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The Effectiveness of Cramer's Rule on the System of Linear Equations (LES) of 2-Loop Electrical Circuits in Improving Mathematical Thinking Ability and Learning Outcomes

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Abstract

Mathematical thinking skills are high-level thinking competencies needed in the 21st century. Many physics problems are expressed in various forms of mathematical equations that require mathematical thinking skills. The low level of mathematical thinking has an impact on physics learning outcomes. Cramer's rule is more attractive to students when solving the Linear Equation System (LES) than the substitution method or other methods. So this study aims to examine the effectiveness of Cramer's rule on the system of linear equations of 2-loop electrical circuits as an alternative solution to problem solving. The research was conducted in the odd semester of 2024/2025 with research subjects as many as 44 students and 30 high school students in Jember using Pre-Experimental Design and One Group Pretest-Posttest form. The results showed the improvement of college students' mathematical thinking skills in the high category for indicators of specializing (72.7%), generalizing (95.5%), and conjecturing (86.4%) while the convincing indicator was in the medium category (65.9%). The improvement of college student learning outcomes (79%) and high school students (81%) in the high category with a very positive response or 82.14%. So Cramer's rule is effective in improving mathematical thinking skills and learning outcomes.

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INTRODUCTION

21st century learning is a learning process that represents experience and reflects conditions and situations in the future (futuristic) so that the problems learned are problems that will be experienced by students in the process of living in the future. Therefore, the learning process in 21st century learning must integrate knowledge with creativity, communication, critical thinking, and collaboration to equip learners with the skills needed in work and life (Peña-Ayala, 2021). Engaging learners in an interactive learning process and applying certain mathematical methods in solving mathematically-assisted physics problems is an effective strategy to train them to think critically, improve their ability to filter and evaluate information and problems, and solve problems holistically (Harahap, 2023). This strategy aims to hone learners' mathematical abilities and develop 21st century skills (Szabo et al., 2020). Three main aspects of 21st learning are: (1) learning skills, including creativity, innovation, critical thinking, problem-solving, communication, and collaboration; (2) literacy skills, consisting of information literacy, media literacy, and information and communication technology (ICT) literacy; and (3) life skills, such as flexibility and adaptability (González-Pérez & Ramírez-Montoya, 2022). In addition, 21st century education also focuses on developing higher-order thinking skills, including mathematical thinking skills, which are one of the important competencies in this era (Rizki & Priatna, 2019).

Mathematical thinking is the mental activity of students when solving problems is an indispensable aspect of learning and is very influential in problem solving which has an impact on learning outcomes. Mathematical thinking involves all skills such as logical and analytical thinking and quantitative reasoning so that it is not only thinking related to numbers and abstract mathematical concepts (Devlin, 2021). Mathematical thinking ability is the ability to understand a mathematical concept, solve mathematical problems, and construct a theory or problem by applying it to a mathematical model (Onal et.al. 2017). Mathematical thinking involves exploring, questioning, working systematically, visualizing, conjecturing, explaining, generalizing, justifying, and proving (Devlin, K., 2021). This is an important component of learners' cognitive activity, therefore the development of mathematical thinking in students is very important to shape their competencies and is a component of cognitive activity (Sadieva, M., 2020). Mathematical thinking skills are needed to help learners solve complex problems, make data-based decisions, and innovate in various fields.

Physics is closely related to mathematics because many physics problems or phenomena are expressed in various forms of mathematical equations. Mathematical thinking ability is a very important ability in physics because mathematical thinking ability is the ability to describe the relationship between quantities of measurement results in tables or graphs and the ability to formulate a conclusion (Geyer & Kuske-JanBen, 2019). Physics problems are formulated in the form of simple mathematical equations to complex mathematical equations (Siombone & Niwele, 2023). So understanding physics is very dependent on the ability to think mathematically (Woitkowski, 2020). Based on PISA 2019 data, only 1% of Indonesian students can model complex situations mathematically, select, compare, and evaluate appropriate problem-solving strategies, far below the global average of 11% and below the average percentage of other countries such as China 44%, Singapore 37%, Hong Kong 29%, Macao 28%, Taipei 23% and Korea 21% (OECD, 2019). This low mathematical thinking ability hurts student learning outcomes, including in physics subjects. Kawuri et al. (2019), stated that as many as 60% of students have not met the Minimum Completeness Criteria (MCC) in physics subjects.

One of the physics materials that has low learning outcomes because it is considered difficult and requires high mathematical ability is dynamic electricity. The average value of dynamic electricity learning outcomes is very low, namely 34 (Utusan & Buhungo, 2023) The biggest factor causing low learning outcomes in dynamic electricity is that students have difficulty in completing mathematical calculations (Halim & Lestari, 2019). The application of Kirchoff's law in electrical circuits (dynamic electricity) produces a Linear Equation System (LES) of at least two variables known as a System of Linear Equations with Three Variables (SLETV). Solving problems about electric current and voltage in integrated circuits results in complex simultaneous equations with various variables (Batarius & Samane, 2021). Generally, solving the LES of electrical circuits uses the method of substitution and elimination. This method is quite effective for a Two-Variable Linear Equation System (TVLES) and less effective for LES with more than 2 variables because it is not time-efficient and has a high error rate (Ilmi et al., 2023).

A 2-loop electrical circuit produces a System of Linear Equations of Three Variables

(LESTV). LES with ≥3 variables can be expressed in the form of matrix multiplication.

$$\begin{aligned} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{aligned} \Rightarrow A\varphi = B \quad (1)$$

with

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}; \varphi = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } b = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Several methods can be used to solve n-variable LES. Batarius & Samane, (2021) showed that the Gauss-Jordan method can solve the 5-loop Electrical circuit at the 5th iteration and 11th Step. The Matlab-based OBE method also successfully calculates the amount of current in an electrical circuit (Ilme et al., 2023). The Cayley-Hamilton theorem and eigen characteristic equation can calculate the amount of electric current in a 2-loop and 3-loop electrical circuit (Supriadi et al., 2024). shows that the application of the Cayley-Hamilton theorem and the third-order eigen matrix characteristic problem can be used to solve the amount of current flowing in a two-loop electrical circuit. Luo et al. (2021) stated that Cramer's rule is a good choice for solving inhomogeneous LES because each unknown variable is calculated independently compared to the substitution-elimination and Gauss-Jordan elimination methods.

The solution of LES using Cramer's rule consists of 3 steps, namely:

(1) Transform the LES into matrix multiplication as in equation (1).

(2) Determine the determinants of matrices A, E, F and G, with

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad E = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$F = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} \quad G = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

(3) Determine values of variables x, y, and z with Cramer's rule (Alhasan, 2021)::

$$x = \frac{\det E}{\det A}, y = \frac{\det F}{\det A}, z = \frac{\det G}{\det A}$$

Maharani (2020) stated that Cramer's rule is 75% more desirable to use in solving SPL than the substitution and elimination methods. The application of Cramer's rule in solving 2-loop electrical circuits analytically needs to be seen for its effectiveness so that it can be used as an alternative method of solving problems in the learning process. Effectiveness describes the extent to which an action or method can achieve optimal results (Norman et al., 2022). So the purpose of this study is to apply and measure the effectiveness of using Cramer's rule in solving the system of linear equations in 2-loop electrical circuits in terms of improving mathematical thinking skills and learning outcomes and student responses.

METHOD

The research was conducted at Jember High School and Jember University in the odd semester of the 2024/2025 academic year. The research design was a Pre-Experimental Design with the form of One Group Pretest-Posttest as shown in Table 1.

Table 1. One Group, Pretest and Posttest Design

Group	Pre Test	Treatment	Post Test
Eksperimen	O_1	X	O_2

(Source: Manly, 1992)

Description:

O_1 = Giving the initial test to the experimental class

O_2 = Giving the final test to the experimental class

X = Learning with the implementation of Cramer's rule on Linear Equation System (LES) 2 Loop Electrical Circuit

Mathematical thinking ability is reviewed from 4 indicators including, specializing, generalizing, conjecturing, and convincing (Ferdianto et al. 2022). The 4 indicators of mathematical thinking were assessed from the results of the pretest and posttest with the help of 5 essay questions. Data were analyzed using the Normalized Gain (g) test with equation (3)

$$N - Gain \langle g \rangle = \frac{N_{post} - N_{pre}}{N_{max} - N_{pre}} \quad (3)$$

Description:

N_{post} = Average posttest score

N_{pre} = Average value of pretest

N_{max} = Maximum value

The effectiveness criteria based on the N-Gain <g> test results use categories of Table 2.

Table 2. N-Gain Level Category

Limits	Category
<g> > 0.7	High
0.3 ≤ <g> < 0.7	Medium
<g> < 0.3	Low

(Source : Yustina et al., 2020)

In addition to seeing the improvement of mathematical thinking and learning outcomes of students, researchers also provided a response questionnaire to find out students' responses to Cramer's rule in a 2-loop electrical circuit, which consists of 5 indicators, namely interest, motivation, readability, satisfaction, and response (Andriani et al., 2021). The response questionnaire uses the answer criteria according to the Likert scale in Table 3.

Table 3. Response result score criteria

Criteria	Score
Strongly agree	5
Agree	4
Undecided	3
Disagree	2
Strongly disagree	1

(Source: Sugiyono, 2016)

The data from the learner response questionnaire is converted into a percentage with equation (4) with the categories in Table 4.

$$Precentage = \frac{total\ score\ obtained}{highest\ score} x\ 100\% \quad (4)$$

Table 4. Categories of Percentage of Response Result

Percentage (%)	Category
80% < P ≤ 100%	Very positive
60% < P ≤ 80%	Positive
40% < P ≤ 60%	Moderately positive
20% < P ≤ 40%	Less positive
0% < P ≤ 20%	Not very positive

(Source: Arikunto, 2010)

RESULT AND DISCUSSION

Cramer's Rule for the LES of a 2-Loop Electrical Circuit

The 2-loop electrical circuit is solved by applying Kirchoff's 1st law, $\Sigma I_{in} = \Sigma I_{out}$ and Kirchoff's 2nd law, $\Sigma\varepsilon + \Sigma IR = 0$. From the electrical circuit of Figure 1.

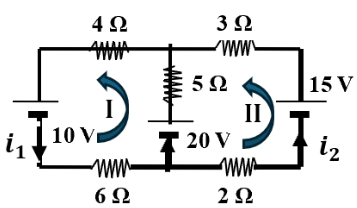

Figure 1. 2-Loop Electrical Circuit

The following LES is obtained:

(i) $2I_1 + I_3 = 2V$

(ii) $I_2 - I_3 =- 1V$

(iii) $I_1 - I_2 - I_3 = 0$

Equation (5) above can be expressed in the form of 3rd order matrix multiplication as follows:

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad (6)$$

Description

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}; x = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \ \&\ B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

The values $I_1, I_2, and\ I_3$ are obtained using Cramer's rule, i.e.:

$$I_1 = \frac{\det A_1}{\det A}, I_2 = \frac{\det A_2}{\det A}, I_3 = \frac{\det A_3}{\det A}$$

Description

$$\det A = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 5$$

$\det A_1 =$ determinant of matrix A with the first column replaced by column matrix B

$$= \begin{vmatrix} 2 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & -1 \end{vmatrix} = 3$$

$\det A_2 =$ determinant of matrix A with the second column replaced by column matrix B

$$= \begin{vmatrix} 2 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & -1 \end{vmatrix} =- 1$$

$\det A_3 =$ determinant of matrix A with the third column replaced by column matrix B

$$= \begin{vmatrix} 2 & 0 & 1 \\ -1 & 1 & -1 \\ 0 & -1 & -1 \end{vmatrix} = 4$$

The determinant of a 3rd order matrix can use the (1) Sarrus method; (2) Laplace Expansion (cofactor); and (3) Gauss Elimination (Operations of Row Elementary or ORE). So that we get

$$i_1 = \frac{3}{5}A; \quad i_2 =- \frac{1}{5}; \text{ and } i_3 = \frac{4}{5}A$$

Mathematical Thinking Ability

The results and discussion of students' mathematical thinking skills seen from the pretest answers (A1) and posttest answers (A2) using Cramer's rule on each indicator, namely: (1) Specializing; (2) Generalizing; (3) Conjecturing; dan (4) Convincing.

Specializing

Students are considered to meet the specializing indicator well if students can identify problems by writing down what information is known and asked in the problem (Ferdianto et al., 2022a). Writing information from the problem in problem-solving is very influential because it will have an impact on the next step. Specializing ability in problem-solving can be seen from the answers of A1 and A2 in Figure 2 and Figure 3.

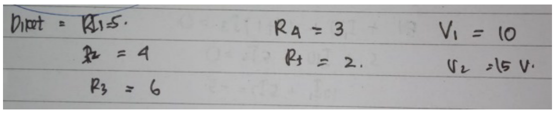

Figure 2. Answer A1 (Pretest)

Based on Figure 2, students are able to identify problems quite well. This is indicated by writing the values of V their units correctly. However, in answer A1, students do not include information about problem asked in the problem.

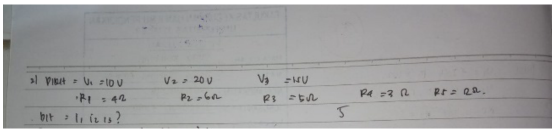

Figure 3. Answer A2 (Postest)

Based on Figure 3, students can identify problems well. This is shown by writing the values of and , the values and and their units correctly. Students also write information about the problem asked in the problem. The mathematical thinking ability of students on the Specializing indicator in one class is shown in Figure 4.

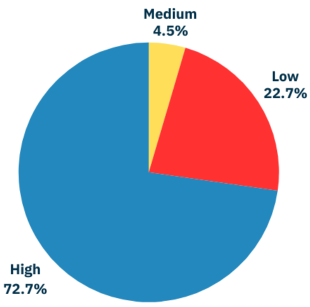

Figure 4. Percentage improvement Specializing

Based on Figure 4, the increase in specializing indicators from 44 students has a percentage of 22.7% in the low category, 4.5% of students in the medium category, and 72.7% of students reaching the high category.

Generalizing

Learners are considered to meet the generalizing indicator well if students can explain the ideas made and expand the scope of the results obtained (Iswari et al., 2019). In solving electrical circuit problems, students are considered to meet the generalizing indicator when students can determine the direction of loop 1 and loop 2, write the loop 1 and loop 2 equations based on Kirchoff's Law II, and write Kirchoff's Law I equation. Generalizing ability in problem-solving can be seen from the answers of A1 and A2 in Figure 5 and Figure 6.

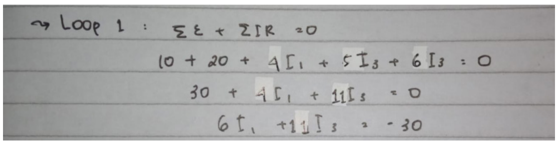

Figure 5. Answer A1 (Pretest)

Based on Figure 5, students are categorized as less able to explain ideas and expand results obtained from the first indicator (specializing). This is indicated by learners lacking in describing loop analysis, Kirchoff's Law I, and Kirchoff's Law II. In Figure 5, students are only able to determine the direction and equation of loop 1.

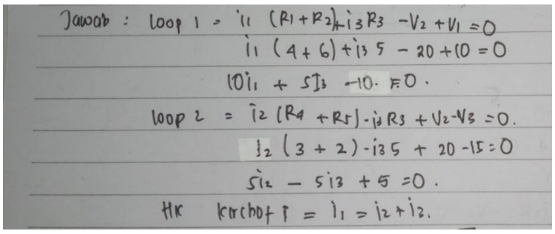

Figure 6. Answer A2 (Postest)

Based on Figure 6, students are categorized as able to explain ideas and expand the results obtained from first indicator (specializing). This is indicated by students being able to explain loop analysis, Kirchoff's Law I, and Kirchoff's Law II. In Figure 4 learners can determine direction of loops 1 and 2, able to analyze Kirchoff's Law 1, and analyze Kirchoff's Law II. The mathematical thinking ability of students on the Generalizing indicator in one class is shown in Figure 7.

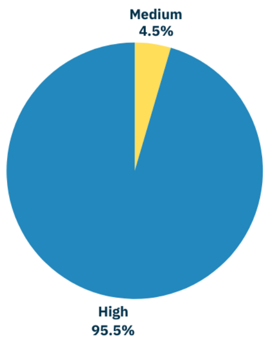

Figure 7. Percentage increase Generalizing

Based on Figure 7, the increase in generalizing indicators from 44 students has a percentage of 4.5% in the medium category and 95.5% of students reach the high category.

Conjecturing

The third indicator, namely conjecturing, is an important indicator of mathematical thinking ability because this indicator shows the ability of students to determine solutions by involving mathematical formulas or methods to solve problems in the given problem (Isa, N., & Ibrahim, H., 2023). Learners are categorized as meeting the Conjecturing indicator if students can convert LES in a 2-loop electrical circuit into a matrix multiplication form and use Cramer's rule as a solution step. The ability of students before the use of Cramer's rule in solving LES in 2-loop electrical circuits can be seen in Figure 8.

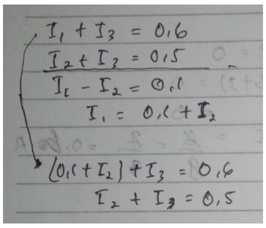

Figure 8. Answer A1 (Pretest)

Based on Figure 8, students are still unable to solve LES on 2-loop electrical circuits using the substitution-elimination method properly, this is indicated by most students only writing linear equations but not writing the solution process to find the information asked, a small number of students have written the substitution-elimination method but the answers written are still wrong. The ability of students after the use of Cramer's rule in solving LES on 2-loop electrical circuits can be seen in Figure 9.

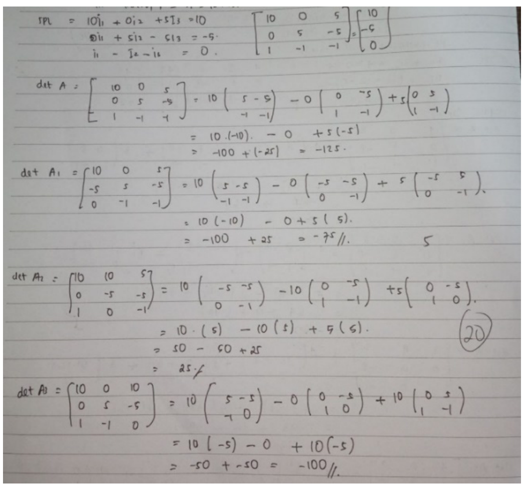

Figure 9. Answer A2 (Postest)

Based on Figure 9, students can solve 2-loop electrical circuit problems using Cramer's rule. This is addressed by students being able to convert LES in the form of matrix multiplication (matrix A) and find the information asked by Cramer's rule. The completion step of the 2-loop electrical circuit using Cramer's rule is calculated by dividing the determinant of matrix A by the determinant of the matrix in question, for example, to find the value of calculated by dividing the determinant of matrix A by the determinant of matrix A1 (column 1 is replaced by column b), to find the information in question also has the same solution steps. The mathematical thinking ability of students of conjecturing indicators in one class is shown in Figure 10.

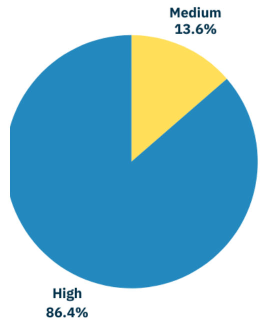

Figure 10. Percentage increase in conjecturing

Based on Figure 10, the increase in conjecturing indicators in one class with 44 students has a percentage of 15% reaching the medium category and 85% reaching the high category.

Convincing

Convincing indicator is the last indicator to convince about the pattern of the results obtained (Ferdianto et al., 2022b). Learners are categorized as meeting the Convincing indicator if students can provide conclusions about the information in question equipped with the correct units. The ability of students in the Convincing indicator can be seen in Figure 11 and Figure 12.

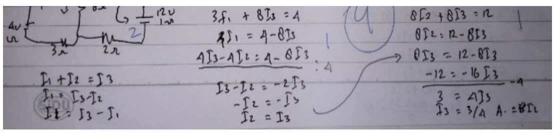

Figure 11. Answer A1 (Pretest)

Based on Figure 11, A1's answer before applying Cramer's rule to the completion of the 2-loop electrical circuit, students still do not fulfill the Convincing indicator, this is indicated by A1's answer only writing the final result of the calculation without making a conclusion equipped with the unit.

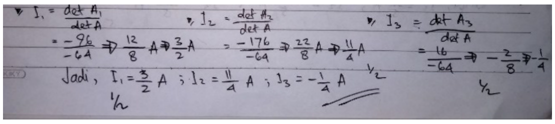

Figure 12. Answer A2 (Postest)

Based on Figure 12, A2's answer after applying Cramer's rule to the completion of the 2-loop electrical circuit, students are categorized as being able to fulfill the indicator. This is indicated by the answer A2 has been able to make a conclusion about the information asked, namely the strong current (I) equipped with the correct unit. The mathematical thinking ability of students on Convincing indicators in one class is shown in Figure 13.

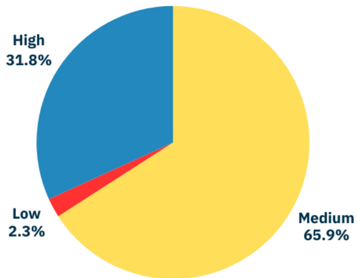

Figure 13. Percentage improvement Convincing

Based on Figure 13, the increase in Convincing indicators in one class with 44 students has a percentage of 2.5% in the low category, 72.5% in the medium category, and 25% reaching the high category.

Overall, the increase in mathematical thinking with the results of the N-Gain analysis was 0.81 with a high category, this is shown in Figure 14.

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
NGain	4	.69	.88	.8100	.09208
Valid N (listwise)	4				

Figure 14. N-Gain SPSS

This proves that using Cramer's rule to solve LES on 2-loop electrical circuits effectively improves students' mathematical thinking skills. The steps of Cramer's rule in solving the LES of a 2-loop electrical circuit involve various important aspects of mathematics (matrices), starting to classify coefficients, variables, and results in matrix notation, determining determinants, and applying Cramer's rule in a structured and sequential manner without cascading and repetitive. This is also to the research of Kazunga, C., & Bansilal, S., (2020) which states that Cramer's rule in solving LES requires analysis and application of rules in a structured manner by involving systematic steps in calculating the determinant of the main matrix and modified matrix, to train students' accuracy, problem-solving skills, and reasoning. According to Blaschke, L. M., & Hase, S. (2016), Cramer's rule gives learners flexibility in choosing the method for determining the determinant of the matrix (Sarrus method, cofactor expansion method, row elimination, or Gauss elimination) to allow exploration and application of strategies that suit their preferences, freedom in choosing methods provides room for creativity and increases confidence in solving problems. Thus, Cramer's rule not only teaches the technique of solving linear equations, but also trains logic, analysis, creativity, and flexibility of thinking in mathematical problem solving and has an impact on improving students' mathematical thinking skills. This is in line with the research of Puspitasari, et al (2018) which explains that students who have high mathematical thinking skills show fluency, flexibility, and originality in solving problems.

Learning Outcomes

The improvement of physics learning outcomes on two-loop electrical circuit material is measured through pretest and posttest. The pretest was conducted before learning by using Cramer's rule, while the posttest was conducted after the implementation of Cramer's rule in learning two-loop electrical circuit material. The following is a recapitulation of the limited trial results.

Table 5. College Student Learning Outcome Data

Description	Value	
	Pretest	Posttest
Number of College Students	44	44
Lowest Score	15	73
Lowest Score	42	100
Average Score	19.9	83.7
Average N-Gain	0.79	
Category	High	

Table 5. shows that the lowest pretest score was 15, while the highest pretest score was 42. For the post-test score, the lowest score was 73 and the highest score reached 100. The average pretest score was 19.9, while the average posttest

score was 83.7. This data shows that the average post-test score is higher than the average pretest score. Based on the results of the N-Gain test analysis, an average score of 0.79 was obtained, which indicates that the increase in college student physics learning outcomes on the material of two-loop electrical circuits after the implementation of Cramer's rule is included in the high category.

Table 6. Data on student learning outcomes

Description	Value	
	Pretest	**Posttest**
Number of Learners	30	30
Lowest Score	10	76
Lowest Score	30	96
Average Score	16.8	84.9
Average N-Gain	0.81	
Category	High	

Table 6 shows that the lowest pretest score was 10, while the highest pretest score was 30. For the post-test score, the lowest score was 76 and the highest score was 96. The average pretest score was 16.8, while the average posttest score was 84.9. This data shows that the average post-test score is higher than the average pretest score. Based on the results of the N-Gain test analysis, an average score of 0.81 was obtained, which shows that the increase in students' physics learning outcomes on the material of two-loop electrical circuits after the implementation of Cramer's rule is included in the high category ().

Mathematical thinking skills can improve students' learning outcomes in physics, especially the 2-loop electrical circuit material, because this concept relies heavily on logic, analysis, and solving systems of linear equations. In studying 2-loop electrical circuits, students are involved in mathematical relationships between current, voltage, and resistance in various branches of the circuit. Problems in electrical circuits are often modeled in the form of a system of linear equations that can be solved using Cramer's rule. Based on the results of the analysis of learning outcomes in high school and college students, it can be proven that Cramer's rule can improve the learning outcomes of students, especially in the material of 2-loop electrical circuits. This is by research conducted by Simamora (2022) which states that matrices can be used to make it easier to determine the solution to a system of linear equations. Apart from that, Hyndriuk, V., & Yurchenko, N. (2022) stated that Cramer's rule can improve students' mathematical skills. Supported by Saraswati et al. (2020) also stated that basic mathematics abilities influence physics learning outcomes.

This research shows that Cramer's Rule on 2-loop electrical circuits can improve the quality of science learning, especially physics. Physics often involves solving systems of linear equations, such as in Kirchoff's Law on electrical circuits. Common methods such as substitution and elimination are often inefficient, especially for equations with more than two variables. By applying Cramer's Rule, students can solve the system of equations more systematically and quickly, thus improving their understanding of the basic concepts of electricity. This application also helps students in developing mathematical thinking ability as evidenced by the improvement of specialization, generalization, and analysis skills, which contribute directly to solving science problems more effectively. In addition, this study showed that this method can significantly improve physics learning outcomes, proving that the integration of mathematical methods in science can accelerate the understanding of complex concepts. The integration of this mathematical approach makes physics concepts easier to understand and apply in real life.

Mathematical thinking ability and learning outcomes have a close relationship in science learning. Mathematical thinking skills help students understand science concepts in depth, improve analytical skills, and facilitate solving scientific problems, which according to Syaiful et al (2021) states that students with good analytical thinking skills tend to have better science process skills. Meanwhile, optimal learning outcomes indicate the effectiveness of the science learning process which is supported by various factors, including students' mathematical abilities. Therefore, strengthening mathematical thinking skills in education is needed to improve the quality of science learning. This is in accordance with Hakim, D (2023) also stated that mathematical ability has a considerable influence on the overall value of science learning outcomes.

Response

The response of students is obtained through a response questionnaire after students are introduced to how to solve the LES of a 2-loop electrical circuit using Cramer's rule. The analysis of response data for 5 indicators is presented in Table 7.

Indicator	Percentage (%)	Category	Average/ Category
Interest	80.84	Very Positive	82.14 % Very Positive
Motivation	80.24	Very Positive	
Readability	80.72	Very Positive	
Satisfaction	84.05	Very Positive	
Response	84.89	Very Positive	

Based on the data in Table 7, it can be seen that the student's response to the use of Cramer's rule in solving the LES of Electrical circuit 2 for 5 indicators is in a very positive category with an average percentage of 82.14%.

Interest Indicator

The indicator of interest received a very positive response from students with a percentage of 80.84%. This shows that students are more excited, happy to learn, and interested in using Cramer's rule to be applied to other materials.

Motivation Indicator

The motivation indicator received a very positive response from students with a percentage reaching 80.24%. So the use of Cramer's rule to solve the LES of 2-loop electrical circuits significantly builds motivation, enthusiasm, and confidence, while triggering the desire to deepen understanding of the material.

Readability indicator

The readability indicator received a very positive response from students with a percentage reaching 80.72%. This shows that Cramer's rule makes it easier to solve problems, increases curiosity, and makes participants more diligent in learning LES in Kirchoff's Law.

Satisfaction Indicator

The satisfaction indicator received a very positive response from students with a percentage reaching 84.05%. This proves that Cramer's rule not only facilitates problem-solving but also broadens horizons, triggers exploration of solving electrical problems, and is to the needs and expectations of students.

Response Indicator

The response indicator received a very positive response from students with a percentage reaching 84.89%. This proves that the use of Cramer's rule on the LES of Kirchoff's Law is easier to understand, very effective, more concise, and fast, and can generate enthusiasm for learning problems in the 2 Loop Electrical Circuit.

This result is to the research of Krisnawanto et al. (2024) which states that the student's response to the use of Cramer's rule assisted by Sarrus' method in solving 2-loop electrical circuit problems is very positive with an average percentage of 81%. Aida et al. (2023) stated that 62.2% of students prefer Cramer's method using Microsoft Excel to solve direct current electrical circuits compared to other solution methods. Ndlovu, Z., & Brijlall, D. (2019) also stated that Cramer's rule is more effective for solving matrices from the results of a system of linear equations. However, Cramer's rule also has weaknesses if used to solve electrical circuits of $n \geq 3$ loops because the resulting LES will have large and complex variables.

CONCLUSION

The application of Cramer's rule in solving the Linear Equation System (LES) on 2 Loop Electrical Circuits is proven to improve students' mathematical thinking skills and learning outcomes, this is indicated by an increase in college students' mathematical thinking skills obtaining an average N-Gain of 0.81 with a high category and an average N-Gain of student learning outcomes of 0.79 and an average N-Gain of student learning outcomes of 0.81. In addition, the application of Cramer's rule in solving LES on 2 Loop Electrical Circuits received a very positive response with an average percentage of 82.14%. Based on the increase in the increase in students' mathematical thinking and the increase in college student and high school student learning outcomes and college student responses after applying Cramer's rule in solving LES on 2-loop electrical circuits, it can be concluded that Cramer's rule is effective for solving LES on 2-loop Electrical Circuits. Based on these conclusions, it is suggested that Cramer's rule in solving the Linear Equation System (LES) in a 2-loop electrical circuit can be used as an alternative method in addition to the substitution-elimination method. It is also necessary to conduct further research for other mathematical methods to solve the 2-loop electrical circuit and apply this rule to other physics materials.

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