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Solving Bernoulli's Equations Using Python: Enhancing Student Understanding Through Inquiry-Based Learning

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Abstract

This study explores using Python to solve Bernoulli's equations with the goal of enhancing student understanding. Inquiry-Based Learning (IBL) is integrated into it. The research included second-year students majoring in mathematics from Srinakharinwirot University. The teaching approach began with an introduction to Bernoulli's equations in theory. These equations were then converted into linear form and solved using Python programming. Among the practical tasks in which students took part were post-test assessments, group problem-solving, and pre-test evaluations. Group members used Bernoulli's equations to real-world scientific problems including fluid mechanics and population dynamics using Python to generate solutions and present discoveries. Inquiry-based learning (IBL) principles were used in the study, where students posed questions, looked into problems, and assessed the solutions using Python. The results demonstrated a greater understanding of mathematical concepts as well as computational techniques. The decrease in standard deviation between the pre- and post-test data showed how well IBL and Python combine to foster critical thinking and practical problem-solving skills. This method has the potential to assist student teachers acquire computational abilities and a deeper understanding of mathematics, which will better prepare them for careers as teachers.

1. Introduction

Bernoulli's equation, a well-known first-order nonlinear differential equation, is extensively applied across numerous scientific disciplines, including fluid mechanics, population dynamics, and chemical processes (Boyce et al., 2017). The equation is expressed as

$$y' + P(x)y = Q(x)y^n$$

where P(x) and Q(x) are continuous functions, and n is a constant. Its nonlinearity, particularly when $n \neq 1$, makes it difficult to find a universally applicable analytical solution. Translating the problem into a linear form and using first-order linear differential equation methods can explain it. The transformation process is crucial for math students, in particular those who desire to teach, because it prepares them for theoretical understanding and practical application. Understanding Bernoulli's equation, including its translation and application, is essential to solving real-world scientific problems. Recently, classrooms have prioritized computational tools. Python has helped solve tough mathematical issues like Bernoulli's equation, say Dangeti et al. (2017). Python's user-friendly interface lets students explore differential equations' theoretical features, view solutions, and test their behaviour. It has robust numerical, symbolic, and data visualization modules like Numpy, Sympy, and Matplotlib (Harris et al., 2020; Fuhrer, 2021). These computational skills help students understand and solve nonlinear problems like Bernoulli's equation by connecting theory and practice. In inquiry-based learning, students cooperate to create questions and perform in-depth investigation. A rise in popularity has been observed (Archer-Kuhn & MacKinnon, 2020; Juan Rubio & GarcÃa Conesa, 2022). IBL encourages conversation outside of class. IBL enables students to develop

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critical thinking by exploring real-world scientific issues, building mathematical models, and solving problems in Python. Student math and problem-solving skills increase with this strategy. This project teaches IBL Bernoulli equation with Python. A mixed strategy improves students' Python and problem-solving skills. The study involved 32 Srinakharinwirot University second-year math students. They examined IBL and taught Bernoulli's equation and Python implementation to increase critical thinking and problem-solving. Group exercises, reflective conversations, and pre- and post-tests tested Python and IBL. Mathematics education and prospective teachers were evaluated.

To grasp and apply theoretical principles to real-world issues, science and math students must convert. Bernoulli's equation has many effects. Fluid dynamics pressure, velocity, and height can be predicted, as well as population growth rates. Despite its versatility, Bernoulli's equation's nonlinearity challenges students. Solving and understanding this problem typically involves computation. Math teachers use Python to solve tough differential equations. Python is commonly used in mathematics education because to its libraries, such as Matplotlib, Numpy, and Sympy (Harris et al., 2020; Muqri, 2021; Guniš, 2020). Bernoulli's equation and Python's problem-solving can be examined by students. Research shows that utilizing Python in mathematics instruction improves students' problem-solving and understanding of complex concepts (Ng & Cui, 2021). Python's adaptability enables students solve differential equations numerically and algebraically. Students can use Python to linearize and solve nonlinear problems with integrating factors. Students can see solution behavior and how Bernoulli's equation describes real-world events with Python's visuals. Students understand complex equation relationships by plotting solution behaviors over parameter ranges. Knowledge of concepts increase critical thinking and arithmetic comprehension over tedious computations. IBL is a significant active learning approach for mathematics (Sen & Güler, 2021). IBL encourages students to analyze, question, and solve mathematical issues together. Research shows IBL promotes critical thinking, engagement, and recall (Khalaf & Mohammed Zin; Laursen, 2014; Gholam, 2019). Python's IBL with Bernoulli's equation allows students explore real-world events and analyze. Students can start IBL by applying Bernoulli's equation to fluid dynamics and population growth. Students solve mathematically modeled problems with Python, studying math and problem-solving (Khalaf & Mohammed Zin, 2018; Husni, 2020); Wale & Bishaw, 2020). Students discuss, assess, and share ideas in groups to fully understand the equation and its applications in IBL. An IBL architecture with Python gives several educational benefits. Python simplifies Bernoulli's equation calculations and lets students assess results. Students evaluate computational mechanics and conceptual analysis solution outputs to enhance critical thinking and problem-solving. Python's graphical output features assist students grasp Bernoulli's equation's complex variable interactions. Another method Python increases IBL student autonomy. Students examines scientific problems, build mathematical models, and use Python for exploratory problem-solving. Collaboration improves learning as students create, examine, and critique solutions. Bernoulli's equation, critical thinking, communication, and collaboration are presented here. It is the goal of this study to evaluate the impact that teaching Bernoulli's equation through inquiry-based learning (IBL) combined with Python programming has on the problem-solving abilities of students, as well as to evaluate the improvement in students' conceptual understanding of mathematics that takes place as a result of integrating Python with IBL in the method for learning Bernoulli's equation.

2. Methods

The objective of this study was to explore the impact that teaching Bernoulli's equation using IBL based on Python had on the students' capacity to provide solutions and their conceptual understanding of mathematics. The participants in the study were the second year mathematics students from Srinakharinwirot University. Including a variety of diverse instructional methods, such as group exercises, assessments, computational practice, and theoretical education, was one of the objectives of the research project that was being carried out. An all-encompassing review of the students' progress in learning was the goal of this action, which was taken with the objective of performing the evaluation.

2.1 Participants and Setting

There were thirty-two undergraduate student teachers who were in their second year of study and had majored in mathematics over the period of the research. They were in their second year of study. Despite the students did not have a great deal of experience with Bernoulli's equation and its applications, there were certain fundamental differential equations that they were already familiar with. These equations were essential to the students' comprehension of differential equations. During the course of the academic semester, the research was carried out as a component of a more comprehensive differential equations course that the students were attending. This course was being taken by the students. Students have the chance to interact with computational tools at any point during the course. This is made possible by the reason that the courses were held in a computer lab that was supplied with Python programming software. This is due to the fact that the sessions were held in a location that was created with a specific objective of fulfilling this purpose.

2.2 Research Design and Instructional Approach

The instructional approach was divided into three main phases.

- Theoretical Introduction: Students received a theoretical refresher of Bernoulli's equation at the start of the semester. They received this evaluation in our delivery. The students learned about the problem's nonlinearity, how to reduce it to a linear differential equation, and its essential framework during the lecture. The students were also given an explanation at the start of the lesson. The audience discussed transformative tactics and integrating components throughout the talk. Students were guided through the process of creating a linear equation that could be solved conventionally. The students were also building an equation that could be solved using standard procedures. This is achieved by dividing the initial equation by y^n and adding v = y^{1-n} to the linear equation. The process is repeated often.
- Python Programming for Solving Bernoulli's Equation: After demonstrating their comprehension of the theoretical change, students received computational instructions. Python is the most popular computer programming language due to its simple syntax and high-quality libraries. These include Sympy, Numpy, and Matplotlib. Python's flexibility is one aspect. The instructor presented Bernoulli's equation using Python. Students were taught how to write Python code that takes an equation, modifies it, and calculates the result. They also used Python charting to demonstrate how parameter values affected results. This was done for clarification.

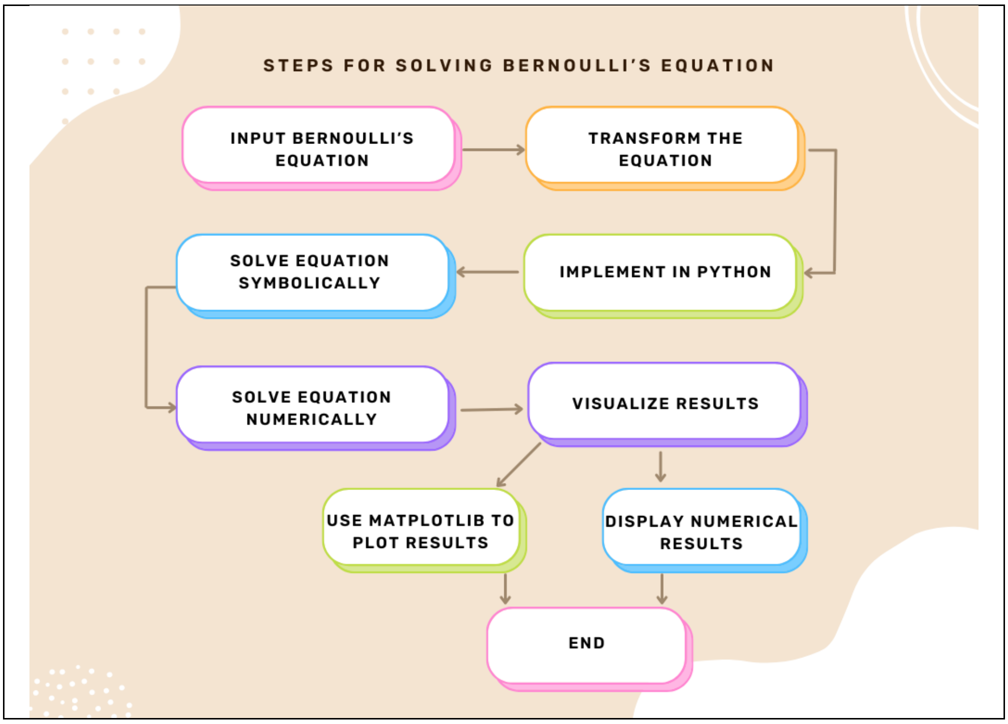

Figure 1. Steps for solving Bernoulli's equations using Python

```python
import sympy as sp

# Define symbols
x = sp.symbols('x')
y = sp.Function('y')(x)
# Define the Bernoulli differential equation
# y' + 2y = xy^2
equ = sp.Eq(y.diff(x) + 2*y, x*y**2)
# Solve the Bernoulli differential equation
general_sol = sp.dsolve(equ, y)
# Display the general solution
print("ผลเฉลยทั่วไปของสมการคือ:")
sp.pprint(general_sol)
# Apply initial condition y(0) = 1
C1 = sp.symbols('C1')
in_eq = general_sol.subs({x: 0, y: 1})
C1_value = sp.solve(in_eq.rhs - in_eq.lhs, C1)[0]
# Substitute back the constant into the general solution
specific_sol = general_sol.subs(C1, C1_value)
# Display the specific solution
print("ผลเฉลยเฉพาะกับค่าเงื่อนไขเริ่มต้น y(0) = 1 คือ:")
sp.pprint(specific_sol)
```

Figure 2. Example Python code for solving Bernoulli's equation

```python
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
# กำหนดค่าคงที่
r = 0.5  # อัตราการเติบโตภายในของประชากร (ต่อปี)
K = 1000  # ความสามารถในการรองรับของสิ่งแวดล้อม (ตัว)
P0 = 100  # จำนวนประชากรเริ่มต้น (ตัว)
# ฟังก์ชันที่กำหนดสมการเชิงอนุพันธ์
def logistic_growth(P, t, r, K):
    return r * P * (1 - P / K)
# จุดเวลาที่ต้องการคำนวณ
time_points = np.linspace(0, 10, 100) # เวลาตั้งแต่ 0 ถึง 10 ปี
# แก้สมการเชิงอนุพันธ์
P_solution = odeint(logistic_growth, P0, time_points, args=(r, K))
# แสดงผลลัพธ์จำนวนประชากรเมื่อเวลาผ่านไป 10 ปี
P_at_10_years = P_solution[-1][0]
print(f"Population size after 10 years: {P_at_10_years:.2f} individuals")
# วาดกราฟแสดงพลวัตประชากร
plt.plot(time_points, P_solution)
plt.xlabel('Time (years)')
plt.ylabel('Population size')
plt.title('Population dynamics according to the logistic model')
plt.grid(True)
plt.show()
```

Figure 3. Example of Python code for finding the solution to Bernoulli's equation in relation to a scientific problem

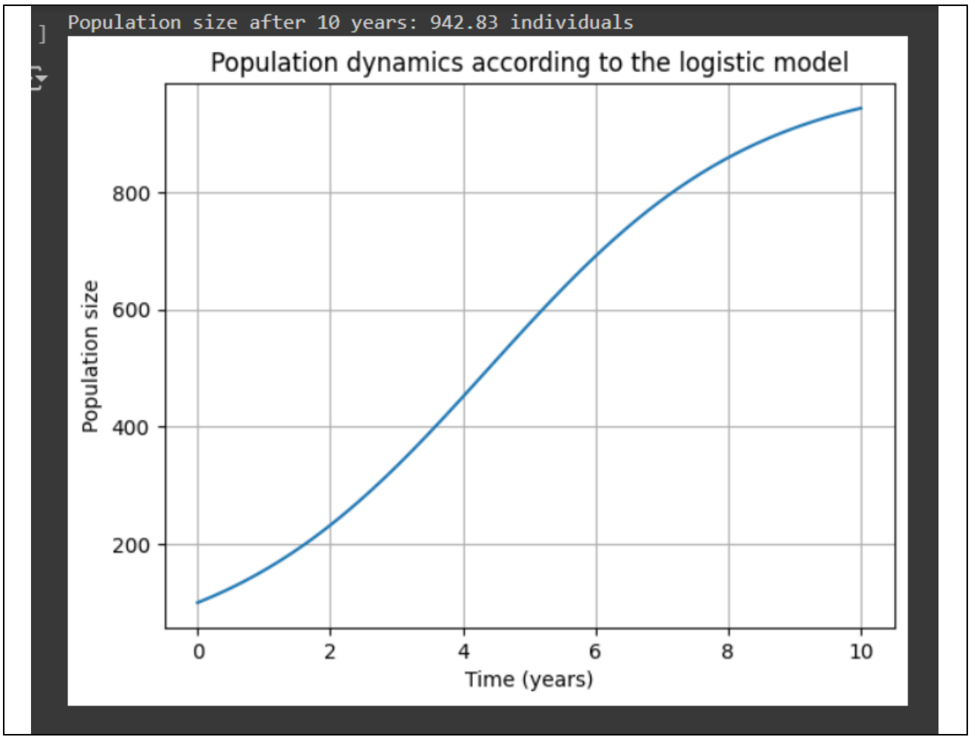

Figure 4. Graph showing the results obtained from Figure 3

- Inquiry-Based Learning Group Activities: The The development of IBL, which would eventually become the educational strategy, was the foundational strategy piece that was developed during the final phase of the educational strategy. In each of the six student groups, the assignment was to determine a scientific problem that might be solved by applying Bernoulli's equation. The problem could be based on a real-world scenario. I was given this task to complete. Every single one of the groups was tasked with the responsibility of finishing this specific project. In the course of the conversation, a wide range of subjects were brought up for discussion. Models of population increase, chemical processes, and fluid dynamics were discussed in engineering and physics. We also discussed population growth models. It should be noted that this list does not cover all conversation topics. The talk also expanded to cover many other issues. It was essential to fully consider each of these topics. Each group was given a challenge and had to come up with one, explain it theoretically using Bernoulli's equation, then solve it using Python. This was their assignment. This activity was planned and expected to be completed to meet mission criteria. Both of these tasks were necessary in order to finish the work. In addition to being required to devise a solution, students were also had to deliver a presentation to the class that was not only informative but also comprehensive. This was a requirement that they were required to fulfill. These presentations provided a full discussion of the Python code that was used, as well as graphical representations of the results, an explanation of the mathematical adjustments that were involved, and any other material that was necessary. The use of this collaborative approach made it possible for students to interact with both theoretical mathematical concepts and actual computing technologies. This was made possible by the fact that students were given the option to engage with both. Through the process of working in groups, they were able to develop their Python skills and acquire a more in-depth understanding of the mathematical subject. This was made possible by the interaction between the groups. These are a few examples problems that are used in the group projects. The objective of these projects is to provide students with the opportunity to demonstrate their knowledge and skills.

Problem 1: The Spread of Disease

Assume a population in a town where a disease is spreading, with the following information available:

The rate of disease spread is $r = 0.3$ per year.
The carrying capacity of the healthcare system is $K = 5000$ people.
The initial number of infected individuals is $I(0) = 50$ people.

We aim to find the number of infected individuals $I(t)$ over time t using the logistic growth model:

$$\frac{dI}{dt} = rI - \left(\frac{r}{K}\right) I^2$$

Question: After 5 years, how many people will be infected?

Figure 5. Scientific problems assigned for students to work on in groups

Problem 2: The Growth of Bacteria

Assume there is a culture of bacteria in a laboratory with the following information:

The growth rate of the bacteria is $r = 0.8$ per hour.
The carrying capacity of the food supply in the laboratory is $K = 10,000$ bacteria.
The initial number of bacteria is $B(0) = 200$ bacteria.

We aim to find the number of bacteria $B(t)$ over time t using the logistic growth model:

$$\frac{dB}{dt} = rB - \left(\frac{r}{K}\right) B^2$$

Question: After 24 hours, how many bacteria will there be?

Figure 6. Scientific problems assigned for students to work on in groups

2.3 Assessment and Evaluation

The study included formative and summative evaluations to figure out if participants met their learning objectives. This evaluated students' understanding of Bernoulli's equation and Python competencies. Tests were given before and after the lecture. To assess conceptual understanding, students have to take a pre-test before learning the theoretical underlying. This guaranteed student readiness. After teaching and collaborative tasks, students took a post-test. The post-test assessed students' problem-solving and comprehension progress. In addition to pre- and post-tests, students had to turn in their Python code and group reports explaining the real-world problems they solved, their solutions, and their computational and mathematical methods. Other exams were given before and after the course. The precision of the Python code, the accuracy of the answer, and the clarity with which the group presented the problem and solution were used to evaluate these reports.

2.4 Data Analysis

A mixed-methods study was carried out with the purpose of evaluating the growth of students. This study included both quantitative and qualitative evaluations in order to accomplish its objective. During the same time period, evaluations of both a qualitative and quantitative nature were carried out. The quantitative research study compared pre and post-test scores of thirty-two mathematics students. To get a conclusion, two sets of scores were compared. After evaluating all the results, the comparison was conducted. This comparison was performed to find out if students' understanding of Bernoulli's equation enhanced from what they had experienced. The standard deviation was needed to determine how much students performance varied. A study examined these exams' average scores to accomplish the objective. After the educational intervention was carried out, there was a decrease in the standard deviation, which is an indication that the students' knowledge became more precise. This was a sign that the intervention was productive. This study reveal that integrated Python programming instruction enhanced conceptual comprehension across a wide range of prior knowledge levels. Information was gathered through group discussions and feedback sessions for qualitative analysis. After concluding their group projects, the students discussed using Python to solve Bernoulli's equations, their problems, and how IBL impacted their learning. Students also described their challenges. Transcribed discussions were submitted to theme analysis to gather insights into students' problem-solving processes and ability to apply theoretical concepts to real issues. This was done for understanding. Qualitative data added context to the findings, confirming the quantitative study. These studies also showed the value of combining computational and active learning technologies. We expect this study will help us understand how instructional methods affect student learning. We utilize statistical analysis of pre and post-test scores and theme analysis of group discussions to achieve this. Python programming and IBL improve calculus students' critical thinking and problem-solving skills, and the mixed-methods approach validates this. This is because the technique improves students' abilities.

2.5 Inquiry-Based Learning Framework

The IBL dominated this project's instructional design. Students were encouraged to participate in their education by examining real-world problems, finding solutions, and working with others. Success was the goal. Group activities were planned to enhance critical thinking, problem-solving, and communication skills. Students should experiment with Python code parameters to discover how they impact Bernoulli's equation. We encouraged students to undertake this because experiments are important. A proposition was considered. After using this strategy, students showed greater mathematical understanding. Students also learned to apply these concepts to scientific problems.

2.6 Ethical Considerations

The research was carried out in a manner that was consistent with the ethical principles that govern educational research. This was done in accordance with the principles that govern the research. Prior to the beginning of the research study, each and every participant supplied their informed consent, and it was made very clear to the students that their participation would not in any way impact their overall mark in the class. The outcome of this matter occurred students' right to personal privacy was protected by keeping secret each and every piece of information that was obtained from the examinations and group projects in which they took part. This was done for the sake of protecting the students' right to privacy, which was the motivating factor behind our actions.

2.7 Limitations

The most major restriction of the study was the extremely small size of the sample, which may have an effect on the amount to which the findings may be generalized. This was the most significant deficiencies of the study. It is also important to note that the research carried out within a single college institution, which limited the amount to which it could be relevant to different environments. In the future, it is possible that studies will be conducted to investigate the effects of combining Python and IBL with student groups that are both larger and more diverse.

3. Results and Discussion

The study's findings are based on the analysis of student reflections, pre- and post-test data, and group activity outcomes. The primary objective was to assess how students' problem-solving and conceptual understanding of mathematics improved when Bernoulli's equation was taught using Python in InquiryBased Learning (IBL). At the beginning of the research, a pre-test was administered to 32 second-year mathematics student teachers to determine their basic understanding of Bernoulli's equation and their ability to solve it using traditional methods. The students' low pre-test average demonstrated their limited comprehension of nonlinear differential equations and the challenges associated with solving them by hand. With a standard deviation of 12.57, the pre-test results showed a significant degree of diversity in the students' initial aptitudes. Following the lesson, a post-test including theoretical justifications, Python-based computational exercises, and group work supervised by IBL was administered. The results of the post-test showed that the students' comprehension and computational skills had considerably improved. The effectiveness of the instructional technique may be seen in the much better average post-test scores. Furthermore, the standard deviation of the post-test results decreased to 8.37, indicating that students had reached a more consistent understanding level. This decrease in score variability demonstrates how well Python and IBL were used to address students' learning disabilities and level the playing field in terms of their comprehension of Bernoulli's equation (Table 1).

Table 1. Summary of Average Scores and Standard Deviations for Pre-test and Post-test Results

Metric	Pre-test	Post-test
Average Score	**58.77**	**81.10**
Standard Deviation	**12.57**	**8.37**

Table 1 shows how effectively the inquiry-based learning (IBL) approach works with Python by comparing the average scores and standard deviations from the pre-test and post-test. The results demonstrate the effectiveness of this strategy by highlighting significant increases in student comprehension and decreased variability. The employment of IBL and Python improved students' conceptual understanding, computational abilities, and teamwork. This approach also enhanced their capacity to use their knowledge in real-world situations and solve challenging difficulties. The introduction of Python-based computing into the IBL framework greatly improved students' comprehension and problem-solving skills. It may be difficult for traditional procedures to get comparable results, emphasizing the value of fusing technical approaches with inquiry-based methods to solve real-world issues. These results highlight how Python and IBL can assist students in becoming competent in difficult mathematical applications while encouraging teamwork and critical thinking.

The group activities were crucial in supporting the reinforcement of the students' learning. Students were required to choose real-world scientific problems, such as population dynamics or fluid mechanics, that might be explained using Bernoulli's equation for these activities. They used Python to solve the equations and created images to illustrate how the outcomes reacted in various settings. Every group presented their findings, including the required mathematical procedures and demonstrating how they arrived at and used Python to represent the findings. Students made important links between their theoretical knowledge and practical skills because of the collaborative nature of the group work and the real usage of computational tools. Together with the quantifiable increases in test scores, students' qualitative contributions during reflective talks were contributed. Many students observed that using Python made solving Bernoulli's equation easier and more evident since the computational tool made the complex transformations and calculations simpler. They also stressed the value of working in groups to solve problems together as it gave them the opportunity to try out different strategies and learn from each other's approaches. Students engaged with the material at a higher level because they could design their own questions and research applications of Bernoulli's equation. This was made possible via the IBL framework. Overall, the results of the research demonstrate that teaching Bernoulli's equation inside an IBL framework and in Python significantly enhanced students' understanding of the mathematical theory and its computational applications. Test scores have improved, student performance variability has decreased, and positive feedback from group projects and reflections has reinforced the effectiveness of this teaching method. Students strengthened their conceptual understanding and acquired practical computational skills, preparing them for future challenges in mathematics.

The study's findings demonstrate the effectiveness of using Python programming and other inquiry-based learning (IBL) techniques to teach Bernoulli's equation. The rise in students' post-test outcomes and the reduction in score variability make it evident that combining computational tools with active learning strategies may significantly enhance students' understanding of difficult mathematical concepts. The primary outcome of the research was an improvement in the students' post-test scores, which demonstrated their enhanced proficiency in solving Bernoulli's equation. The pre-test results showed that most students first struggled to understand the nonlinear nature of the problem and how to transform it into a linear form. However, after instruction on the theoretical foundations and the use of Python-based computational solutions, students demonstrated a significant improvement in their comprehension and problem-solving skills. The lower standard deviation of the post-test results suggests that the Python and IBL approach was effective in closing the skill gap amongst students and helping them all to grasp the material more consistently. With the addition of Python, students now have a helpful tool to handle complex calculations and visualizations that would be difficult to do by hand. This computational technique allowed students to study how solutions behaved in various settings in addition to helping them understand the transformation process needed to solve Bernoulli's equation by altering parameters. The ability for students to see the answers in Python helped them to understand how different variables affected the equation's response. It might be challenging to learn this talent using traditional teaching methods alone. In addition, the IBL framework significantly improved the students' educational experience. The IBL approach helped students develop their critical thinking and cooperative problem-solving skills by using Bernoulli's equation to solve realworld problems. By creating scientific questions, researching different methods to problem-solving, and applying what they learned in group projects, students were given the freedom to take ownership of their education. Because they were able to integrate theoretical concepts with practical applications, the active engagement of the students made the learning process more engaging and relevant to their future careers as teachers. The qualitative data obtained from the students' answers supports the efficacy of this method. Students said that Python simplified complicated computations and offered instantaneous visual feedback, making mathematics easier to learn. Additionally, since the group activities were collaborative, students were able to experiment with various approaches to problem-solving and gain knowledge from one other, which enhanced their comprehension. The advantages of integrating Python programming and IBL in mathematics education are emphasized in the research's conclusion. The results suggest that this kind of training might improve students' conceptual understanding, help them advance their computer skills, and provide a more engaging and collaborative learning environment. By integrating technology with active learning, educators may better equip their students to tackle difficult mathematical problems in both academic and professional contexts.

4. Conclusion

This study demonstrates that 32 second-year math students can learn Bernoulli's equation using IBL and Python. Statistics showed that students' understanding and computational skills improved, as shown by higher post-test scores and less performance variation. Python's complicated transformations and visualizations helped clarify Bernoulli's equation and its practical uses. IBL encouraged students to study real-world scientific subjects, enhancing engagement and comprehension.

Research findings: Researchers found that IBL and Python programming raised students' analytical thinking, problem-solving, and theoretical knowledge application. Python helped students comprehend Bernoulli's equation by illustrating its operations.

Research Limitations: The 32-student sample size could limit the generalizability of this study. However, the study focused only on second-year mathematics students at Srinakharinwirot University, limiting its relevance.

Implications for Practice: Python and active learning may assist students understand challenging mathematical concepts. Teachers should use programming and inquiry-based methods to connect theoretical principles to real-world applications.

Recommendation for future research: Future studies should include bigger and more varied student populations to validate these findings across educational contexts. A mixed-methods approach that includes qualitative data like student interviews could reveal more about student learning and issues. Further research should examine how IBL and computational tools affect student learning and retention over time.

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