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# Relational Thinking Test Instrument Analysis: Transition from Arithmetic to Algebra

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Abstract

Relational thinking, a cognitive approach bridging arithmetic thinking and algebraic thinking, has been developed by numerous researchers through mathematics learning processes to address students' difficulties in learning algebra. This study aims to develop a relational thinking test instrument for students and conduct both quantitative and qualitative analyses to measure relational thinking abilities in a valid and reliable manner. The research methodology adapts the development model proposed by Gall and Borg (2003) regarding the process of developing and validating products (test instruments), involving 64 seventh-grade students. Data analysis involved validity and reliability tests as well as analysis of students' written responses. The results indicate that the test instrument is valid (except for test item P8) and reliable, with a coefficient of α = 0.69 (high). Revisions were made to several items that were invalid or less valid based on quantitative and qualitative analysis. The revised instrument can be considered a good relational thinking test instrument, though it is recommended to support it with non-test results (such as interviews) to enhance accuracy and observe students' thinking consistency.



## 1. Introduction

Algebra is often regarded as the most critical "gatekeeper" in the field of mathematics (Cai et al., 2005). Research on algebra is often conducted through the development of algebraic thinking. One of the urgencies of algebraic thinking is to analyze deeper mathematical structures (Kiziltoprak & Kose, 2017). Hence, early algebraic thinking, as an early phase of algebraic thinking, frequently receives special attention from researchers due to the common difficulties students face in learning early algebra, such as challenges related to applying arithmetic operations (Jupri & Drijvers, 2014), difficulties in solving algebraic expressions (Muchoko, Jupri, & Prabawanto, 2019), and challenges in solving equations (Jupri, Drijvers, & Heuvel-panhuizen, 2014). However, understanding algebra is crucial for mastering advanced mathematical domains. Therefore, Lenz (2022) concluded that algebraic thinking is essential even at the elementary school level.

Difficulties in understanding early algebra are often linked to students' understanding of arithmetic. There are characteristic differences between arithmetic and algebra. Arithmetic is closely related to calculation, whereas algebra focuses on relations (Carpenter et al., 2005). On the other hand, arithmetic and early algebra are closely connected, particularly in the basic properties of number operations. For some students, learning algebra is a natural progression based on their understanding of arithmetic (Harbour et al., 2017); however, for many students, learning algebra feels entirely different from their experience with arithmetic, and they encounter various difficulties during the transition (from arithmetic to algebra) (Cai & Moyer, 2008).

The distinct characteristics encourages students to transition from arithmetic to algebra. From a cognitive perspective, this transition is believed to involve moving from the knowledge required to solve arithmetic problems involving operations on numbers to the knowledge required to solve algebraic

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problems involving variables (Warren, 2003). Boulton-Lewis et al. (1998) studied students' understanding of algebraic concepts when transitioning from arithmetic to algebra, which includes understanding the properties of operations, interpreting the equal sign, and understanding unknowns and variables.

Relational thinking has been a focus of research for several decades and is frequently regarded as a pivotal solution to the challenges students face during the transition from arithmetic to algebra. Carpenter and Levi (2005) define relational thinking as the ability to perceive mathematical expressions and equations holistically, rather than treating them as a sequence of procedural steps. This form of thinking encompasses interpreting the equal sign meaningfully, leveraging the fundamental properties of operations, recognizing interrelationships among numbers (Carpenter & Levi, 2005; Harbour et al., 2017; Jacobs et al., 2007; Kiziltoprak & Kose, 2017), and making strategic decisions in problem-solving (Harbour et al., 2017; NCTM, 2017). Furthermore, Kindrat and Osana (2018) emphasize that relational thinking is integral to algebraic reasoning, involving the coordination of quantities within mathematical expressions, often bypassing calculations, and applying flexible reasoning to transform expressions into equivalent forms. From the perspective of mathematical understanding, Nahdawiyah et al. (2023) highlight that individuals employing conceptual understanding to solve problems are actively engaging in relational thinking.

Recognizing the critical importance of fostering relational thinking from an early age, numerous researchers have emphasized its development among elementary school students (Blanton et al., 2019; Carpenter et al., 2005; Kiziltoprak & Kose, 2017). Meanwhile, studies by Usodo et al. (2020) and Darsono et al. (2018) have concentrated on exploring relational thinking in middle school students. To assess the levels or profiles of relational thinking in students effectively, a well-validated and reliable test instrument is essential. Such an instrument serves as a foundational tool for supporting the development of relational thinking through students' mathematical learning processes in school. Instrument validity is commonly defined as "the extent to which an instrument measures what it purports to measure" (Kimberlin & Winterstein, 2008), while reliability refers to the consistency of a method and its results (Budiastuti & Bandur, 2013). Both validity and reliability evaluations form integral components of the quantitative analysis conducted in this study.

Other research on relational thinking, particularly regarding the use of number sentences (either true or false or open number sentences), has been successful in developing relational thinking (Banerjee, 2011; Carpenter & Levi, 2005; Kindrat & Osana, 2018; Kiziltoprak & Kose, 2017; Pang & Kim, 2018), which has been analyzed qualitatively. This success forms the basis for the development of relational thinking test instruments using number sentences. However, this study includes both quantitative and qualitative data analysis. Darsono et al. (2018) also developed an algebra module based on relational thinking, while this research focuses on a relational thinking test instrument. Therefore, this study will develop a relational thinking test instrument for students (in the form of number sentences) and analyze the instrument both quantitatively and qualitatively, with the expectation that it will accurately and reliably measure relational thinking abilities.

## 2. Methods

The development of the relational thinking test instrument in this study adapted the development model by Gall and Borg (2003) concerning the process of developing and validating products. Beyond merely developing a product, the study aims to discover new knowledge through research. Therefore, quantitative and qualitative analyses of the trial results were also conducted. Briefly, the research began with initial data collection (literature review) to identify objectives (relational thinking), instrument planning, field testing, and instrument analysis and refinement.

The blueprint for the relational thinking test instrument was developed based on theoretical insights from experts and previous studies, resulting in the formulation of the following test indicators for relational thinking: (1) interpreting the equal sign as equivalence, (2) using relations between numbers, (3) applying the fundamental properties of number operations, and (4) making strategic decisions. Subsequently, test items were constructed based on these indicators, scoring guidelines were formulated, and a content validity test was conducted. The content-validated instrument was then tested with students, and the results were analyzed.

The trial was conducted with 64 seventh-grade students from two different schools who had studied integer material. The test results were assessed for validity and reliability and qualitatively analyzed to evaluate the instrument's feasibility. Revisions were made to items that did not meet the feasibility criteria, such as validity, reliability, or other considerations based on the analysis results.

## 3. Results & Discussions

The developed instrument, based on predetermined indicators, consists of two sections; section 1 consists of four items (P1-P4) involving closed numerical sentences (true-false), and section 2 consists of four items (P5-P8) involving open numerical sentences (questions in unknown numbers). The distribution of indicators across the test items is presented in Table 1, while student responses to the relational thinking test were evaluated using a scoring rubric that takes into account both the final answers and the provided justifications or solution processes, as shown in Table 2.

**Table 1.** Distribution of Relational Thinking Indicators

| Indicator | Items |
|---|---|
| Interpreting the equal sign as equivalence | P5 |
| Using relations between numbers | P4 |
| Applying the fundamental properties of number operations (including commutative, assosiative, and distributive properties) | P1, P2, P3, P7 |
| Making strategic decisions | P6, P8 |

**Table 2.** Scoring Rubric for Relational Thinking Test

| Final Answer | Justification | Score |
|---|---|---|
| Correct | Correct reasoning demonstrating relational thinking | 4 |
| Correct | Correct reasoning but not demonstrating relational thinking | 3 |
| Correct | Correct reasoning but with improper or unclear justification | 2 |
| Incorrect | Logical reasoning or correct steps, but wrong answer | 1 |
| Incorrect | Incorrect or unclear reasoning; no justification | 0 |

Based on student responses, the distribution of scores for each item is shown in Figure 1. The items, labeled P1 through P8, indicate that most scores ranged between 0 and 3, with very few achieving a score of 4, demonstrating relational thinking. Scores of 4 were achieved only on P3 (2%), P4 (3%), P5 (2%), P6 (2%), and P7 (2%).

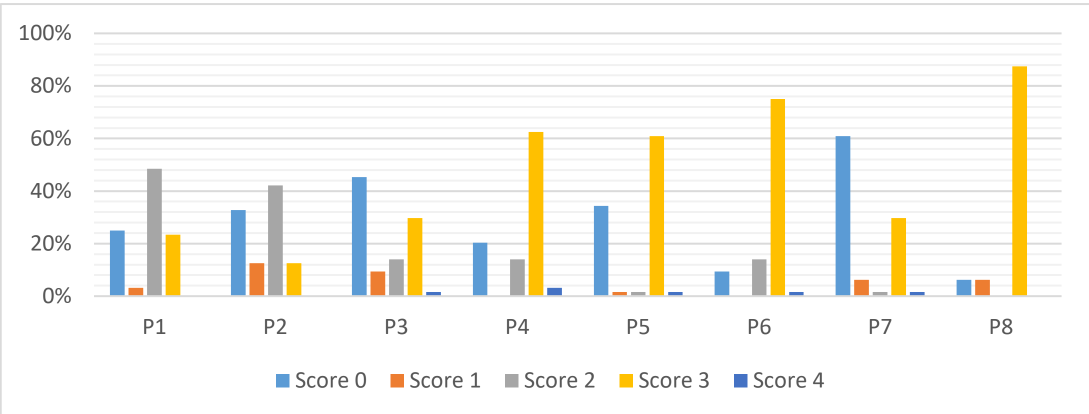



**Figure 1.** Percentage Distribution of Relational Thinking Test Scores

### *3.1. Instrument Validity and Reliability Test*

The instrument validity test included content validity and empirical validity. Content validity was assessed by expert reviewers (academic supervisors) and readability tests by a seventh-grade mathematics teacher. After minor revisions, the instrument was deemed content-valid with no significant changes, allowing for student trials.

For empirical validity, student responses were scored using Table 1 and analyzed using Pearson correlation in SPSS ver.27 ($\alpha = 0.05$). The results are summarized in Table 2. It can be concluded that all items were categorized as valid, except for P8 ($r = 0.003$) which failed to measure relational thinking indicators. Additionally, items P2 ($r = 0.275$) and P6 ($r = 0.264$) showed low validity coefficients.

**Table 2.** Empirical Validity Results for Relational Thinking Test

| Item | *r*-Value | Criteria |
|---|---|---|
| P1 | 0.496 | Valid |
| P2 | 0.275 | Valid |
| P3 | 0.504 | Valid |
| P4 | 0.518 | Valid |
| P5 | 0.649 | Valid |
| P6 | 0.264 | Valid |
| P7 | 0.575 | Valid |
| P8 | 0.003 | Not Valid |

In addition to the validity test, a reliability test was also conducted, aimed at assessing the consistency of the instrument. Using Cronbach's Alpha in SPSS ver.27 ($\alpha = 0.05$), the reliability coefficient was 0.69 as showen in Figure 2. Based on *Guilford*'s classification, this coefficient falls within the range of 0.60 to 0.80, categorized as high reliability. This indicates that the instrument is sufficiently consistent for measuring relational thinking skills.

**Reliability Statistics**

| Cronbach's Alpha | N of Items |
|---|---|
| .690 | 11 |

**Figure 2.** Cronbach's Alpha Reliability Test Results

### *3.2. Revisions to the Relational Thinking Test Instrument*

Item P8 (99 × 3 = …) categorized as not valid. Most students provided uniform responses (correct answers using conventional multiplication). The majority scored 3, with no scores of 2 or 4. Revision suggestion: Include a directive requiring students to solve the problem using two different methods, as shown in Table 3.

**Table 3**. Revision to Item P8

| Before Revision | After Revision |
|---|---|
| 99 × 3 = … | 99 3 = … (Solve using two different methods) |

Item P2 (25 – 8 + 7 = 8 – 25 + 7) displayed low validity ($r = 0.275$). Most students scored 2, indicating unclear or incorrect justifications despite correct final answers. Misconceptions about the order of operations were common. Revision suggestion: Simplify the sentence structure to focus on subtraction (e.g., "25 – 12 = 12 – 25"). Item P6, similar to P2, low validity was observed. Detailed analysis and suggestions for improvement are provided in Figure 3.

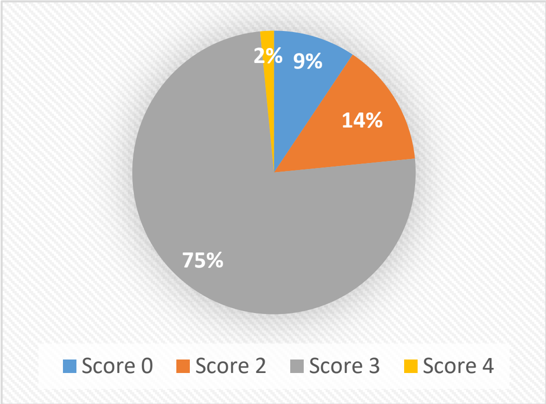


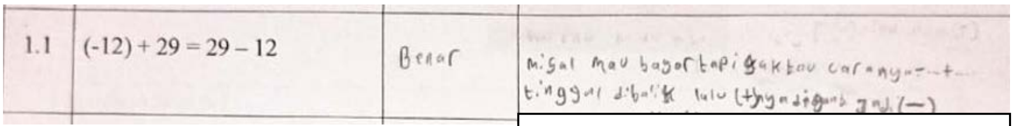

**Figure 3**. Score Distribution Diagram for P6

Approximately 75% of students scored 3, with the majority arriving at correct answers by substituting the unknown number into the equation. It is suspected that students relied on a "guess and check" strategy since the numbers in the question were relatively simple to estimate. To discourage this approach, the question can be revised to include larger numbers that are not easily guessed. The revised version is shown in Table 4.

**Table 4**. Revision of Question P6

| Before Revision | After Revision |
|---|---|
| 56 + ... – 2 = 57 | 567 + ... – 2 = 568 |

In addition to revising items with invalid or low validity scores, Question P1, which asks for the validity of (−12) + 29 = 29 − 12 and has a validity score ($r$ = 0.496), is also recommended for revision. This is because many student responses tended to focus on operations involving negative integers (as shown in Figure 4) rather than on the structure (application of basic arithmetic properties). To address this issue, it is suggested that the numbers be changed to positive integers, revising the question from" (−12) + 29 = 29 − 12" to "12 + 29 = 29 + 12".




For example, if I don't know how - … + …, I just switch the position, then change the positive (+) to negative (-)

**Figure 4**. Example of Student Responses for Question P1

*3.3. Answer Analysis Based on Question Items*

For Questions P1 and P2, most students scored 2. This indicates that while they answered correctly, their reasoning was either correct but not sufficiently justified (e.g., solving the operation correctly for only one side of the equation), incorrect or unclear (e.g., "because negative and positive signs can be swapped left to right"), or missing entirely. Figure 5 shows an example of a student response to Question P2, where the student understood the concept of equivalence but made a procedural error during the calculation process.

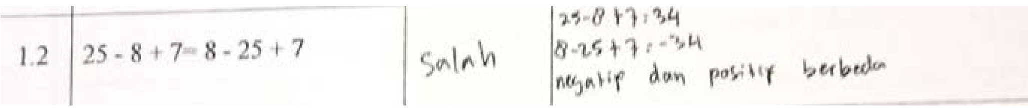



**Figure 5**. Example of a Response with a Score of 2 for Question P2

For Question P3, most students scored 0, meaning their final answers were incorrect, and their reasoning was either incorrect, unclear, or absent. Although students demonstrated an understanding of number sentences as equivalence, they often performed operations on both sides to compare results and made arithmetic errors, leading to different outcomes on each side. These errors could have been avoided if students had viewed the equation holistically, applied the associative property, or understood the concept of inverses (e.g., recognizing 35 – 9 + 9 as 35 + (−9) + 9 rather than simplifying it to 35 − 18. Figure 6 provides an example of such errors.

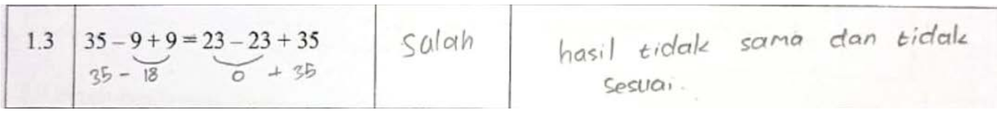
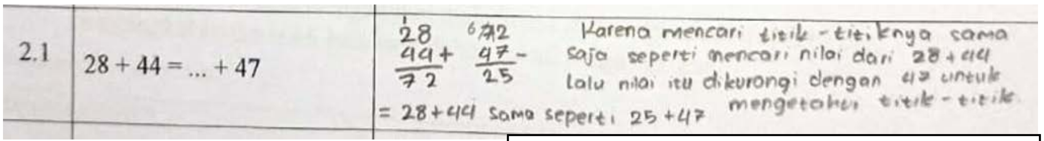

| | | | |
|---|---|---|---|
| 1.3 | $35 - 9 + 9 = 23 - 23 + 35$ <br> $35 - 18 \quad 0 + 35$ | Salah | hasil tidak sama dan tidak sesuai. |

**Figure 6**. Example of a Response with a Score of 0 for Question P3

In Question P4, 63% of students scored 3, indicating correct answers with accurate reasoning, though relational thinking was not demonstrated. Students tended to solve Question P4 procedurally to achieve equivalence, and similar patterns were observed for Questions P5 and P6. Figure 7 illustrates an example of such procedural work. While this demonstrates a good understanding of the equal sign as equivalence, most students did not yet recognize the mathematical structure or the interrelation between numbers.


| | | |
|---|---|---|
| 2.1 | 28 + 44 = ... + 47 | 28 + 44 = 72, 72 − 47 = 25 <br> = 28+44 sama seperti 25+47 <br> Karena mencari titik-titiknya sama saja seperti mencari nilai dari 28+44 lalu nilai itu dikurangi dengan 47 untuk mengetahui titik-titik |

Because finding the blank is the same as finding the value of 28 + 44, then subtracting 47 from that value to find the blank.

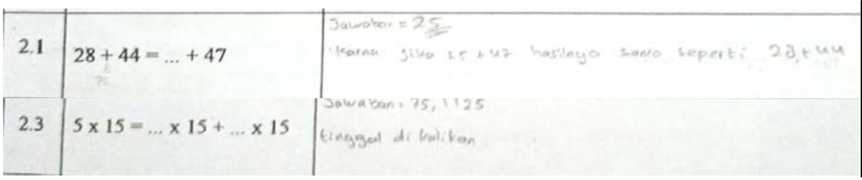
**Figure 7**. Example of a Response with a Score of 3 for Question P5

However, this level of understanding did not carry over to Question P7. Many students struggled with Question P7, with 61% scoring 0 (incorrect answers and reasoning). While some students showed an understanding of the equal sign in previous questions, 23% interpreted the equal sign as an indicator of the result, writing answers such as 75 and 1.125 (the results of 5 × 15 and 75 × 15, respectively). This suggests that students were unfamiliar with the question format 5 × 15 = ... × 15 + ... × 15 or lacked a genuine understanding of the equal sign as equivalence.

Question P8 followed a similar pattern to Questions P4, P5, and P6. Approximately 88% of students answered correctly and used appropriate methods, though relational thinking was not evident. Most students used conventional multiplication, such as long multiplication. However, six students demonstrated creativity by writing solutions like 99 × 3 = 2 × 99 + 1 × 99, 99 × 3 = 33 × 9, 99 × 3 = 99 + 99 + 99, or 99 × 3 = 100 × 2 + 10 × 9 + 7. While these answers were correct and showed creativity, they did not exhibit strategic decision-making, such as transforming 99 into (100 − 1) and rewriting (100 − 1) × 3 as 100 × 3 – 1 × 3.

For the indicator related to interpreting the equal sign as equivalence, most students showed a good understanding, as evidenced in Question P5 (28 + 44 = ... + 47). Approximately 61% of students successfully compared the operations on both sides, while 34% (22 students) scored 0. Among these, 16 students interpreted the equal sign as a symbol for placing the result (e.g., writing "72" as the answer). This pattern is consistent with responses to Question P7 (5 × 15 = ... × 15 + ... × 15), where students who scored 3 in Question P5 tended to score 3 in Question P7, and those interpreting the equal sign incorrectly in Question P5 also did so in Question P7.

Interestingly, there was inconsistency among some students. For example, five out of the 16 students who failed to interpret the equal sign in Question P7 successfully solved Question P5, likely due to exposure to similar models or questions (e.g., Question P4). This suggests these students may still be at the operational stage rather than demonstrating true relational thinking. Figure 10 illustrates the inconsistency observed in Subject S7’s responses to Questions P5 and P7.




Because 25 + 47 gives the same result as 28 + 44."

Just multiply

**Figure 8**. Example of Inconsistent Responses to Questions P5 and P7 by Subject S7

This inconsistency highlights the need for varied question formats (more than one question) for the same indicator. Furthermore, qualitative or mixed-method approaches with triangulation (e.g., observation or interviews) could strengthen the analysis of students' relational thinking.

Overall, based on the diagram in Figure 1, the majority of students scored between 0 and 3 for each question item, with very few (less than 5%) scoring 4 (relational thinking). This finding aligns with Usodo et al. (2020), who found that less than 12.5% of junior high school students demonstrated relational thinking, with most relying on computational thinking to solve mathematical problems. Similarly, Darsono et al. (2018) noted that problem-solving in schools predominantly involves applying algorithms to arrive at final answers, rather than encouraging students to think critically or independently discover concepts of equivalence relations.

Based on the quantitative and qualitative analyses presented, the relational thinking test instrument can be considered valid and reliable. The revisions made by the researchers provide a reference for further analysis. However, this study has limitations, including a qualitative analysis that relied solely on written tests. Future research could include contextual problems or story-based questions, as the current instrument only involves number sentences. Additionally, student response analysis could be enhanced through interviews or observations. Assessing students' thinking processes more accurately could help teachers develop students' relational thinking skills in the context of mathematics teaching and learning at schools.

### *3.4. Relational Thinking Test in Identifying the Transition from Arithmetic to Algebraic Thinking*

The results of the relational thinking test revealed important patterns in how students approached numerical expressions and equations, reflecting a shift in their mathematical thinking. Students who demonstrated the ability to view expressions holistically, recognize numerical relationships, and justify their solutions without relying solely on computation exhibited characteristics of relational thinking. These traits are essential indicators of a cognitive transition from arithmetic thinking, focused on procedural operations, to algebraic thinking, which involves structural understanding and symbolic reasoning (Jacobs et al., 2007; Carpenter et al., 2005; Kieran, 2004; Kaput, 2008).

This transition was evident, for example, when students identified equivalencies in expressions such as $28 + 44 = \ldots + 47$ (P5) by analyzing the relationship between the numbers rather than performing step-by-step calculations. Such responses suggest that the relational thinking test can serve as a diagnostic tool for identifying students who are beginning to engage in early algebraic reasoning. The findings support previous research emphasizing the role of relational thinking as a bridge between arithmetic fluency and algebraic understanding (Blanton & Kaput, 2005; Warren, 2003; Blanton et al., 2015). Furthermore, understanding the equal sign relationally, as a symbol indicating equivalence rather than merely a signal to compute, has been shown to play a key role in developing algebraic thinking (Knuth et al., 2006).

## 4. Conclusion

Based on the findings and discussion of the study, the quantitative analysis indicates that all test items are categorized as valid, except for Question P8 ($r = 0.003$), which means it failed to measure relational thinking indicators. Additionally, two questions have low correlation coefficients: Question P2 ($r = 0.275$) and Question P6 ($r = 0.264$) so they need to be revised. The reliability test shows that the instrument is reliable, with a Cronbach's alpha coefficient of 0.69 (high) which indicates that the instrument is sufficiently consistent for measuring relational thinking skills, so it can be used to identify student's transition from arithmetic to algebraic thinking.

From the quantitative analysis, supported by qualitative analysis of student responses, four test items (P8, P2, P6, and P1) are recommended for revision. The revised instrument can be considered a strong relational thinking test; however, it has limitations in terms of question variation and data triangulation.

To address these limitations, the instrument could be further developed to include contextual problems or story-based questions. Additionally, data analysis beyond written test results could involve interviews or observations. Accurately assessing students' stages of thinking can provide a valuable reference for developing their relational thinking through mathematics teaching and learning processes in schools.

## References

Banerjee, R. (2011). Is Arithmetic Useful for the Teaching and Learning of Algebra?. *Contemporary Education Dialogue*, 8(2), 137–159. https://doi.org/10.1177/097318491100800202.

Blanton, M., Stephens, A., Knuth, E., Gardiner, A., Isler, I., & Kim, J. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention. *Journal for Research in Mathematics Education*, *46*(1), 39–87.

Blanton, M., Stroud, R., Stephens, A., Gardiner, A. M., Stylianou, D. A., Knuth, E., … Strachota, S. (2019). Does Early Algebra Matter? The Effectiveness of an Early Algebra Intervention in Grades 3 to 5. *American Educational Research Journal,* 56(5), 1930–1972. https://doi.org/10.3102/0002831219832301.

Boulton-Lewis, G., Cooper, T. J., Atweh, B., Pillay, H., & Wills, L. (1998). Arithmetic, prealgebra and algebra: A model of transition. In C. Kanes, M. Goos, & E. Warren (Eds.), Teaching mathematics in new times (*Proceedings of the 21st annual conference of the Mathematics Research Group of Australasia*) (pp. 114- 120). Gold Coast: MERGA.

Budiastuti, D. dan Bandur, A. (2018). *Validitas dan Reliabilitas Penelitian Dengan Analisis dengan NVIVO, SPSS dan AMOS*. Jakarta: Mitra Wacana Media.

Cai, J., Lew, H. C., Morris, A., Moyer, J. C., Fong Ng, S., & Schmittau, J. (2005). The development of studients' algebraic thinking in earlier grades. *Zentralblatt Für Didaktik Der Mathematik*, 37(1), 5–15. https://doi.org/10.1007/bf02655892

Cai, J. & Moyer, P. (2008). Developing algebraic thinking in earlier grades: Some insights from international comparative studies. In C. E. Greenes (Ed.), *Algebra and Algebraic thinking in School Mathematics* (pp.169-179). Reston, VA: NCTM.

Carpenter, T. P., Levi, L., Franke, M. L., & Zeringue, J. K. (2005). Algebra in Elementary School: Developing Relational Thinking. *ZDM - International Journal on Mathematics Education,* 37(1), 53–59. https://doi.org/10.1007/BF02655897 .

Darsono, Santia, I., & Jatmiko. (2018). Peningkatan Kemampuan Pemecahan Masalah Aljabar Siswa SMP Melalui Pembelajaran Modul Aljabar Berbasis Relational Thinking. *Jurnal Ilmiah Pendidikan Matematika),* 6(2), 119–124. http://journal.unipma.ac.id/index.php/jipm

Harbour, K. E., Karp, K. S., & Lingo, A. S. (2017). Inquiry to Action Diagnosing and Addressing Students ' Relational Thinking About the Equal Sign. *Teaching Exceptional Children,* 49(2), 126–133. https://doi.org/10.1177/0040059916673310.

Gall, M.D, Gall, J.P, dan Borg, W.R. (2003). *Educational Research: An Introduction edisi 7*. Allyn and Bacon.

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Levi, L., & Battey, D. (2007). Professional Development Focused on Children ' s Algebraic Reasoning in Elementary School of. *Journal for Research in Mathematics Education*, 38(3), 258–288.

Jupri, A., & Drijvers, P. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Group of Australasia*, (229). https://doi.org/10.1007/s13394-013-0097-0

Jupri, A., Drijvers, P., & Heuvel-panhuizen, M. Van Den. (2014). Student Difficulties in Solving Equations from an Operational and a Structural Perspective. *Mathematics Education,* 9(1), 39–55.

Kaput, J.J. (2008). What Is Algebra? What Is Algebraic Reasoning?. *Algebra in the Early Grades*, 5–17. doi: https://doi.org/10.4324/9781315097435-2.

Kieran, C. (2004). Algebraic Thinking in The Early Grades: What is it. *The Mathematics Educator,* 8(1), 139–151.

Kimberlin, C. L., & Winterstein, A. G. (2008). Validity and reliability of measurement instruments used in research. In *American Journal of Health-System Pharmacy* (Vol. 65, Issue 23, pp. 2276–2284). American Society of Health-Systems Pharmacy. https://doi.org/10.2146/ajhp070364

Kindrat, A. N., & Osana, H. P. (2018). The relationship between mental computation and relational thinking in the seventh grade. *Fields Mathematics Education Journal*, 3(1). https://doi.org/10.1186/s40928-018-0011-4

Kiziltoprak, A., & Kose, N. Y. (2017). Relational thinking : The bridge between arithmetic and algebra. *International Electronic Journal of Elementary Education*. 10(1), 131-145. https://doi.org/10.26822/iejee.2017131893

Knuth, E. J., Stephens, A. C., McNeil, N. M., dan Alibali, M. W. (2006). Does Understanding The Equal Sign Matter? Evidence from Solving Equations. *Journal for Research in Mathematics Education*, 37, 297–312.

Lenz, D. (2022). The role of variables in relational thinking: an interview study with kindergarten and primary school children. *ZDM - Mathematics Education*, 54(6), 1181–1197. https://doi.org/10.1007/s11858-022-01419-6

Muchoko, C., Jupri, A., & Prabawanto, S. (2019). Algebraic visualization difficulties of students in junior high school. *Journal of Physics: Conference Series*, 1157(3). https://doi.org/10.1088/1742-6596/1157/3/032108

Nahdawiyah, S. A., Nusantara, T., & Rahardi, R. (2023). Berpikir Relasional Mahasiswa Pada Materi Fungsi Polinomial. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 12(3), 3246-3258.

NCTM. (2017). Relational Thinking : What ' s the Difference ? Author ( s ): Ian Whitacre , Robert C . Schoen , Zachary Champagne and Andrea Goddard. *Published by : National Council of Teachers of Mathematics*, 23(5), 302–308. http://www.jstor.org/stable/10.5951/teacchilmath.

Pang, J., & Kim, J. (2018). Characteristics of Korean Students' Early Algebraic Thinking: A Generalized Arithmetic Perspective. In C. Kieran (ed.), *Teaching and Learning Algebraic Thinking with 5- to 12-Year-Olds, ICME-13 Monographs*, (pp.141-165). Switzerland: Springer International Publishing. https://doi.org/10.1007/978-3-319-68351-5_6.

Warren, E. (2003). The role of arithmetic structure in the transition from arithmetic to algebra. *Mathematic Education Research Journal*, 15(2), 122-137.