

Physical Analysis of a Fishing Rod and Development of an Elasticity E-Module as a Supplement for Mechanics Learning

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Abstract: Mechanics instruction at the university level, particularly for students in the Physics and Physics Education study programs, is often hindered by the abstract nature of the content, complex formulas, and unengaging modes of delivery. A preliminary study showed that 83.3% of students found mechanics difficult to understand, partly because of the limited variety of learning media. One interesting topic in mechanics is the dynamics of elastic bodies, including fishing rods. These dynamics involve numerous physical concepts and complex mathematical equations, so an enrichment module is needed to facilitate students' understanding. This study aimed to develop an elasticity e-module as a supplement for mechanics learning, focusing on the analysis of the curvature of a fishing rod (an elastic rod) and a comparison of the acceleration multiplication factor between elastic and non-elastic rods subjected to a force impulse. This study used a Research and Development (R&D) approach with the 4D development model (Define, Design, Develop, Disseminate). The physical analysis was conducted theoretically and experimentally on tapered elastic rods. In the curvature experiment, the load was varied, whereas in the acceleration experiment, the acceleration response was observed using the accelerometer sensor of a smartphone. The developed e-module was validated by subject-matter and media experts and tested with students as users. The validation results showed that the e-module was categorized as "very feasible" in both content and media aspects. Meanwhile, students' responses fell into the "very good" category for material presentation (90.53%), language (91.58%), visual display (88.84%), usefulness (91.89%), technical aspects (91.75%), and evaluation (90.53%). Therefore, the elasticity e-module is feasible for use as a supplement in Mechanics courses.

Keywords: Elasticity, E-Module, Mechanics, 4D Development, Fishing Rod

Introduction

Mechanics is a core course in the Physics and Physics Education study programs and provides the foundation for understanding other areas of physics, such as thermodynamics, electromagnetism, and modern physics. Despite its importance, many students struggle to understand this subject. A preliminary study conducted by the researchers with 24 students in the third and fifth semesters of the Physics and Physics Education study programs at the Faculty of Mathematics and Natural Sciences, Universitas Negeri Semarang, showed that 83.3% of the students reported that mechanics was difficult to understand. The contributing factors included poorly varied learning media, abstract concepts, numerous formulas, and complex technical terminology. These conditions may hinder students from constructing physical representations and applying conceptual models to understand mechanical phenomena. This finding is consistent with studies showing that students often encounter difficulties in using representations and physics modeling when solving mechanics problems (Van Heuvelen, 1991; Docktor & Mestre, 2014). Therefore, learning media that can connect abstract concepts with real phenomena are needed so that students can understand mechanics concepts more easily in context.

One mechanics topic regarded as challenging is elasticity. Physical concepts such as elasticity and Hooke's law require a learning approach that emphasizes not only theoretical aspects but also contextual application in real life (Fitrian et al., 2025). Elasticity concerns how a body deforms when subjected to an external force and returns to its original shape once the force is removed. Everyday phenomena such as the bending of a fishing rod can serve as a concrete example of elasticity. The tapered shape of a fishing rod, in

which the diameter at the base is larger than that at the tip, produces a characteristic bending response when a force is applied (Silverman & Farrah, 2018). This phenomenon is rarely used as a learning medium, even though it is highly relevant for explaining the concepts of curvature, bending moment, and rod-tip acceleration.

The transformation of modern physics instruction emphasizes the importance of using interactive media to improve conceptual understanding (Wieman & Perkins, 2005). The use of digital learning media such as e-modules can be an alternative for helping students understand the concept of elasticity in a more contextual way. An e-module is a digital teaching material that is systematically organized to help learners achieve learning objectives independently and in a guided manner (Rahdiyanta, 2016). The use of e-modules in physics learning is considered capable of helping students visualize abstract concepts and increasing their engagement in the learning process.

Previous studies have examined tapered elastic rods from theoretical and simulation perspectives. Sabrin Manssor et al. (2020) investigated the deflection of a carbon-fiber fishing rod using numerical and experimental approaches. McMillen & Goriely (2003) discussed impulse propagation in an elastic whip, which is also tapered. Goriely et al. (2002) mathematically modeled elastic waves. However, these studies focused mainly on theoretical and numerical analyses and had not yet integrated the physical analysis of real phenomena with the development of elasticity learning media.

Based on these problems, this study aims to analyze the physical characteristics of an elastic fishing rod by examining its curvature and acceleration multiplication factor in comparison with a non-elastic rod, and to develop an elasticity e-module based on the fishing rod phenomenon as a supplement for Mechanics learning. This study not only analyzes the elasticity phenomenon experimentally but also integrates it into the development of a contextual e-module as a learning supplement for students.

Methods

This study used a Research and Development (R&D) approach with the 4D model, which comprises the define, design, develop, and disseminate stages. The study employed purposive sampling a nonrandom sampling technique based on particular characteristics aligned with the research objectives (Lenaini, 2021). The sample (subjects) comprised students of the Physics and Physics Education study programs at Universitas Negeri Semarang who were taking or had completed the Mechanics course, namely students in the fourth, sixth, and eighth semesters. The data were obtained through expert validation questionnaires (subject-matter and media) and student response questionnaires using a Likert scale, as well as data from the physical analysis and experiments, which were obtained by processing the experimental data. The Likert scale used in this study consisted of positive statements scored 5, 4, 3, 2, and 1, with 5 being the highest score and 1 the lowest.

The four stages of the 4D development model are described in more detail as follows (Tamagola, 2022). The define stage was carried out through front-end analysis, learner analysis, task analysis, concept analysis, and specifying instructional objectives. The design stage included designing the e-module framework, selecting the format, and preparing the evaluation instrument. The develop stage involved drafting the e-module, expert validation by subject-matter and media experts, and revising the e-module. The disseminate stage consisted of distributing the e-module to students together with the administration of the student response questionnaire.

The physical analysis in this study consisted of a theoretical analysis that was subsequently validated against experimental results. The first experiment analyzed the curvature curve of an elastic rod (a fishing rod) under varying loads, using photographic methods and data processing in Python. The second experiment measured the acceleration of elastic and non-elastic rods subjected to a force impulse, using the accelerometer sensor of a smartphone together with the Arduino Science Journal software.

The data analysis technique for the validity test (as a means of testing the feasibility of the e-module product) involved calculating the ideal mean (M_i) and the ideal standard deviation (SB_i) through the following steps (Faresta et al., 2020):

Calculating the Total Score of the Assessment Aspect

If there are n validators, the total score of an assessment aspect is calculated using the formula in Equation (1).

$$\sum x = x_1 + x_2 + \dots + x_n \quad (1)$$

dengan:

$\sum x$: total score

x_1 : score of the first validator

x_2 : score of the second validator

x_n : score of validator n .

Calculating the Ideal Minimum and Maximum Scores, the Ideal Mean (M_i), and the Ideal Standard Deviation (SB_i)

After the total score of the assessment aspect has been calculated, the ideal minimum and maximum scores, the ideal mean (M_i) and the ideal standard deviation (SB_i) are calculated next. The formulas used, in sequence, are given in Equations (2), (3), (4), and (5).

$$\text{Ideal minimum score} = \text{minimum Likert scale value} \times \text{number of respondents} \times \text{number of statements} \quad (2)$$

$$\text{Ideal maximum score} = \text{maximum Likert scale value} \times \text{number of respondents} \times \text{number of statements} \quad (3)$$

$$M_i = \frac{1}{2} (\text{ideal maximum score} + \text{ideal minimum score}) \quad (4)$$

$$SB_i = \frac{1}{6} (\text{ideal maximum score} - \text{ideal minimum score}) \quad (5)$$

Determining the Assessment Criteria

The assessment criteria are determined on the basis of the ideal mean and ideal standard deviation calculated using Equations (4) and (5). Once the values of M_i and SB_i have been obtained, the score categories can be determined as shown in Table 1. The variable x in Table 1 represents the total score of the assessment aspect.

Table 1. Quantitative Score Ranges and Their Categories

Quantitative Score Range	Category
$x \geq M_i + 1.5(SB_i)$	Very Good
$M_i + 1.5(SB_i) > x \geq M_i$	Good
$M_i > x \geq M_i - 1.5(SB_i)$	Fair
$M_i - 1.5(SB_i) > x$	Poor

The data analysis technique for the student response questionnaire used the ratio of the obtained score to the maximum score. After the student response data were collected, the maximum score of the assessment items (N) was calculated using the formula in Equation (6).

$$N = \text{maximum Likert scale value} \times \text{number of respondents} \times \text{number of statements} \quad (6)$$

After the total score (f) has been obtained, the percentage score is calculated using the formula in Equation (7).

$$P = \frac{f}{N} \times 100\% \quad (7)$$

where:

- P : Percentage of the score obtained
- f : Total score of the assessment items
- N : Maximum total score of the assessment items

The interpretation of the categories based on the percentage score is presented in Table 2.

Table 2. Score Categorization Based on the Percentage Index

Percentage Index (%)	Category
80 – 100	Very Good
60 – 79.99	Good
40 – 59.99	Fair
39.99 – 20	Poor
0 – 19.99	Very Poor

(Pranatawijaya et al., 2019)

Results and Discussion

Fishing Rod Curvature Analysis

Theoretical Analysis

The analysis of fishing rod curvature begins with the theory of slender-rod elasticity. The relationship among bending moment, elastic modulus, and area moment follows the classical Euler–Bernoulli formulation, with a shear correction as described by Timoshenko (1921). A general mathematical treatment of elastic rod deformation can be found in the theory of continuum elasticity (F., 1920), whereas the concepts of elastic modulus and second moment of area are explained comprehensively in the mechanics of materials literature (Gere & Goodno, 2013).

The analysis of the fishing rod curvature curve begins with the theoretical approach using the curvature equation $\kappa = \frac{1}{R} = \frac{MEI}{EI}$, so that the radius of curvature R can be expressed as $R = \frac{EI}{M}$, where E is Young's modulus (Pa), I is the second moment of area (m^4), and M is the bending moment ($\text{N}\cdot\text{m}$).

Using the equation for the diameter along the coordinate of the tapered fishing rod, the equation $d(s) = D - \left(\frac{D}{L}\right)s$ is obtained, so that the value of $R = \frac{\pi E}{64M} \left(D - \left(\frac{D}{L}\right)s\right)^4$ holds for a circular cross-section. This equation is then converted from polar to Cartesian coordinates through the relations $x = R \cos\theta$ and $y = R \sin\theta$, with the variable θ represented as as . The variable s is the coordinate along the rod, whereas a is a constant that depends on the elastic properties of the rod. This transformation yields the theoretical curve equation given in Equation (8).

$$y = \frac{\pi E}{64M} \left(D - \left(\frac{D}{L}\right)s\right)^4 \sin(as) \quad (8)$$

The theoretical equation was visualized in the Python programming language using the NumPy and Matplotlib libraries, yielding a spiral curve that describes the bending shape of the fishing rod resulting from variations in diameter and force along the length of the rod (Figure 1). This visualization facilitates understanding of fishing rod deformation based on the slender-rod elasticity approach.

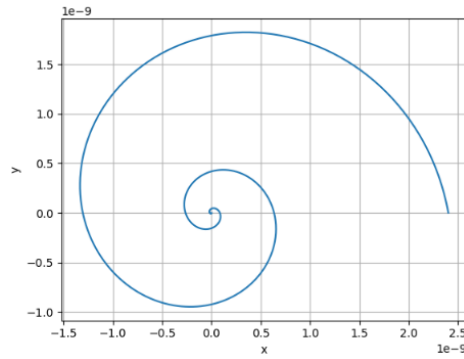


Figure 1. Visualization Result of the Fishing Rod Curvature Curve Using Python Code

Next, the component $\sin(as)$ in the equation y was expanded using a Maclaurin series, yielding a sixth-degree polynomial form (Equation (9)).

$$y = \frac{\pi E}{64M} \left(D - \left(\frac{D}{L} \right) s \right)^4 \left(as - \frac{(as)^3}{3!} + \frac{(as)^5}{5!} - \frac{(as)^7}{7!} \dots \right) \quad (9)$$

This form was then simplified into the explicit equation $y(s)$ which contains polynomial terms up to the sixth order. This equation was used to model the curvature curve within the experimental domain (Equation (10)).

$$y = \left(\frac{\pi E D^4}{64M} \right) \left(\left(\frac{4a^3}{L} \right) \left(-\frac{a^2}{120} + \frac{1}{6L^2} \right) s^6 - (a) \left(\frac{a^2}{L^2} - \frac{a^5}{120} - \frac{1}{L^4} \right) s^5 - \left(\frac{4a}{L} \right) \left(\frac{1}{L^2} - \frac{a^2}{6} \right) s^4 + (a) \left(\frac{6}{L^2} - \frac{a^2}{6} \right) s^3 - \left(\frac{4a}{L} \right) s^2 + (a)s \right) \quad (10)$$

Experiment

In the experiment, six different loads were applied to the tip of the fishing rod: 0.070 kg, 0.112 kg, 0.155 kg, 0.193 kg, 0.236 kg, and 0.280 kg. The resulting deformation was documented using a mobile phone camera and then analyzed using the website graphreader.com to obtain the coordinates of points plotted along the rod. The coordinate data were processed in Microsoft Excel to generate a sixth-order polynomial curve representing the measured deformation.

The theoretical curve and the experimental results were compared through a fitting process to evaluate the suitability of the model. The results show that the smaller the deviation between the theoretical and experimental curves, the higher the accuracy of the model in predicting rod deformation. An example of the comparison between the theoretical and experimental curves is shown in Figure 2, where the theoretical curve appears consistent with the bending pattern observed in the experiment for one of the tested loads..



Figure 2. Fitting of the Fishing Rod Curvature Curve Between Theory and Experiment

Acceleration Multiplication Factor in the Fishing Rod

Theoretical Analysis

The dynamic response of an elastic rod subjected to a force impulse is strongly influenced by the rod's geometry. In a tapered rod, the mass and stiffness distributions vary along the rod, producing vibration characteristics and dynamic responses that differ from those of a rod with a uniform cross-section (Attarnejad et al., 2011; Boreyri et al., 2014). In addition, changes in cross-sectional area alter the wave propagation characteristics and mechanical impedance along the rod, so that the response amplitude at certain sections may be amplified or attenuated as the wave travels toward the tip of the rod (Gan et al., 2016; Kalkowski et al., 2016). These dynamic responses indicate that the distribution of mass and stiffness along the rod plays an important role in determining the vibration behavior and acceleration of the rod. Therefore, a mechanical analysis is needed to describe the forces acting on the rod quantitatively.

The analysis begins with the application of Newton's second law to the translational and rotational motion of the fishing rod. For translational motion, the differential force acting on a small mass element (dm) is expressed as $dF_x = dm a_x$ dan $dF_y = dm a_y$ so that the total force equations are obtained, as given in Equations (11) and (12),

$$F_x' = \rho A \frac{d^2 x}{dt^2} \quad (11)$$

$$F_y' = \rho A \frac{d^2 y}{dt^2} \quad (12)$$

where ρ is the density and A is the cross-sectional area of the rod. The prime symbol $'$ denotes the derivative with respect to ds .

For rotational motion, the differential torque is derived from the translational force components acting on the axis of the rod, as shown in Equations (13), (14), and (15).

$$\tau_1 = -F_x \sin \varphi ds \quad (13)$$

$$\tau_2 = F_y \cos \varphi ds \quad (14)$$

$$\tau_3 = dM \quad (15)$$

so that the total torque satisfies $\sum \tau = I\alpha$.

Combining the two equations yields the general form shown in Equation (16).

$$-F_x \sin \varphi + F_y \cos \varphi + (EI\varphi')' = \rho I \ddot{\varphi} \quad (16)$$

To simplify the solution, this equation is converted into a dimensionless form by scaling the variables (non-dimensionalization). The scaling uses length and time parameters, for example:

$$s = \frac{R_o}{2} \tilde{s} \quad (17)$$

$$t = \frac{R_o}{2} \sqrt{\frac{\rho}{E}} \tilde{t} \quad (18)$$

$$x = \frac{1}{2} R_o \tilde{x} \quad (19)$$

$$y = \frac{1}{2} R_o \tilde{y} \quad (20)$$

which simplifies the calculation and also allows a direct comparison with the experimental results.

After scaling, the rotational equation of motion of the rod becomes Equation (21).

$$\delta^2 \ddot{\varphi} = \delta^2 \varphi'' + \tilde{F}_y \cos \varphi - \tilde{F}_x \sin \varphi \quad (21)$$

where $\delta = \frac{R}{R_o}$. The translational acceleration components are calculated through the derivatives of position with respect to the parameter s , giving the results shown in Equations (22) and (23).

$$\frac{d}{ds} \left(\frac{F_x'}{\delta} \right) = \frac{d^2}{dt^2} \left(\frac{dx}{ds} \right) = \cos \varphi \quad (22)$$

$$\frac{d}{ds} \left(\frac{F_y'}{\delta} \right) = \frac{d^2}{dt^2} \left(\frac{dy}{ds} \right) = \sin \varphi \quad (23)$$

The total acceleration is then expressed in vector form as in Equations (23) and (24).

$$\frac{d^2}{dt^2} \left(\frac{\vec{r}}{\delta} \right) = \frac{d^2}{dt^2} \left(\frac{dx}{ds} \right) \hat{e}_x + \frac{d^2}{dt^2} \left(\frac{dy}{ds} \right) \hat{e}_y \quad (23)$$

$$\frac{d^2}{dt^2} \left(\frac{\vec{r}}{\delta} \right) = (\cos \varphi) \hat{e}_x + (\sin \varphi) \hat{e}_y \quad (24)$$

Equation (24) provides the basis for analyzing the acceleration distribution along the rod and for relating it to the internal forces and the dynamic response of the fishing rod.

The conservation of mechanical energy and angular momentum was examined for four variations of the boundary conditions of the fishing rod. This analysis aims to determine how differences in constraints at the ends of the rod affect the conservative properties of energy and angular momentum.

In Case I, the rod is rigidly clamped at both ends so that it cannot bend, translate, or rotate at its ends. This condition makes the system closed, so that the mechanical energy and angular momentum are entirely stored in the rod. The conservation of mechanical energy is expressed by the equation $\frac{d}{dt} \mathcal{H}(t) = 0$, whereas the conservation of angular momentum is expressed by $\frac{d\mathcal{L}}{dt} = F_y(a) \int_a^b \cos \varphi ds + F_x(a) \int_a^b \sin \varphi ds$.

In Case II, the base of the rod is held fixed while the other end is left free, so that it can bend or move freely. The presence of a constant external force makes the system open, and energy can enter the rod continuously. The conservation of mechanical energy is expressed by the equation $\mathcal{H}(t) = -\alpha x(a, t) + \mathcal{H}(0)$, whereas the conservation of angular momentum is $\frac{d\mathcal{L}}{dt} = -\delta^2(a) \varphi_s(a) + \alpha y(a)$.

In Case III, the rod is fixed at its base, similar to a rod leaning against a wall, so that the base can neither move nor rotate, whereas the free end can move freely. No external force acts, but the base locks the moment, so the mechanical energy remains constant, while the angular momentum may change because of internal torque. This condition is expressed by the equation $\frac{d}{dt} \mathcal{H}(t) = 0$ for the mechanical energy and $\frac{d\mathcal{L}}{dt} = -\delta^2(a) \varphi_s(a)$ for the angular momentum.

In Case IV, the rod is attached at its base by a pivot hinge, with the free end able to move. Because there is no external force or external moment, both the mechanical energy and the angular momentum are conserved. This is shown by the equations $\frac{d}{dt} \mathcal{H}(t) = 0$ dan $\frac{d\mathcal{L}}{dt} = 0$.

From these four boundary cases, it can be concluded that the constraint conditions of the rod strongly affect the conservation properties of its mechanical energy and angular momentum. Fully closed systems, such as a rod clamped at both ends or one attached by a hinge without external forces, show constant energy and angular momentum. Conversely, the presence of an external force at the end of the rod, as in the case of a fixed base and a free end, causes both the mechanical energy and the angular momentum to change over time. This difference is an important basis for understanding the dynamic response of an elastic rod subjected to external forces under different constraint configurations.

First Experiment

Acceleration was measured in this study using the accelerometer sensor embedded in a smartphone. Smartphones have been widely used as physics experiment tools because they are equipped with various internal sensors capable of recording motion data in real-time. In addition to their relatively low cost, the use of smartphones also facilitates data acquisition and analysis through integration with various data-logging applications (Peters, 2010). Various studies have shown that smartphone sensors can be used to support physics experiments with adequate reliability and allow experiments to be conducted more flexibly and accessibly (Monteiro et al., 2019).

The first experiment was conducted to compare the maximum and minimum acceleration ranges of elastic and non-elastic rods at three fulcrum positions (40 cm, 30 cm, and 20 cm from the end). The load used was a brick weighing 1.21 kg. The experimental design is shown in Figure 3

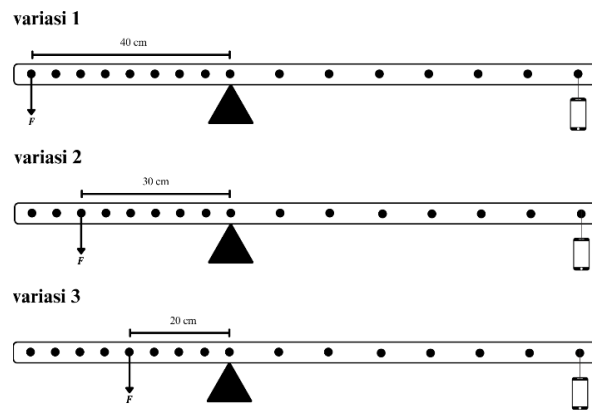


Figure 3. Design of the First Experiment on the Acceleration Multiplication Factor in an Elastic Rod and Its Comparison with a Non-Elastic Rod

The acceleration measurement results of the first experiment are presented in Table 3. The data in Table 3 show that the elastic rod produced a larger acceleration range than the non-elastic rod. In the first variation, the elastic rod reached an acceleration range of 74 m/s^2 , whereas the non-elastic rod reached only 42 m/s^2 .

Let the quantity μ be defined as the ratio of the acceleration range of variation n to that of the next variation $(n+1)$. The quantity μ can serve as a measure of the acceleration multiplication factor. Mathematically, this relationship can be expressed by Equation (25).

$$\mu_n = \frac{\Delta a_n}{\Delta a_{(n+1)}} \quad (25)$$

where,

Δa_n : acceleration range for variation n (m/s^2)

$\Delta a_{(n+1)}$: acceleration range for variation $(n + 1)$ (m/s^2)

Calculation of the acceleration multiplication factor (μ) shows that for the elastic rod, $\mu_1 \approx 1.57$ and $\mu_2 \approx 1.58$ were obtained, whereas for the non-elastic rod, $\mu_1 \approx 1.24$ and $\mu_2 \approx 1.46$ were obtained. This difference indicates that the elastic rod is able to store more elastic energy, thereby producing higher oscillation acceleration than the non-elastic rod.

Table 3. Comparison of the Maximum and Minimum Acceleration Range Values for Elastic and Non-Elastic Rods

Variation	Elastic Rod			Non-Elastic Rod		
	a_{min} (m/s^2)	a_{max} (m/s^2)	Δa (m/s^2)	a_{min} (m/s^2)	a_{max} (m/s^2)	Δa (m/s^2)
1	-16	58	74	-24	18	42
2	-25	22	47	-18	16	34
3	1.2	31	29.8	-6.3	17	23.3

Second Experiment

The second experiment was designed to examine the acceleration multiplication factor under different jerking forces (gentle and strong) and different locations of the acceleration sensor (the base and the tip of the rod). The experimental design is shown in Figure 4.

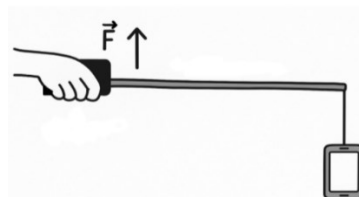


Figure 4. Design of the Second Experiment on the Acceleration Multiplication Factor in an Elastic Rod and Its Comparison with a Non-Elastic Rod

The accelerometer orientation was adjusted to the dominant direction of motion: the smartphone at the base of the rod measured the x-axis, whereas the smartphone at the tip of the rod measured the y-axis, since its dominant motion was vertical, as viewed from the side, owing to the hanging effect. The acceleration data for the elastic rod are presented in Table 4, and those for the non-elastic rod in Table 5.

Table 4. Experimental Data Results for an Elastic Rod

Variations	Sensor Location	Minimum (m/s ²)	Maximum (m/s ²)	Range
Gentle jerk	Base (<i>x</i>)	-0.5	13	13.5
Gentle jerk	Tip (<i>y</i>)	-3.1	24	27.1
Strong jerk	Base (<i>x</i>)	-2.7	22	24.7
Strong jerk	Tip (<i>y</i>)	-2.3	46	69

Table 5. Experimental Data Results for a Non-Elastic Rod

Variations	Sensor Location	Minimum (m/s ²)	Maximum (m/s ²)	Range
Gentle jerk	Base (<i>x</i>)	-3.4	12	15.4
Gentle jerk	Tip (<i>y</i>)	-3.2	21	24.2
Strong jerk	Base (<i>x</i>)	-0.8	15	15.8
Strong jerk	Tip (<i>y</i>)	-5.1	26	31.1

For the elastic rod, a strong jerk produced the highest acceleration range, 69 m/s² at the tip of the rod, whereas for the non-elastic rod the maximum acceleration range reached only 31.1 m/s². The acceleration multiplication factor was calculated from the ratio of the acceleration ranges at the tip and the base of the rod using Equation (26) as follows,

$$\mu = \frac{\Delta a_{tip}}{\Delta a_{base}} \quad (26)$$

Where:

- μ : ratio of the acceleration ranges at the base and the tip of the rod (dimensionless)
 $\Delta a_{(n+1)}$: acceleration range at the base of the rod (m/s²)
 Δa_{tip} : acceleration range at the tip of the rod (m/s²).

The calculation results show that the elastic rod has a higher multiplication factor ($\mu_1 \approx 1.57$; $\mu_2 \approx 1.58$) than the non-elastic rod ($\mu_1 \approx 1.24$; $\mu_2 \approx 1.46$). This trend is consistent with the first experiment, reinforcing the conclusion that the elastic properties of the rod significantly affect the magnitude of the acceleration at the rod tip. The more elastic the rod, the greater the difference in acceleration between the tip and the base when a jerking force is applied.

E-Module Development Stage

The e-module was developed through four systematic stages of the 4D model (Define, Design, Develop, and Disseminate) as proposed by Thiagarajan (1974). The results and discussion of each stage are described below.

Define Stage

The define stage aims to identify and formulate the learning needs underlying the development of the e-module. The main activities included:

a) Front-End Analysis

Difficulty in understanding Mechanics concepts remains a major problem for Physics students at the Faculty of Mathematics and Natural Sciences, Universitas Negeri Semarang. The preliminary study showed that 83.3% of students considered this course difficult. To address this problem, an elasticity-themed e-module was developed as an additional learning resource to strengthen the understanding of Mechanics concepts

b) Learner Analysis

The learner analysis covers the characteristics of the students who were the subjects of the study. The learners in this study were students in the physics cluster of the Faculty of Mathematics and Natural

Sciences, Universitas Negeri Semarang, who were taking or had completed the Mechanics course. That is, this study targeted students from the fourth to the eighth semester of the Physics and Physics Education study programs at the Faculty of Mathematics and Natural Sciences, Universitas Negeri Semarang

c) Task Analysis

The task analysis in the development of this elasticity e-module was conducted to map the skills and knowledge that students need to master in order to understand the material thoroughly. The analysis stage included breaking the concept of elasticity down into prerequisite topics comprising the mechanics of elastic bodies, curvature, the inertia tensor, the second moment of area, and the bending moment, which serve as the theoretical foundation for analyzing the response of elastic rods to forces. After mastering the basic material, students were guided to the core discussion, namely the experimental results and their analysis, including the curvature curve of the fishing rod under varying loads and the acceleration multiplication factor in elastic and non-elastic rods. The material was organized systematically with clear learning objectives so that the process of understanding the concepts proceeds gradually and in a guided manner.

d) Concept Analysis

In developing this elasticity e-module, the main concepts were organized systematically to align with learning needs and instructional objectives. Eight main topics were mapped following a logical progression from fundamental material to experimental application, namely the mechanics of elastic bodies, curvature, the inertia tensor, the second moment of area, the bending moment, the physical analysis of the fishing rod curvature curve, the physical analysis of the acceleration multiplication factor in the fishing rod, and the experiment on the acceleration multiplication factor in elastic rods and its comparison with non-elastic rods. The first five topics serve as the theoretical foundation, whereas the last three constitute the focus of applying the concepts in theoretical and experimental contexts. The material focuses on two main topics: (1) the analysis of the curvature curve of a tapered elastic rod, and (2) the comparison of the tip acceleration of elastic and non-elastic rods when subjected to a force impulse

e) Specifying Instructional Objectives

The learning objectives of this elasticity e-module were formulated on the basis of the structure of the developed material and aligned with the cognitive domain of Bloom's taxonomy, so that students can build understanding gradually, from basic concepts to higher-order thinking skills. These objectives include understanding the mechanics of elastic bodies and Hooke's law, analyzing the relationships among stress, strain, and curvature, mastering the concept of the inertia tensor, and applying calculations of the second moment of area and the bending moment. In addition, students are guided to analyze the curvature curve of the fishing rod both theoretically and experimentally, to evaluate the acceleration multiplication factor between elastic and non-elastic rods, and to develop problem-solving skills through practice problems and the analysis of experimental data. This formulation is expected to guide students to master the content of the module systematically while fostering critical thinking skills regarding elasticity phenomena in mechanics

The result of the define stage is a solid framework of development needs together with a detailed mapping of the topics and objectives of the e-module.

Design Stage

The design stage was carried out to develop the initial prototype (blueprint) of the e-module. The activities at this stage included:

a) Preparation of the E-Module Framework

The framework of the e-module "Kupas Tuntas Elastisitas Tongkat Pancing" (A Thorough Examination of Fishing Rod Elasticity) began with the design of a template in Canva, which was then exported to .pptx format for refinement in Microsoft PowerPoint. The structure of the e-module was designed systematically to support students' independent learning and the attainment of the learning objectives, and it consists of three main parts: the introduction, the content, and the learning support section. The introductory part contains the module identity, foreword, table of contents, learning

objectives, and a concept map as an initial guide. The content part focuses on the analysis of the curvature curve of the elastic rod and the comparison of the acceleration multiplication factor between elastic and non-elastic rods, presented contextually on the basis of experiments and physical analysis and complemented by illustrations, graphs, and formula derivations to clarify the concepts. The learning support part contains evaluation questions, an answer key, a glossary, a bibliography, and appendices that support students' understanding. With this structure, the e-module presents the concept of elasticity not only theoretically but also in an applied manner, thereby providing a contextual learning experience that is relevant to real phenomena.

b) Format Selection

The e-module "Kupas Tuntas Elastisitas Tongkat Pancing" was designed in standard A4 size using the Canva platform, emphasizing a clean, structured, and comfortable-to-read visual aesthetic. Its color palette combines shades of purple, blue, and white to create a modern academic impression that is also visually friendly; primary colors such as Light Lavender (#E6E7FF) were used as the background, Blue Violet (#515492) for text elements, and white as the base of the content pages. This design supports readability and gives the module a distinctive visual identity. For typography, the main font used was Times New Roman 12 pt with 1.5 line spacing for the module content, whereas chapter titles were enlarged to 16 pt. On the cover, modern sans-serif fonts were used, namely Nunito Bold for institutional information and League Spartan for the main title of the module, so that the appearance remains professional while still being attractive.

c) Preparation of the Evaluation Instrument

The evaluation instrument of this e-module was prepared on the basis of the material presented and aligned with the learning objectives, taking into account practicality, measurability, and variety of questions. The evaluation consists of a combination of objective questions, in the form of multiple-choice and true-false items, to measure conceptual understanding quickly, and subjective questions, in the form of essays, designed to hone critical thinking skills, argument construction, and in-depth analysis. The combination of these two question formats provides holistic coverage of assessment, enabling students to test their mastery of factual knowledge as well as their analytical abilities through varying levels of difficulty arranged in a graded manner.

The result of the design stage is the initial (draft) version of the e-module together with the complete evaluation instrument.

Develop Stage

The develop stage focuses on producing the draft e-module, expert validation, and revision of the e-module to ensure its feasibility and effectiveness. The detailed steps include:

a) Preparation of the E-Module Draft

After the e-module framework had been designed, the next step was to prepare the complete draft based on the established structure. This process involved filling in the content systematically, from the introduction and the main material to the evaluation, with a focus on the alignment among the material, the learning objectives, and pedagogical aspects. The researcher adjusted the language, integrated visual elements, and ensured that the appearance was attractive while remaining easy for students to understand. The draft was prepared in Microsoft PowerPoint, adapted from the Canva template for flexibility in layout and in inserting elements such as illustrations, graphs, mathematical formulas, and evaluation questions. This draft served as the initial representation of the e-module before the validation stage, while also ensuring the completeness and coherence of its content.

b) Subject-Matter and Media Expert Validation (E-Module Feasibility Test):

The validation of the e-module "Kupas Tuntas Elastisitas Tongkat Pancing" was conducted simultaneously on 28 April 2025 and involved five validators, consisting of three subject-matter expert validators and two media expert validators. The validators were selected on the basis of the suitability of their competence and the recommendation of the supervising lecturer in order to ensure the quality and validity of the developed e-module. This validation was carried out to assess the feasibility of the content, presentation, and language, as well as media aspects such as visual design and interactivity. Meanwhile, the media expert validation instrument covered five aspects, namely visual design,

interactivity and navigation, readability and typography, aesthetics and professionalism, and technical consistency. The results of the subject-matter and media expert validation are presented in Table 6 and Table 7.

The validation data in Table 6 and Table 7 were then analyzed using the SBI and Mi approach. From the results of this analysis, the feasibility category of the e-module can be determined with reference to the established quantitative score ranges. The quantitative score ranges for the subject-matter expert and media expert assessments are presented in Tables 8 and 9, whereas the validation results of the subject-matter and media experts are shown in Tables 10 and 11.

Table 6. Summary of Subject-Matter Expert Validation Results

No.	Validation Aspect	Validator 1	Validator 2	Validator 3
1.	Content feasibility and material appropriateness	36	38	39
2.	Presentation feasibility	9	9	9
3.	Language feasibility	29	29	29

Table 7. Summary of Media Expert Validation Results

No.	Validation Aspect	Validator 1	Validator 2
1.	Visual design	18	20
2.	Interactivity and navigation	10	8
3.	Readability and typography	10	10
4.	Aesthetics and professionalism	8	10
5.	Technical consistency and accuracy	9	8

Table 8. Quantitative Score Ranges and Assessment Categories for Subject-Matter Experts

Quantitative Score Range			Category
Content feasibility and material appropriateness	Presentation feasibility	Language feasibility	
$x \geq 96$	$x \geq 24$	$x \geq 72$	Very Good
$96 > x \geq 72$	$24 > x \geq 18$	$72 > x \geq 54$	Good
$72 > x \geq 48$	$18 > x \geq 12$	$54 > x \geq 36$	Fair
$48 > x$	$12 > x$	$36 > x$	Poor

Table 9. Quantitative Score Ranges and Assessment Categories for Media Experts

Quantitative Score Range					Category
Visual design	Interactivity and navigation	Readability and typography	Aesthetics and professionalism	Technical consistency and accuracy	
$x \geq 32$	$x \geq 16$	$x \geq 16$	$x \geq 16$	$x \geq 16$	Very Good
$32 > x \geq 24$	$16 > x \geq 12$	$16 > x \geq 12$	$16 > x \geq 12$	$16 > x \geq 12$	Good
$24 > x \geq 16$	$12 > x \geq 8$	$12 > x \geq 8$	$12 > x \geq 8$	$12 > x \geq 8$	Fair
$16 > x$	$8 > x$	$8 > x$	$8 > x$	$8 > x$	Poor

Table 10. Categorization of Subject-Matter Expert Validation Results

No.	Validation Aspect	Score	Category
1.	Content feasibility and material appropriateness	113	Very Good
2.	Presentation feasibility	27	Very Good
3.	Language feasibility	87	Very Good

Table 11. Categorization of Media Expert Validation Results

No.	Validation Aspect	Score	Category
1.	Visual design	38	Very Good
2.	Interactivity and navigation	18	Very Good
3.	Readability and typography	20	Very Good
4.	Aesthetics and professionalism	18	Very Good
5.	Technical consistency and accuracy	17	Very Good

Overall, the results of the subject-matter and media expert validation place this e-module in the “Very Good” feasibility category, so that it is declared feasible for use as supplementary teaching material for learning elasticity in the mechanics’ course. This finding also served as the basis for a minor revision stage before the e-module was published and tested with students.

c) Product Revision

After the validation by the subject-matter and media experts, the e-module was revised on the basis of the validators’ comments and suggestions. This revision aimed to refine both the content and the appearance so that the e-module would be more suitable for use as supplementary teaching material. Through this revision, the e-module was declared to meet the feasibility standards in terms of both material and media and to be ready for use as a supplement for learning elasticity in the mechanics course in the Physics and Physics Education study programs at the Faculty of Mathematics and Natural Sciences, Universitas Negeri Semarang. The revisions were made on the basis of the validators’ input, such as simplifying sentences, adding illustrative simulations, and improving navigation between pages.

The final result of the develop stage is the final e-module “Kupas Tuntas Elastisitas Tongkat Pancing,” which has been validated and is ready to be disseminated to students as research subjects.

Disseminate Stage

The e-module that had been validated by the experts was then tested on a limited basis with students to obtain direct responses from them as end users. The purpose of this stage was to assess the extent to which the e-module is useful, easy to use independently, and visually appealing. The questionnaire was administered on 26–28 May 2025, with a total of 38 respondents from the Physics (23.7%) and Physics Education (76.3%) study programs. The respondents consisted of fourth-semester (5.3%), sixth-semester (34.2%), and eighth-semester (60.5%) students, so the majority already had an academic background relevant to the content of the e-module

The questionnaire instrument covered six main aspects, namely material, language, visual display, usefulness/benefit, technical aspects, and evaluation. The assessment of each aspect included the relevance of the material, the readability of the language, visual aesthetics, benefits for independent learning, ease of technical access, and the effectiveness of the evaluation. The summary of student responses on these six aspects is presented in Table 12.

Table 12. Summary of Student Response Scores for the E-Module

No.	Assessment Aspect	Percentage (%)	Category
1.	Material	90.53	Very Good
2.	Language	91.58	Very Good
3.	Visual Display	88.84	Very Good
4.	Usefulness/Benefit	91.89	Very Good
5.	Technical Aspects	91.75	Very Good
6.	Evaluation	90.53	Very Good

Based on Table 12, all assessment aspects obtained the “Very Good” category. The usefulness/benefit aspect obtained the highest percentage at 91.89%, followed by the technical (91.75%),

language (91.58%), material (90.53%), evaluation (90.53%), and visual display (88.84%) aspects. These findings indicate that the e-module meets good quality criteria in terms of content, language, visual, and technical aspects, and is therefore suitable for use as a supplement for learning in the Mechanics course.

In addition to the quantitative assessment, the questionnaire also included a comment section for students. Most of the comments were positive, highlighting the attractive appearance of the e-module, its comprehensive content, and its easily understood language. Several suggestions for improvement were proposed, including adding color variation, using more attractive and relevant illustrations, increasing font contrast, including the supervising lecturer's name on the cover, and adding explanations to the answer key of the questions. This feedback is constructive and serves as a basis for further refinement without changing the main substance of the e-module.

The disseminate stage ensured that the e-module is beneficial for the research subjects in supporting the learning of elasticity in a contextual and applied manner.

Conclusion

This study produced an elasticity e-module themed on the fishing rod using the 4D model approach. The curvature analysis showed that the theoretical model $y(s) = A \sin(as)$, expanded into a sixth-degree polynomial, is in high agreement with the experimental data. The experimental results also revealed that the elastic rod exhibits a larger acceleration multiplication factor than the non-elastic rod, owing to the role of elastic strain in transferring energy as waves. The developed e-module was validated by experts and received a very good response from students; therefore, it is suitable for use as a supplement for mechanics learning.

Future studies are advised to include a quantitative analysis of the deviation between the experimental and theoretical curves to improve objectivity. Integrating video documentation of the experiments into the e-module through links or QR codes would also strengthen the visual and contextual aspects of learning. In addition, exploring elastic rods made of different materials is recommended to extend the study of the influence of the mechanical properties of the material on the dynamic response of the rod.

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