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Characteristic of Near Ring From Group Object of Categories

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Abstract

Setiap objek pada kategori dengan objek terminal dan produk disebut grup objek jika memiliki beberapa aksioma seperti aksioma grup tetapi didefinisikan oleh diagram komutatif. Aksioma-aksioma tersebut seperti asosiatif, eksistensi elemen identitas dan elemen invers. Untuk setiap objek kelompok G, himpunan endomorfisme dari G ke G dilambangkan dengan Hom (G, G). Hom (G, G) berada tepat di dekat ring pada opersai penjumlahan Å dan operasi perkalian °. Dalam penelitian ini kami menunjukkan bahwa Hom (G, G) dapat dipertimbangkan sebagai cincin B1 di dekat kedua operasi tersebut.

Abstract

Every object on category with terminal object and product is called group object if its have some axioms like group axioms but defined by comutative diagram. Its axioms such as associative, existence identity element and invers element. For any group object G, set of endomorphism from G to G denoted by Hom(G,G). Hom(G,G) is right near ring over addition operation Å and multiplication operation °. In this research we shown that Hom(G,G) can be considering as B1-near ring over both operation.

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PENDAHULUAN

Category consist of class of objects and morphisms from one object to object and satisfies some axioms Schubert (1972). On Categories there are terminal object, initial object, product and coproduct. On Clay (1994) every object on category that consists terminal object and product and fullfil some axioms like group axioms we called as group object. Dual from group object is cogroup object. In this article we need study category and fungtor in Schubert (1972), Adamek *et al* (2004) and Pareigis(1970).

Near ring is generalization from ring. A Set *N* with additive operation "+" and multiplicative operation "□" is called near ring provided $(N,+)$ is Abelian group, $(N,□)$ is semigroup and $(N,+,□)$ satisfied left distributive or right distributive law (Pilz, (1983)). If $(N,+,□)$ satisfied left distributive law we called $(N,+,□)$ as left near ring. On Clay (1994) and Puspita (2007) exlpained that set of endomorphism from group object G denoted by $Hom(G,G)$. On $Hom(G,G)$ defined binary operation "$\bar{\oplus}$" and "$\circ$" i.e $(f \bar{\oplus} g) = (\pi \circ [f,g])$ and "$\circ$" defined like composition function for any $f,g \in Hom(G,G)$

Some research about near ring have been studied by several authors i.e in Pilz (1983), Ashraf and Siddeque (2015), Boua (2012).

On Balakhrisnamet al (2011) introduced *B₁*near ring as special near ring. Near ring $(N,+,□)$ is called B_1-near ring if for every $a \in N$ there exist $x \in N^*$ such as $Nax = Nxa$. In this article shown that for any group object, then left near ring $(Hom(G,G),\bar{\oplus},\circ)$ is B_1-near ring.Silviya et al (2010) researched about strong S1 near ring

RESULT AND DISCUSSION

Near ring dan B_1-Near ring

In this chapter we have definition of near ring as generalization from ring. Beside that we see some examples of near ring.

Definition1 (Pilz 1983).

Non empty set N with two binary operation "$+$" and "$\bullet$" *called **near ring** if*

1. Tuple $(N,+)$ is group,

2. Tuple $(N,\bullet)$ is semigroup,

3. Triple $(N,+,\bullet)$ satisfied left or right distributive

a. Left Distributive:

$$(\forall n_1,n_2,n_3 \in N) n_1 \bullet (n_2 + n_3) = n_1 \bullet n_2 + n_1 \bullet n_3$$

b. Right Distributive:

$$(\forall n_1,n_2,n_3 \in N)(n_1 + n_2) \bullet n_3 = n_1 \bullet n_3 + n_2 \bullet n_3$$

Near ring that satisfied left distributive law called as left near ring. Near ring N over operation "$+$" and "$\bullet$" denoted by $(N,+,\bullet)$.

Example 2: Let group Γ with operation "$+$" with identity element o (*"omykron"*). Ifond Γ defined operation "$\bullet$" i.e $(\forall \delta,\gamma \in \Gamma)\gamma \bullet \delta = o$, then $(\Gamma,+,\bullet)$ is near ring.

Example 3: Let group $(\Gamma,+)$. Suppose $End(\Gamma) = \{f \mid f : \Gamma \to \Gamma\} = \Gamma^\Gamma$ is set of all function from group Γ to group Γ, then $End(\Gamma)$ is near ring over addition operation "$\tilde{\oplus}$" and composition function "$\circ$" defined as below

$$(\forall f,g \in End(\Gamma))(\forall x \in \Gamma)(f \tilde{\oplus} g)(x) = f(x) + g(x)$$

$$(\forall f,g \in End(\Gamma))(\forall x \in \Gamma)(f \circ g)(x) = f(g(x))$$

1. Tuple $(End(\Gamma),\tilde{\oplus})$ is group.

For $f,g \in End(\Gamma)$, and $x \in \Gamma$,

$(f \tilde{\oplus} g)(x) = f(x) + g(x)$,

Because of $f(x) + g(x) \in \Gamma$,

then $f \tilde{\oplus} g \in End(\Gamma)$ ·

For $f,g,h \in End(\Gamma)$, and $x \in \Gamma$,

$$((f\tilde{\oplus}g)\tilde{\oplus}h)(x)=(f\tilde{\oplus}g)(x)+h(x)=f(x)+g(x)+h(x)$$
$$=f(x)+(g\tilde{\oplus}h)(x)=(f\tilde{\oplus}(g\tilde{\oplus}h))(x)$$

Existence identity element

$(\exists o\in End(\Gamma))(\forall x\in\Gamma)(\forall f\in End(\Gamma))(f\tilde{\oplus}o)(x)$
$=f(x)+o(x)=f(x)$

$(\exists o\in End(\Gamma))(\forall x\in\Gamma)(\forall f\in End(\Gamma))(o\tilde{\oplus}f)(x)$
$=o(x)+f(x)=f(x)$

For $f\in End(\Gamma)$, there exist $f^{-1}:\Gamma\to\Gamma$ i.e $f^{-1}=-f$, or $-f:\Gamma\to\Gamma$ $x\mapsto -f(x)$, such as
$(f\tilde{\oplus}(-f))(x)=f(x)-f(x)=-f(x)+f(x)$
$=(-f\tilde{\oplus}f)(x)=o(x)$

So proved that $(End(\Gamma),\tilde{\oplus})$ is group

2. Set $(End(\Gamma),\circ)$ is semigrup.

For any $f,g\in End(\Gamma)$, and $x\in\Gamma$, $(f\circ g)(x)=f(g(x))$, because of $f(g(x))$ is also function from Γ to Γ, then $f\circ g\in End(\Gamma)$.

$(\forall f,g,h\in End(\Gamma))((f\circ g)\circ h)(x)$
$=(f\circ g)(h(x))=f(g(h(x)))$
$((f\circ g)\circ h)(x)=f((g\circ h)(x))=(f\circ(g\circ h))(x)$

3. Set $(End(\Gamma),\tilde{\oplus},\circ)$ is right distributive.

$(\forall f,g,h\in End(\Gamma))((f\tilde{\oplus}g)\circ h)(x)=(f\tilde{\oplus}g)(h(x))$
$=f(h(x))+g(h(x))=(f\circ h)(x)+(g\circ h)(x)$
$=((f\circ h)\tilde{\oplus}(g\circ h))(x)$

From (1) – (3), then it is proved that $(End(\Gamma),\tilde{\oplus},\circ)$ is near ring. □

B1-near ring is special near ring. Definition of B1-near ring giving as follow

Definition 4 (Balakhrisnam *et al* 2011).
Near ring $(N,+,\square)$ *is* B_1 *– near ring if for any* $a\in N$, *there exist* $x\in N\setminus\{0\}$ *such as* $Nxa=Nax$.

We have sufficient condition such that a near ring can be considered as B_1-near ring on this theorem

Theorem 5 (Balakhrisnam et al 2011 and Fraleigh 1994).
If $(N,+,\square)$ *is near ring with unit element then N is* B_1*-near ring*

Proof :
Suppose near ring $(N,+,\square)$ with unit element 1. For any $a\in N$, there exist $1\in N$ such that $N.1.a=N.a.1=Na$. It is shown that N is B_1-near ring. ■

B1 -Near ring from Group Object of category

Suppose a category C . Group object on C is result from define some object like a group on structure algebra with category rules. On a category C, for have a group object we need product and terminal object Schubert (1972). Some axioms that must be have by object on category like group axioms as follow:

1. Associative

   Let $G\in|$ C $|$, and $\pi\in Hom(P(G,G),G)$.

   For prove associative on category C, suppose $f=[[\pi_1,\pi_2],\pi_3]$ dan $g=[\pi_1,[\pi_2,\pi_3]]$,

$\pi_i:P(G,G,G)\to G$, for $i=1,2,3$
$f=[[\pi_1,\pi_2],\pi_3]:P(G,G,G)\to P(P(G,G),G)$ and
$g=[\pi_1,[\pi_2,\pi_3]]:P(G,G,G)\to P(G,P(G,G))$

Then for product $(P(G,G,G),\pi_1,\pi_2,\pi_3)$ will be satisfied commutative diagram on Figure 1

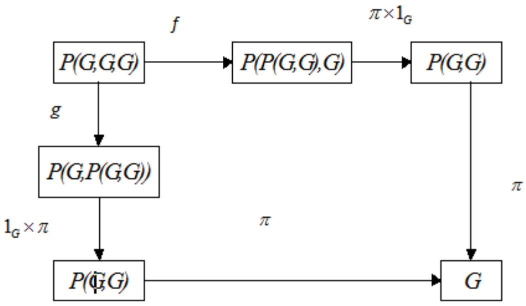

Figure 1. Associative Diagram

Then $\pi\circ(\pi\times 1_G)\circ f=\pi\circ(1_G\times\pi)\circ g$.

Existence Identity Element.

To explain identity element on categories we need terminal object T, and $\mu \in Hom(T,G)$, such that follow diagram is commutative.

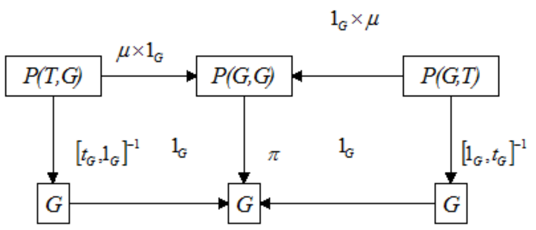

Figure 2. Identity element Diagram

Existence invers element

On last we need to proof existence invers element. We need morphism $\alpha \in Hom(G,G)$, for any $g \in G$, then $\alpha(g) = g^{-1}$, and fullfil commutative diagram as follow

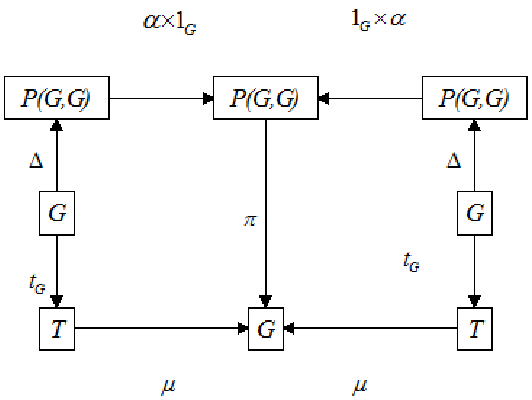

Figure 3. Invers Element Diagram

From 1 – 3 we have definition a group object.

Definition 6 (Clay 1994).

Let category $\mathcal{C}$, $A, B \in |\mathcal{C}|$ *and terminal object* T. *Product A and B denoted by* $P(A,B)$.

Group Object *is quadruple* (G,π,μ,α) *with* $G \in |\mathcal{C}|$, $\pi \in Hom(P(G,G),G)$, $\mu \in Hom(T,G)$, $\alpha \in Hom(G,G)$, *and Figure 1, 2 and 3 commutative .*

Theorem 7 (Clay 1994 and Puspita 2007).

Let group object (G,π,μ,α) *on category* $\mathcal{C}$. *For* $X \in |\mathcal{C}|$, *defined operation* "$\bar{\oplus}$" *on* $Hom(X,G)$, *i.e for any* $f, g \in Hom(X,G)$, $f \bar{\oplus} g = \pi \circ [f,g]$. *Then* $Hom(X,G)$ *is group over* "$\bar{\oplus}$", *with identity element* $\mu \circ t_X$, *and* $-f = \alpha \circ f$.

Proof : (Puspita 2007)

Corollary 8 (Clay 1994 and Puspita 2007).

If (G,π,μ,α) *group object then maka* $(Hom(G,G),\bar{\oplus})$ *is group.*

Proof :

From Theorem 7 we have $(Hom(X,G),\bar{\oplus})$ is group. If $X = G$, then we have proof that $(Hom(G,G),\bar{\oplus})$ ■

Teorema 9 (Clay 1994 and Puspita 2007).

Let Group Object (G,π,μ,α) *on category* $\mathcal{C}$ *and* $X \in |\mathcal{C}|$. *For every* $f \in Hom(X,G)$, *and* $g_1, g_2 \in Hom(G,G)$,

1. $(g_1 \bar{\oplus} g_2) \circ f = (g_1 \circ f) \bar{\oplus} (g_2 \circ f)$.
2. $(g_1 \circ g_2) \circ f = g_1 \circ (g_2 \circ f)$.
3. $1_G \circ f = f$.

Proof : (Puspita 2007).

Corollary 10 (Puspita 2007).

For object group (G,π,μ,α) *on category* $\mathcal{C}$. $(Hom(G,G),\bar{\oplus},\circ)$ *is right near ring*

Proof:

From Corollary 8 shown that $(Hom(G,G),\bar{\oplus})$ is group.

If $X = G$, for group object (G,π,μ,α) on *category* $\mathcal{C}$ we have for any $f \in Hom(G,G)$, $g_1, g_2 \in Hom(G,G)$, by Theorem 9 (2), then $Hom(G,G)$ closed and associative over "$\circ$". From Theorem 9 (1) satisfied right distributive law $(g_1 \bar{\oplus} g_2) \circ f = (g_1 \circ f) \bar{\oplus} (g_2 \circ f)$.

So we have proved that for any group object (G,π,μ,α), then $\left(Hom(G,G),\overline{\oplus},\circ\right)$ is right near ring.

Corollary 11.

Near ring $\left(Hom(G,G),\overline{\oplus},\circ\right)$ *is right B_1* –near ring.

Proof :

From Corollary 10 we have shown that $\left(Hom(G,G),\underline{\oplus},\circ\right)$ is right near ring. From Theorem 9 (3) there is $1_G \in Hom(G,G)$ for any $f \in Hom(G,G)$ such that $f \circ 1_G = 1_G \circ f = f$ or $1_G \in Hom(G,G)$ is unit element on $\left(Hom(G,G),\overline{\oplus},\circ\right)$. Furthermore based on Theorem 5 we have $\left(Hom(G,G),\overline{\oplus},\circ\right)$ is B_1 –near ring.■

CONCLUSION

Group object is object on category C with terminal object and product that satisfied some axioms like group axioms on algebra structure. Near ring is generalization from ring by reduces some ring axioms. Then every ring is near ring. One of special near ring is B_1 near ring that form by add one axiom on near ring.

From an group object G can be constructed a near ring i.e near ring from set all endomorphism from G to G over binary opearation $\overline{\oplus}$ and $\circ$ denoted $\left(Hom(G,G),\overline{\oplus},\circ\right)$. Near ring $\left(Hom(G,G),\overline{\oplus},\circ\right)$ have identity element over multiplication i.e 1_G . It is implies that near ring $\left(Hom(G,G),\underline{\oplus},\circ\right)$ can be considering as *B_1*-near ring. So every near ring from object group can be considering as *B_1*-near ring.

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