis is carried out on y_1-y_7 using non-parametric statistical tests, namely in sub-section 3.4.4.

3.4. Inferential analysis of students' conceptual understanding ability (y_1-y_7) in LT and non-LT classes

3.4.1. Concept understanding of component y_1

By using the Jonckheere-Terpstra test statistics, from Table 12, Asymptotic Sig. = 0.31 > 0.05 then it can be concluded that the concept understanding of component y_1 does not significantly affect both classes. This means that the understanding of the concept of the component of students' ability to restate a concept and students' ability to classify objects according to certain properties according to the concept in the LT and non-LT classes is equally good.

Table 12. Independent-samples Jonckheere-Terpstra test for variable response y_1

	Value
Total N	63
Test Statistics	430.50
Standard Error	64.86
Asymptotic Sig. (2-sided test)	0.313

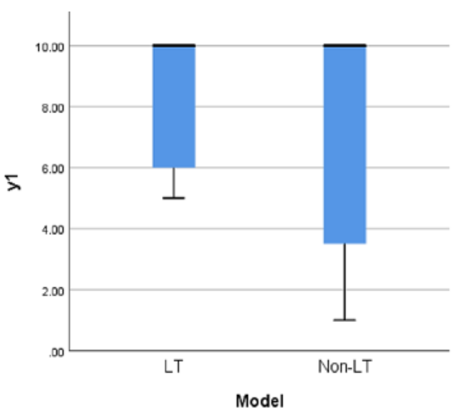

Figure 3. y_1 on LT and non-LT

Visually, the ability to understand the concept of component y_1 can be depicted in Figure 3. In Figure 3, it appears that the rank of y_1 in the LT class is indeed higher than in the non-LT class, but from the results of Table 11 and Table 12, this rank difference is not significant. This means that in the LT class, y_1's ability is the same as in the non-LT class.

3.4.2. Concept understanding of component y_2

From Table 13, since Asymptotic Sig. = 1.00 > 0.05, then it can be said that the understanding of the concept of the y_2 component does not significantly affect both classes. This means that the understanding of the concept of the component of students' ability to classify objects according to certain properties according to the concept in the LT and non-LT classes is equally good.

Table 13. Independent-samples Jonckheere-Terpstra test for variable response y_2.

	Value
Total N	63
Test Statistics	496.00
Standard Error	00.00
Asymptotic Sig. (2-sided test)	1.00

Visually, y_2 in the LT and non-LT classes can be seen in Figure 4.

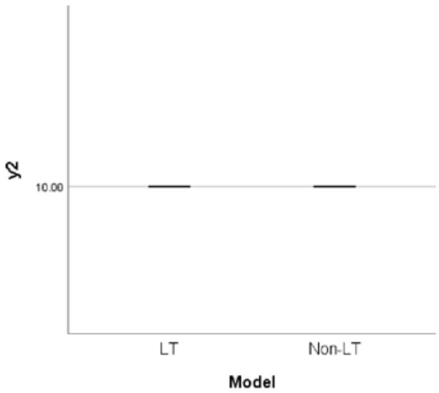

Figure 4. y_2 on LT and non-LT.

From Figure 4, it appears that the rank of y_2 in the LT and non-LT classes are the same. Concerning Table 11 and Table 13, it is clear that y_2 in the LT and non-LT classes is not significant. This means that with or without LT, students' conceptual understanding ability in the component y_2 is equally good.

3.4.3. Concept understanding of component y_3

From Table 14, Sig. = 0.000 > 0.05, it can be concluded that the ability to give examples and non-examples of a concept (y_3) in the two classes is significantly different.

Table 14. Independent-samples Jonckheere-Terpstra test for variable response y_3.

	Value
Total N	63
Test Statistics	0.00
Standard Error	69.88
Asymptotic Sig. (2-sided test)	0.00

It can be seen in Figure 5 that the boxplot shape of y_3 in both classes is quite different, both in terms of the average rank and its distribution. The ability of y_3 in the LT class appears to have quite high dispersion. This indicates that the ability of y_3 in the LT class has a high dispersity compared to the non-LT class. When associated with Table 11 and Table 14, the difference in the ranking of y_3 in the LT and non-LT classes is significantly different. In other words, y_3 in the LT class is better than y_3 in the non-LT class.

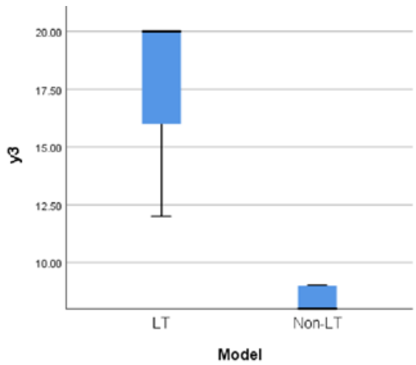

Figure 5. y_3 on LT and non-LT

3.4.4. Concept understanding of component y_4

From Table 15, Sig. = 0.000 < 0.05, it can be concluded that the ability to present concepts in the form of mathematical representation in the two classes is significantly different, and regarded Table 11, it can be analyzed that this ability in the LT class is better than in the non-LT class.

Table 15. Independent-samples Jonckheere-Terpstra test for variable response y_4

	Value
Total N	63
Test Statistics	217.50
Standard Error	71.40
Asymptotic Sig. (2-sided test)	0.00

In Figure 6, it appears that the shape of the boxplot of y_4 in the two classes is quite different. In the non-LT class, there are students with ID 47 and 45 who have very low y_4 abilities, far away from the mean of y_4 ability in the class

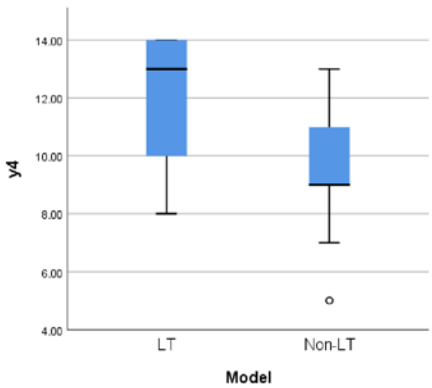

Figure 6. y_4 on LT and non-LT

3.4.5. Concept understanding of component y_5

From Table 16, Sig. = 0.01 < 0.05, it can be concluded that the ability to understand concepts in developing necessary or sufficient conditions of a concept in both classes is significantly different. When associated with Table 11, it can be analyzed that this ability in the LT class is not better than the conventional one.

Table 16. Independent-samples Jonckheere-Terpstra test for variable response y_5

	Value
Total N	63
Test Statistics	692.50
Standard Error	70.85
Asymptotic Sig. (2-sided test)	0.01

Based on Figure 7, the y_5 of the non-LT class is higher than the LT. Concerning Table 16, this indicates that the concept understanding of component y_5 is better than in LT.

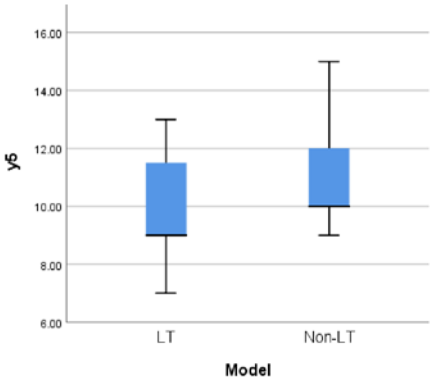

Figure 7. y_5 on LT and non-LT

3.4.6. Concept understanding of component y_6

From Table 17, Sig. = 0.000 < 0.05, it can be concluded that the ability to use and utilize and choose certain procedures or operations in both classes is significantly different. When associated with Table 11, it can be analyzed that this ability in the LT class is better than non-LT.

Table 17. Independent-samples Jonckheere-Terpstra test for variable response y_6

	Value
Total N	63
Test Statistics	174.00
Standard Error	71.94
Asymptotic Sig. (2-sided test)	0.00

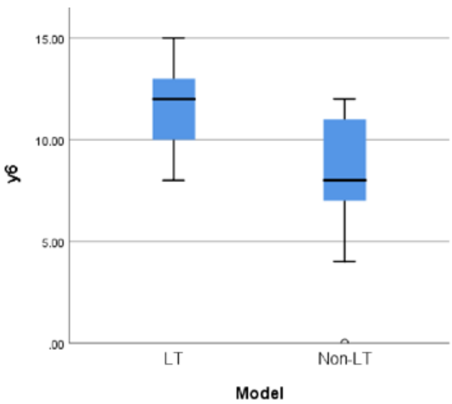

Figure 8. y_6 on LT and non-LT

From Figure 8, a descriptive analysis is obtained that there are students with ID 58 who have y_6 abilities far away below the mean of y_6 in the non-LT class. From the non-LT boxplot, it also appears that the y_6 ability in the non-LT class has a high dispersion compared to the LT class. Regarded to Table 17, the y_6 ability in the LT class is better than the non-LT class. Thus, it indicates that the y_6 ability is greatly assisted by the presence of LT in learning.

3.4.7. Concept understanding of component y_6

From Table 18, Sig. = 0.000 < 0.05, it can be concluded that the ability to apply concepts or algorithms to problem-solving in both classes is significantly different. When associated with Table 11, it can be analyzed that this ability in the LT class is better than the non-LT class.

Table 18. Independent-samples Jonckheere-Terpstra test for variable response y_7

	Value
Total N	63
Test Statistics	232.00
Standard Error	71.84
Asymptotic Sig. (2-sided test)	0.00

From Figure 9, it appears that the y_7 ability in both classes has high dispersion. Students with ID 30 and 15 in the LT class have y_7 abilities far from the mean of y_7, while students with ID 43 are far above and students with ID 38 are far below the average of the non-LT class. Associated with the analysis of Table 18, the y_7 ability of the LT class is better than that of the non-LT class. Furthermore, the y_7 ability tends to need attention from teachers, although the use of LT improves this ability, there exist students with abilities below the mean of y_7 of class.

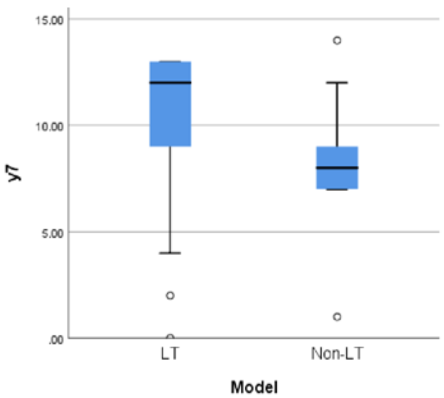

Figure 9. y_7 on LT and non-LT

Overall, the ability to understand concepts in each component can be summarized in Table 19, with an explanation as follows:

- The sign = shows the component of y is the same in the LT and non-LT classes;
- The sign * shows the component of y which should be an attention in the learning process. This claim is based on the outlier observation that came out in the data.
- The sign shows the component of y in the LT is better than in the non-LT.

Table 19. The summary of y_1- y_7

	y_1	y_2	y_3	y_4	y_5	y_6	y_7
LT	=	=	√	√*		√*	√*
Non-LT					√		

Based on the discussion on sub-section 3.2, sub-section 3.4, and Table 19, we can deduce that

- The students' conceptual understanding abilities in the LT is better than in the non-LT class,
- The components of students' conceptual understanding abilities in the LT class that are better than in the non-LT, namely giving examples and non-examples of a concept, presenting a concept in the form of mathematical representation, using and utilizing and selecting certain procedures or operations, applying concepts or algorithms to problem-solving.

4. Conclusion

Based on the discussion, in the view of quantitative research, the learning trajectory is necessary for junior high school students' conceptual understanding abilities. The students' conceptual understanding abilities in the LT classes are better than non-LT classes. Moreover, the component of the student's conceptual understanding abilities can be improved by the LT, namely the ability on giving examples and non-examples of a concept; presenting a concept in the form of mathematical representation; using and utilizing and selecting certain procedures or operations; applying concepts or algorithms to problem-solving. Excluding a component of restating a concept; giving examples and non-examples of a concept; and developing necessary or sufficient conditions of a concept, a teacher can focus on other components of conceptual understanding ability. Hence, this study provides evidence of the essential learning trajectory for junior high school in improving the students' conceptual understanding abilities.

The learning trajectory effectively can improve the students' conceptual understanding abilities. The learning trajectory is important to be applied in junior high school students' mathematics learning, especially to improve students' conceptual understanding ability. The practical implication of the study is the learning trajectories can be applied in the mathematics junior high school students in the point of view to improve the conceptual understanding abilities in the term of giving examples and non-examples of a concept; presenting a concept in the form of mathematical representation; using and utilizing and selecting certain procedures or operations; applying concepts or algorithms to problem-solving.

This study is limited to the use of LT in the Conceptual Understanding Procedures learning. In the future, researchers can conduct research on the use of learning trajectories in models, methods or learning approaches other than CUPs, both in conceptual understanding ability and in other measurements.

Acknowledgment

The authors thank the reviewers and the associated editor for their valuable comments. The author would also like to extend thanks to Universitas Sebelas Maret, LPPM UNS, Surakarta Indonesia for the research funding in the scheme HGR-UNS 14 A with Grant Number 194.2/UN27.22/PT.01.03/2024

References

Bela Talita Yuli Nastiti. (2024). Pengembangan learning trajectory pada model pembelajaran conceptual understanding procedures (CUPS) materi Teorema Phytagoras di SMP Negeri 20 Surakarta Tahun Ajaran 2023/ 2024. (Undergradute-thesis). *Unpublished*. Universitas Sebelas Maret. Surakarta, Indonesia.

Confrey, J., Toutkoushian, E., Shah, M. (2020). Working at scale to initiate ongoing validation of learning trajectory-based classroom assessments for middle-grade mathematics. *The Journal of Mathematical Behavior, 60*: 100818. https://doi.org/10.1016/j.jmathb.2020.100818.

Corcoran, T., Mosher, F. A., & Rogat, A. (2009). Learning Progressions in Science: An Evidence-Based Approach to Reform. (CPRE Research Report). Columbia University. https://doi.org/10.12698/cpre.2009.rr63.

Fauziyah, N. (2023). Learning trajectory in problem-based mathematics learning with literacy and numeracy reinforcement: an implementation of lesson study at junior high school. *JTAM (Jurnal Teori dan Aplikasi Matematika), 7*(2): 384–397. https://doi.org/10.31764/jtam.v7i2.12883.

Hendrik, A. I., Ekowati, C. K., & Samo, D. D. (2020). Kajian hypothetical learning trajectories dalam pembelajaran matematika di tingkat SMP. *Jurnal Matematika dan Pendidikan Matematika*, *1*(1): 1–11. https://ejurnal.undana.ac.id/fraktal.

Isnawan, M. G., & Wicaksono, A. B. (2018). Model desain pembelajaran matematika. *Indonesian Journal of Mathematics Education*, *1*(1), 47–52. jurnal.untidar.ac.id/index.php/ijome.

Kartika, Y. (2018). Analisis kemampuan pemahaman konsep matematis peserta didik kelas VII SMP pada materi bentuk aljabar. *Jurnal Pendidikan Tambusai*, *2*(4): 777–785. https://doi.org/https://doi.org/10.31004/jptam.v2i4.25.

Kharis, S. A. A., Salsabila, E., & Haeruman, L. D. (2021). Effect of mathematical concept understanding and mathematical reasoning on mathematical literacy Abilities. *Journal of Physics: Conference Series*, *1747*(1), 1–8. https://doi.org/10.1088/1742-6596/1747/1/012042.

Munirah. (2018). Peranan Guru dalam Mengatasi Kesulitan Belajar Siswa. *Tarbawi Jurnal Pendidikan Agama Islam*, *3*(2), 111–127. https://doi.org/10.26618/jtw.v3i02.1597.

Mutaqin, E. J. (2017). Analisis learning trajectory matematis dalam konsep perkalian bilangan cacah di kelas rendah sekolah dasar. *Dwijacendekia Jurnal Riset Pedagogik*, *1*(1), 19–33. https://jurnal.uns.ac.id/jdc.

Surya, A. (2018). Learning trajectory pada pembelajaran matematika sekolah dasar (SD). *Jurnal Pendidikan Ilmiah*, *4*(2), 22–26. https://jurnal.fkip.uns.ac.id/index.php/jpi/article/view/11692.

Wilson, P., H., Mojica, G., F., Confrey, J. (2013). Learning trajectories in teacher education: Supporting teachers' understandings of students' mathematical thinking. *The Journal of Mathematical Behavior*, *32*(2):103-121.