IMPACT OF REPRESENTATIONAL APPROACH ON THE IMPROVEMENT OF STUDENTS’ UNDERSTANDING OF ACCELERATION

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ABSTRACT

This research examined the impact of representational approach on the improvement of students’ understanding of acceleration. Subject consisted of 24 prospective physics teacher students of State University of Malang, enrolling Selected Topic of School Physics course in Semester I of 2011/2012 academic year. Using mixed-methods design, this study concluded that (1) students’ mean score of conceptual test increased significantly from 50.8% to 85.0% with very large d-effect size (1.67) and high normalized-gain (0.71), (2) students became able to use operational definition of acceleration to analyze multi-flash motion diagram, and (3) students remediated their misconceptions of acceleration. However, few students stuck in their misconception that acceleration of a shot-up object is decreasing with its elevation, and the tension in the rope of a swinging pendulum is equal to the weight of its bob.

Keywords: students’ conception; acceleration; representational approach; prospective physics teacher

INTRODUCTION

Acceleration is a very essential and fundamental concept in mechanics. Singh & Schunn (2009) argued that the conceptual areas in mechanics are particularly important targets for educational interventions in K-12, since these are the fundamental concepts in all sciences and related to diverse physical phenomena in everyday experiences. These concepts also provide the backbone on which many other science concepts are developed. In Indonesia setting, the force and motion concepts, including acceleration, are regarded as so fundamental that should be taught since middle school (MONE regulation No 22 year 2006, about Content Standard).
Unfortunately, many studies over past three decades show that students have pre-instructional conceptions or beliefs about the concepts that are usually scientifically incorrect (see, for example, bibliographies in Rosenblatt and Heckler, 2011). Students’ misconceptions are usually resistant to change and cause students difficulty in grasping the accepted scientific view. Rosenblatt and Heckler (2011) investigate student conceptual understanding of the relationships between the directions of net force, velocity, and acceleration in one dimension. They found that a significant fraction of students chose partially correct or misconception responses. Thornton and Sokoloff (1998) reported that many students believe that the net force on an object and its velocity must be in the same direction. Some researchers showed that students often have difficulty to distinguish velocity and acceleration (Hake, 1998; Reif & Allen, 1992). Shaffer and McDermott (2005) reported that only about 30% of undergraduate students (n = 125), about 5% of preservice high school teachers (n = 18), and about 15% of graduate and Ph. D students (n = 22) at University of Washington and Montana State University, gave correct response when they were asked to define the approximate direction of acceleration of a pendulum bob at various points after it is released. Similarly, such deficient understanding of acceleration also occurs for some experts as reported by Reif and Allen (1992). They found that only one out of five experts exhibited perfect understanding, while another one exhibited quite marked deficiencies. These experts were professors of physics who had recently taught introductory physics at a major university. However, it is critical to anticipate whether our prospective physics teachers have any difficulty on the concept of acceleration. Our previous work (Sutopo et al., 2011) showed that the students indeed had a markedly deficient understanding even though they had learned more in-depth the concept in Introductory Physics and Mechanics courses.

This research is guided by a vision that the future physics teachers need for grasping deep understanding of physics concepts they will teach in school. It is believed that the mastery and deeply understanding of physics content knowledge is a precondition for becoming effective physics teacher. If a teacher does not understand the nuances of a concept, including the relationships with other concepts and the ways through which the concept was constructed by the physics community, then translating these nuances into student understanding is probably impossible (Etkina, 2010). Physics teachers should have well-organized conceptual knowledge not only within a particular branch of physics (e.g. mechanics, electromagnetics, etc), but also across branches of physics (e.g. mechanics and thermodynamics). In addition, teachers need to understand the processes used to establish new knowledge and determine the validity of claims (Eylon and Bagno, 2006). Teachers also need able to communicate and clarify scientific ideas effectively to students. It implies that, in describing scientific ideas, teachers need able to use multiple modes of representation that lead to the opportunity to exchange and clarify meanings (Ainsworth, Prain, & Tytler, 2011).

As mentioned in advance, students had a markedly deficient understanding of acceleration even though they had learned more in-depth the concept through Introductory Physics and Mechanics courses. To overcome this situation, we have implemented a representational approach in Selected Topics of School Physics (STSP) course. We have reported a global impact of this approach (Sutopo et al., 2012). The approach improved both students’ reasoning ability and understanding of mechanics concepts with very high effect size and with average normalized gain that was in upper-medium category according to Hake’s categorization. Students’ competence on mechanics shifted significantly from ‘under-competent’ level of problem-solving competence to ‘mastery’ level on basic Newtonian mechanics. Because of the importance of acceleration concept, it is critical to investigate the impact of this approach on the improvement of students understanding of acceleration. Accordingly, this paper is intended to address the following two main research questions. (1) To what extent does this approach overcome students’ difficulty on acceleration? (2) To what extent does this approach improve students score on conceptual acceleration test?

METHODS

This paper is a part of research that examines the impact of representational approach, implemented in STSP course, on the improvement of students’ reasoning ability, generic science skills, representational approach, and conceptual understanding of mechanics. The research used mixed-methods (Creswell & Clark, 2007) with a quasi experiment, one...
Theme 1

Refer to the diagram on the right in answer the following questions. The diagram shows a block that is sliding along a frictionless ramp and then leaves the ramp into the air. Neglect the air resistance.

1. What is the arrow that best represents the acceleration of the block when the block is at the position of I?
   (A) 1  (B) 2  (C) 4  (D) 5  (E) None of the arrows; the acceleration is zero.

2. What is the arrow that best represents the acceleration of the block when the block is at the position of II?
   (A) 1  (B) 3  (C) 5  (D) 7  (E) None of the arrows; the acceleration is zero

3. What is the arrow that best represents the acceleration of the block when the block is at the position of III?
   (A) 2  (B) 3  (C) 5  (D) 6  (E) None of the arrows; the acceleration is zero.

Theme 2

X and Z mark the highest and Y the lowest positions of a 50.0 kg bob swinging as illustrated in the diagram.

What is the tension in the rope at point Y if we neglect the mass of the rope and air resistance?

(A) 200 N  (B) 300 N  (C) 500 N  (D) 625 N  (E) 700 N

Theme 3

The figure represents a multi-flash photograph of a small ball being shot straight up by a spring. The spring, with the ball atop, was initially compressed to the point marked X and then released. The ball left the spring at the point marked Y, and reaches its highest point at the point marked Z. If air resistance is negligible, which statement that best describes the acceleration of the ball?

(A) The acceleration of the ball was greatest at point Y.
(B) The acceleration of the ball was decreasing on its way from point Y to point Z.
(C) The acceleration of the ball was zero at point Z.
(D) All of the above responses are correct.

(F) The acceleration of the ball was the same at all points in its trajectory, from points Y to Z.

Figure 1. A cluster of five items used to assess students conceptual understanding of acceleration. Theme 1 consists of three items, whereas Theme 2 and 3 consists of one item for each.
group pretest-posttest design. Pretest and posttest used the same instrument, i.e., integrated test developed from Mechanics Baseline Test (Hestenes & Wells, 1992). By integrated test, we mean that this test was designed to assess those learning outcomes simultaneously. Pretest was conducted at beginning of semester and posttest at the end of semester. Topics being discussed included linear motion, parabolic motion, circular motion, simple harmonics oscillator, physics behind roller coaster, sound wave (musical instruments), and macroscopic behavior of gas. However, in this paper we focus on students’ conceptual understanding of acceleration. For this purpose, we analyzed students’ responses on a cluster of items relating to the concept. This cluster consists of five items as shown in Figure 1. Items in Theme 1 ask students to define the direction of acceleration of a given trajectory. Theme 2 requests students to define acceleration in order to successfully calculate the tension in the rope on a pendulum-like problem. Theme 3 asks students to describe qualitatively the magnitude of acceleration of a ball that is shot upward.

Our analysis was basically based on students’ responses on pretest and posttest. We used a normalized gain (Bao, 2006; Hake, 1998) to measure the improvement (if any). This quantity has been widely used in assessing the effectiveness of a course based on pretest and posttest (e.g., Finkelstein & Pollock, 2005; Coletta, Phillips, & Steinert, 2007; Smith & Wittmann, 2007; Malone, 2008; Kurnianto, Dwijananti, & Khumaedi, 2010; Nieminen, Savinainen, & Viiri, 2010; ). We also made a qualitative analysis based on students’ reasons in responding the test, students’ responses on relevant informal formative assessments, and students’ responses on questionnaire.

This study was conducted during the first semester of 2011/2012 academic year (September to December 2011). Subject consisted of 24 undergraduate students of physics education program in State University of Malang, enrolling the STSP course offered for prospective physics teacher of international school. The teaching was presented in bilingual, English and Bahasa Indonesia. As in other educational universities in Indonesia, this course is intended to facilitate students to learn more in-depth the school physics concepts before they took internship program. Students had learned mechanics concepts including acceleration through Introductory Physics and Mechanics courses presented in the previous semesters.

RESULTS AND DISCUSSION

Descriptive statistics of students’ achievement in pretest and posttest is summarized in Table 1. It is clear that students’ achievement improved from pretest to posttest. To justify the statistical significance of this improvement, we employed paired-samples t-test since skewness of these data sets lay in the interval of [−1, 1]. According to Morgan, Leech, Gloeckner, and Barrett (2004), a data set with such skewness can be assumed to be normally distributed or at least approximately normal. The test resulted $t\ (df = 23) = 10.38, p = 0.000$ (two-tailed), suggesting that mean difference between posttest and pretest is statistically significant at $p = 0.01$. It means that students’ achievement in posttest was significantly higher than that in pretest. The Cohen’s d-effect size (Ellis, 2010) is $1.67$ and Hake’s average normalized gain (Hake, 1998) is $0.71$. This d-effect size is in very high or “higher than typical” category (Morgan et al., 2004), whereas the normalized gain is in high category (Hake, 1998).

It is useful to look at the distribution of students’ responses to these problems. Figure 2 shows the shiftiness of this distribution from pretest to posttest. The figure shows that the number of students choosing correct answer was markedly increase for all problems. This change (in %) is respectively from 67 to 96, 0 to 100, 59 to 100, 0 to 54, and 21 to 75. The smallest shift was on problem 1 of Theme 1 (figure (a)), since most students had chosen cor-

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The correct answer on pretest. In addition, the figure shows that the variance of students’ responses in pretest was markedly more spread out than in posttest, unless for problem in Theme 2. It means that students’ learning acquired throughout the lessons have reduced the variance of their conception about acceleration.

In our previous work (Sutopo et al., 2011), we have made in-depth analysis of students’ reasons on pretest. Therefore, we will describe here the summary of the results.

In Problem 1, sixteen students (67%) chose the correct option C. However, their reasons did not fully justify their responses. Eleven students of them (46%) used the conception that “acceleration is always in the same direction as the motion”, or “acceleration is proportional (in both direction and magnitude) to velocity”; one student argued that “direction of acceleration is that to which the block tends to move”, and the rest two students did not give any reason. Five students (21%) chose option D. Most of them used their own conception, “because of gravity, the block’s acceleration is equal to gravitational acceleration, i.e. in downward direction”. One student referred to Newton’s second law and argued, “The direction of acceleration is parallel to the net (resultant) force acting on the block”. Unfortunately, he did not notice the existence of normal force in this problem so that he claimed that the force acting on the block is just gravitational force. The rest students (12%) chose option E. They argued that, “The acceleration is zero since there is no friction acting on the block during the motion”.

In Problem 2, no student chose the correct answer A. Most students (11 students or 46%) chose option B. Most of them used the reason, “Acceleration is always in the same direction as the motion”, one student used argument, “Direction of acceleration is that to which the block tends to move”, and one student did not give any reason. Students choosing option C (5 students, or 21%) argued, “Because of gravity, the block’s acceleration is equal to gravitational acceleration, i.e. in downward direction”. One student (4%) selected option D and argued, “At this point, block tends to move back so that its acceleration is in backward direction”. Students choosing option E (7 students, or 29%) argued, “The block is momentarily at rest so that its acceleration is zero”, or “The acceleration is zero since there is no friction acting on the block”, or “There is no net force acting on the block”.

In Problem 3, 14 students (about 58%) selected the correct option C. Eight of them (57%) wrote an appropriate reason, but the reasons of the other six students (43%) were not fully valid. Some examples are “Just after leaving the ramp, the block falls downward.
in the direction of $g$", "It stops momentarily at this position and then moves under influence of gravitation", and "Block tends to move in this direction".

No student chose the correct answer E. 67% students chose option C, 8% chose option B, and the rest students (25%) did not give response. All students choosing option C wrote: "$ T = mg = 500 \ N $", whereas students selecting option B wrote: "$ T = mg \times \frac{2}{5} = 50 \times 10 \times \frac{2}{5} = 300 \ N $.

It suggests that most students thought that the bob has zero acceleration at this point, or if it is non-zero, its direction is perpendicular to the rope. This claim is in agreement with students’ responses on Problem 3 in Theme 1. Two students that left their answer sheets blank wrote an appropriate formula: $ T - mg = a $, but they failed to find the value of acceleration $a$. This failure was probably due to their deficient understanding of acceleration. When solving the problems in Theme 1, for instance, they applied an inconsistent framework. It seems that they simply used the "prior examples" that they had gotten from textbooks or previous lectures (Ogilvie, 2009), or used “memory-based approach” (Walsh et al., 2007).

Figure 2 shows that students’ responses to this problem were spreaded over all the given alternatives, including blank ("option‘ K). Only five students (21%) chose the correct answer E. Four of these five students gave a relatively correct reason, “There is no external force acting on the ball except the gravitational force … Its acceleration is $g$, constant at all point … directed to the Earth”. One student confused with her own conception that acceleration should be in the same direction as (or proportional to) the velocity on one hand, and any free falling body has constant acceleration of $g$, on the other hand. She wrote, “Its acceleration is constant at any point, but it is zero at point Z since the velocity is zero at the point”.

As a summary, most students believed that acceleration is always in the same direction as the motion, or proportional to the velocity. It means that most students confused the concept of acceleration with velocity. Some students misunderstood the relationship between acceleration and force as stated in Newton’s second law of motion, $F = ma$. They thought this equation as causality rather than covariance relationship.

Based on the student’s reasons on pre-test, observation on students’ discourses, and short questionnaire administered in the early meetings, there was evidence that students did not deeply understand the operational definition of acceleration $a = \frac{dv}{dt}$ or $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$. Most students thought and believed that acceleration is always in the same direction as the motion (or velocity); even most students believed that acceleration is proportional (in both direction and magnitude) to velocity. It means that most students confused the concept of acceleration with velocity. Moreover, students tended to omit the vector nature of acceleration, and velocity as well. When they were asked whether the acceleration is a vector, they confidently answered, “Yes, acceleration is a vector”. During the first lesson, when a kinematics problem in one dimension was being discussed, we observed a markedly deficient understanding shown by most students. We then required students to describe their conception about the relationship between direction of acceleration, velocity, and motion. Their responses were as follows.

(1) Most students (21 out of 24) wrote, “Both velocity and acceleration are in the same direction as the motion … tangent to the path”. Two of these students added information, “… but, really, I didn’t care with these directions so far; I more concerned on these values rather than their direction”. (2) Two students (out of 24) wrote, “I really confuse about them, since I didn’t care with these directions so far”. (3) Only one student wrote a relatively appropriate statement even though still not fully valid, “Velocity is always in the same direction as motion, but acceleration is not. Acceleration is in the direction to which the object tends to move”.

Another cause is the deficiency of students’ understanding of Newton’s second law of motion. Most students believed that the law is a causality relationship in that the force acts as cause and the acceleration as effect. Therefore, they tended to overemphasize on the left side of $F = ma$ to find acceleration. Students’ reasoning on problems in Theme 1 and 2 give the evidence. Students did not recognize that acceleration could not be simply determined from dynamics when one of the forces is the normal force (as in Problem 2 Theme 1) or the tension in a rope (as in Theme 2). In such cases, the direction and magnitude of acceleration must be inferred using kinematics method.

Superimpose of the two kinds of deficient understanding indeed led students’ difficulties in solving qualitative problems of acceleration. Since students involved in this study had under-
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ses, it means that such deficient understanding persist beyond introductory physics and are not adequately addressed in later courses.

To this problem, 23 students (96%) selected the correct answer (option C). Only one student (4%) selected incorrect option B. Therefore, it is useful to describe first the reasons of students selecting option C, then that of students selecting option B.

Most students (18 out of 23 students) justified their responses based on the kinematics point of view and used at least two kind of representations (verbal and diagram). Students’ reasons were typically as the following.

“The direction of acceleration is the same as that of \( a \), where \( v_{a} \) and \( v_{b} \) is velocity at a point ‘just after’ and ‘just before’ the point at which the acceleration to be defined. In this case, \( v_{a} \) and \( v_{b} \) are in the same direction, i.e. parallel to the track. Therefore, the direction of \( \Delta v \), and hence of \( a \), is parallel to the track”.

Most students implicitly argued that the magnitude of \( v_{a} \) is larger than that of \( v_{b} \) and concluded that the option C is the correct answer. Some students (5 of 18) explicitly justified \( v_{a} > v_{b} \) using principle of mechanical energy conservation. Some other students (3 of 18) gave additional statement such as, “the acceleration is in the same direction as the motion or velocity since the motion is speeding up in a straight line”. Figure 3 shows an example of student’s reason that represents this kind of reasoning.

Two students (out of 23) justified their responses based on kinematics, but simply in verbal representation. For example, they argued: “Its velocity is in constant direction ... downward along the ramp. Since \( a \) is in the same direction as \( \Delta v \), then \( a \) is in the same direction as motion/velocity”.

Three students (out of 23) justified their responses using a force diagram and claimed (implicitly) that the net force acting on the block is in downward direction, parallel to the ramp (as shown by option C). Therefore, the acceleration is in this direction too. Students’ force diagrams were typically as shown in Figure 4. One of these three students also justified her reason using kinematics, in addition to this dynamics approach. However, they did not explain why normal force \( N \) and \( mg \cos \theta \) cancels each other.

![Figure 3](image)

**Figure 3.** Example of students’ posttest reason on Problem 1 Theme 1. This student used symbol \( \vec{v}_{a} \) for ‘just after’ velocity and \( \vec{v}_{b} \) for ‘just before’ velocity.

![Figure 4](image)

**Figure 4.** A force diagram typically constructed by students to define the direction of acceleration.

Student’s reasons leading to option B

One student failed to get the correct answer. She used kinematics method to solve this problem. Really, her reason was similar to that developed by most of their classmates. Unfortunately, she made misinterpretation about the problem. She thought that the path around position I is a curved path, rather than a straight line as it should be. Hence, her reason led to option B.

All students selected the correct option A. Eighteen students (75%) used kinematics approach by drawing a vector that describes the change in velocity, \( \Delta v \), between two sym-
metrical points about the lowest position (i.e. position II as required by the problem). They draw these velocities in equal magnitude and got \( \Delta v \) in upward direction (as represented by arrow 1 in the problem), and hence concluded that the acceleration at position II is in upward direction since acceleration is in the same direction as \( \Delta v \). Most students explained this equality using principle of mechanical energy conservation, but some other students did not give any reason. Figure 5 shows an example of student’s reason.

Three students (12.5%) utilized both kinematics and dynamics of circular motion to justify their responses. They argued that the block moves in circular motion so that it undergoes a centripetal acceleration. Since centripetal acceleration is always toward the center of the circular path, the block’s acceleration is in upward direction. In addition, their kinematics analyses were typically similar to that described in advance. Figure 6 shows an example of these students’ reasons.

Three other students (12.5%) justified their responses based on the dynamics of circular motion. However, their reasons were simply in verbal. The typical student’s reason was, “The block moves in circular motion, so its acceleration is a centripetal acceleration … in upward direction”.

All students selected the correct option C. Most students (96%) justified their responses by stating, “Upon out of the track, block undergoes free falling … parabolic motion … the only force acting on the block is gravitational force, \( mg \) … then its acceleration is equal to \( g \), downward to the Earth”. Most of them complemented their reasons using diagram illustrating the change in velocity. Figure 7 represents an example of these student’s reasons. One student simply wrote, “Upon out of the track, the block goes down. Therefore, its acceleration is downward, to the Earth”.

In responding this problem, 13 (out of 24) students chose the correct answer E, one student chose option B, 8 students chose option C, and two students did not choose any option.

All students selecting option E used formula \( T = mg + ma \) where \( a = \frac{v^2}{R} \), to justify their responses. They argued that the bob’s acceleration is a centripetal acceleration directed to the center, along the rope. Using mechanical energy conservation, they found \( v^2 = 20 \) (m/s)\(^2\) then set \( R = 5 \) m and got \( a = 4 \) m/s\(^2\). They successfully found \( T = 700 \) N. They also made a force diagram typically as shown in Figure 8.
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Figure 8. A typical force diagram drawn by 13 students that selected option E and two students that left their answer sheets blank.

Two students that left their answer sheets blank used similar procedure employed by students selecting option E. However, these students set $R = 6$ m (instead of 5 m as it should be) and found a value of $T$ that was unavailable in the given alternatives. Therefore, they left their answer sheets blank.

Student selecting option B used formula $T = mg - ma$, where $a = \frac{v^2}{R} = 4 \text{ m/s}^2$, and found $T = 300$ N. She also made force diagram as in Figure 10. It means that this student having difficulty in using force diagram, i.e. in translating force diagram into equation of Newton’s law $F = ma$, rather than in understanding of acceleration itself. Instead of writing $T - mg = ma$, she wrote $T + ma = mg$ then $T = mg - ma$.

All of eight students (25%) selecting option C solved this problem using formula $T = mg = 500$ N. Most students did not explain why they used this formula. However, some students argued that there is no acceleration along the rope, since the bob does not move to this direction. It implies that they stuck at their prior conception that acceleration is always in the direction of motion.

In responding this problem, 18 students (75%) chose the correct option E, two students (8%) chose option B, one student (4%) chose option C, and three students (13%) chose option D. Their reasons were as the following.

Most students selecting the correct option E (16 out of 18 students) argued that, “The only force acting on the ball is gravitation (its weight, mg) so that its acceleration is equal to $g$. … In such short distance, $g$ can be considered constant … ball’s acceleration is the same at all point in its trajectory”. Two other students justified their responses using velocity diagram and found that $\Delta v$ is constant, directed downward, along the motion. One student choosing option B argued, “The ball’s acceleration is decreasing from $Y$ to $Z$ since the motion is slowing down … the velocity at point $Y$ is larger than that at point $Z$”. Another student justified her response using motion diagram. She drew a series of vectors joining any two adjacent points and found that their magnitude is smaller and smaller. She called the vector as acceleration, rather than velocity as it should be. It means that these students confused the concept of acceleration with velocity.

Student selecting option C argued, “The ball acceleration is zero at $Z$, and riches its maximum value at the middle point between $Y$ and $Z$”. Whereas, students selecting option D argued, “The ball’s acceleration is decreasing from $Y$ to $Z$; this acceleration is zero at $Z$ since velocity is zero at that point”. It implies that these students also confused the concept of acceleration with velocity.

As a summary, all students have no longer held their own previous misconception that acceleration is always in the same direction as the motion. Moreover, in solving conceptual problems on acceleration, most students tended to apply kinematics method and utilize more than one mode of representation, rather than in verbal only as they did on pretest. However, some students (34%) still had difficulty in solving problem that involves such “passive force” as tension on the rope of the swinging pendulum. They thought that this tension is equal to the weight of the ball and implicitly stated that the net force is zero and thus the acceleration is zero too. About 25% students still had difficulty in describing the acceleration of a ball thrown upward around the Earth, where the air friction is being neglected. They stuck in their misconception that this acceleration is decreasing with elevation, rather than constant
as it should be.

The previous analyses give evidence that most students have changed their conception about acceleration. Students have removed their following previous conceptions. (1) Acceleration is always in the same direction as the motion or velocity. (2) The magnitude of acceleration is proportional to the magnitude of velocity. (3) Acceleration is in the direction to which the object tends to move. (4) In any frictionless track, the object’s acceleration is zero. (5) If an object is in under influence of gravity then its acceleration is equal to the gravitational acceleration g. On the other hand, based on the previous analyses and students reflection, there is evidence that most students have constructed a better conceptual understanding. That is, “The direction of acceleration is not always equal to that of motion or velocity. Direction of velocity is always equal to the path of motion, whereas acceleration is in the same direction as the change in velocity, $\Delta v$. Most students have been able to use operational definition of acceleration $a = \frac{\Delta v}{\Delta t}$ or $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ in non-calculus work (e.g. defining velocity and acceleration using vector diagram, and analyzing a motion based on the given motion diagram).

We argue that such conceptual change resulted from the learning outcomes that students acquired throughout the lesson. Therefore, it is useful to describe students learning in implementing such calculus-based operational definition of acceleration, and of velocity as well, to a motion diagram. In the early stage of lesson, students did not have any opinion about it. They did not able to draw the change of velocity or displacement during an interval of time that is closely equal to zero, $\Delta t \to 0$. Here are some pieces of scaffolding to assist the students learning (L stands for lecturer, and S for students).

L: “Look at the series of number: 1, 3, 5, 7, 9. What is the average value of this data?”
S: “5”
L: “Look at the position of number 5 in the series … Ok. You have learned statistics. Where is the position of the average value of a data set?”
S: “Mostly at the middle of the data series …. If the data is normally distributed, the average value is always in the middle of distribution”.
L: “Excellent … Now, assume that position of a moving particle is $x(t) = c_1 t + c_2 t^2$ where $c_1$, $c_2$, and $c_3$ is a constant. What is the particle’s average velocity between $t = 0$ and $t = t_f$? … What is the particle’s velocity at $t = t_f/2$, that is $v(t_f/2)$?”
S: “Average velocity $\vec{v}(t_{0: t_f}) = \frac{x(t_f) - x(t_0)}{t_f - t_0} = c_1 + c_2 t_f$ … Instantaneous velocity: $v(t_0) = c_1 + 2c_2 t_0$ … $\to v(t_{0/2}) = c_1 + c_2 t_0$”
L: “Okay, now replace position with velocity, and velocity with acceleration. Do with the same mathematical model. What do you get?”
S: “Average acceleration during the interval time of $t_f$ is equal to instantaneous acceleration at $t = t_f/2$.”
L: “Excellent. But, be careful to make overgeneralization. Now, use $x(t) = c_1 t + c_2 t^2 + c_3 t^3$ as your model. What will you get?”
S: “The result is different … average velocity during $t_f$ is not equal to instantaneous velocity at $t = t_f/2$.”
L: “Well. Please calculate the error if you force to equalize them, that is $|v(t_f/2) - \bar{v}(t_{0: t_f})|$”
S: “$\frac{1}{4} c_3 t_f^2$ … it is a very small value for small $t_f$”
L: “Well. Thus, if you take $t_f$ so small that you can neglect this error, then it is still reasonable to use the previous result. That is, the instantaneous velocity at the middle interval is equal to average velocity during this small interval. The same situation is true for acceleration”

Students successfully implemented this procedure to analyze multi-flash motion diagram, including drawing velocity and acceleration vectors. Firstly, they implemented it to solve problem of linear motion (Figure 9a); then to problem of parabolic motion (Figure 9 b) and circular motion (Figure 9c) in the next two lessons. However, students showed their deficient skill in drawing vectors of $\Delta v$ and $\Delta a$ during the first two lessons. They could do it more fluently in the third lesson. Using this method, students successfully found that the direction of velocity is always tangent to the path and the particle’s acceleration is constant directed to the Earth in parabolic motion and toward the center of circle in circular motion with constant speed. They were also able to describe the motion using various kinds of representation including graph of velocity-time and acceleration-time as well, and able to analyze the self-consistence
between the results.

Those students’ success confirms the claim of recent science education researchers that to learn science effectively students need to understand the different representations of science concepts and processes, be able to translate a representation into one another, and understand their coordinated use in representing scientific knowledge (Hubber, Tytler, & Haslam, 2010; Prain, Tytler, & Peterson, 2009; Waldrip, Prain, & Carolan, 2010). The ability to use multiple representations is considered as a key to learning physics (Kohl, Rosengrant, & Finkelstein, 2007). Students with higher representation ability have higher chance to solve complex problems successfully (Malone, 2008). Rosengrant, Heuvelen, and Etkina (2009) found evidence that students who frequently use multiple representations are successful in FCI (force concept inventory), MBT (mechanics baseline test), and CSEM (conceptual survey of electrostatics and magnetism) tests.

CONCLUSIONS

In this research, we implemented a representational approach in which students negotiate their understanding of science concepts by collaborative work in a small group then shared their works through whole class discussion. Students both construct and critique their own representation through processes that include the posing of questions, the construction of claims, and the communication of evidence to support the claims. The role of lecturer is to provide prompts to clarify students’ statement (claim, rebuttal, or backup), encourage students to propose claim and/or backup, and require students to re-represent, critique, and argue their understanding of the concept. The approach encourages students to think critically and reason logically as they negotiate understanding through talking, modeling, writing, drawing, and graphing. We then examine the effect of this approach on the improvement of students’ understanding of acceleration.

The approach indeed improved students’ understanding of acceleration. The students’ average score on conceptual test increased significantly ($p = 0.000$) from 50.8% (SD = 25.0%) on pretest to 85.0% (SD = 15.9%) on posttest with Cohen’s d-effect size of 1.67 (in ‘larger than typical’ category) and Hake’s average normalized gain of 0.71 (in high category). Most students have become able to use operational definition of acceleration $a = \frac{dv}{dt}$ or $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$ in non-calculus works (e.g. defining velocity

Figure 9. Multi-flash motion diagrams for which students should describe the change of position, velocity, and acceleration of motion using various kinds of representation including word, diagram, graph, and mathematical representation. (a) Linear motion with constant negative acceleration, (b) parabolic motion without any friction, (c) circular motion with constant speed.
and acceleration using vector diagram, and analyzing a motion based on the given motion diagram).

The approach was able to remediate some students’ misconceptions. Up to the early stages of lessons, students held the following misconceptions. (1) Acceleration is always in the same direction as the motion or velocity. (2) The magnitude of acceleration is proportional to the magnitude of velocity. (3) Acceleration is in the direction to which the object tends to move. (4) If an object moves in a frictionless track, its acceleration is zero. (5) If an object is in under influence of gravity then its acceleration is equal to the gravitational acceleration (g). Eventually, students completely removed these misconceptions and constructed new better understanding, “The direction of acceleration is not always equal to that of motion or velocity. Direction of velocity is always tangent to the path of motion, whereas acceleration is in the same direction as the change in velocity, $\Delta v$.”

However, few students stuck in their misconception that the acceleration of a shot-up object is decreasing with its elevation, rather than constant (i.e. equal to $g$) as it should be. Some students still thought that the tension in the rope of a swinging pendulum is equal to the weight of its bob. We argued that this phenomenon is not surprising since we did not focus to remediate such misconception throughout the lessons. Therefore, it is critical to further study intended to remediate these misconceptions. Student’s difficulty in understanding Newton’s second law, $F = ma$, still need to further investigation.

REFERENCES


