



## Description of Junior High School Students in Generlizing Patterns Based on Semiotic Perspective

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### Abstract

Generalization is a reasoning in making conclusions that contain symbolism and are general. Semiotics have an important role in the process of generalization, in which generalize the pattern not only seen from the work of students but based on the process of students in understanding and making things. This paper aims to describe how junior high school students generalize patterns based on semiotic perspectives. The type of this research is descriptive qualitative in which data obtained by test, interview and observation. The subjects consist of 3 students of grade VIII junior high school students of 1 high, moderate, and low mathematics students. The findings of this research are at the factual stage the three subjects have the same gesture and word. But in the contextual phase each subject has a different way or step of work in accordance with the understanding possessed by each subject and the last stage of the symbolization of the three subjects can create the same formula. This paper is expected to be used as a reference for teachers to understand the ability of students in the generalization process.

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## INTRODUCTION

Mathematics learning is a process of providing learning experiences to learners through a series of planned activities so that learners acquire competence on mathematics materials studied (Gatot, 2008). Based on Permendiknas No. 22 Year 2006, one of the objectives of mathematics learning is to use reasoning on patterns and traits, perform mathematical manipulations in generalizing, compiling evidence, or explaining mathematical ideas and statements. This is in line with Vogel (2003) which states that pattern analysis, descriptions of order, and properties are one of the goals of mathematics.

The mathematics curriculum of the year 2013 for junior high school has contained numerical pattern material. In the matter of number patterns students are asked to make a generalization of a pattern. Nur Indha (2016) found that the seventh grade students of junior high school still had difficulties in the matter of number patterns. Most students experience errors in determining generalizations in a number pattern. Siti Inganah (2005) found that out of 19 junior high school students, only 5 students could continue the pattern in the form of still-affordable images and could not determine the general rule, 8 students can define the general rule of the pattern by using the sentence, and 6 students can define the general rule Pattern by using symbols. So from the research can be concluded that junior high school students still find difficulties in generalizing a pattern.

According to Ernest (2006) mathematics is a field of human work and knowledge known for all the unique signs and signs of activity. The theory of learning about signs is called semiotics. The word Semiotics comes from the Greek semeion which means sign. Semiotics is defined by Ferdinand de Saussure as the study of signs as part of social life (Praptomo, 2007). A sign is anything that can be attached (interpreted) as a significant substitute for something else. According to Pierce there are three factors that determine the existence of a sign that is the sign itself, the thing marked, and a new sign that occurs in the recipient's mind (Asep, 2009).

The semiotic perspective provides a conceptual way of learning mathematics. The main focus in a semiotic perspective is on communicative activities in math utilizing signs that involve both acceptance of sign and understanding through listening and reading, as well as marking production through speaking and writing or sketching (Ernest, 2006). The importance of semiotics for mathematics education lies in the use of signs: this use is in every branch of mathematics.

There is an approach in which signs are a fundamental part of mathematical activity called the theory of objectification. Objective theory is an attempt to understand learning not as a result of student work but the process of students in understanding and making things. Objects, tools, linguistic devices and marks are deliberately used by the individual for the process of making social meaning to carry out the action in order to achieve that goal called semiotic objectification. The objective semiotic approach focuses on gestures, words (words), and signs (symbolic) when students refer to mathematical objects.

One of the learning materials containing the signs is the pattern of numbers in which the pattern material has been taught from pre-kindergarten to intermediate level. According to Walle (2008), learning to discover patterns and how to explain, translate, and expand patterns is part of doing mathematics and algebra thinking. From about four and up to the intermediate level, students can deepen patterns of prolongation from one step to another. In developing patterns students not only develop patterns but also seek generalizations or algebraic relationships that will give an idea of the umpteenth number. The process of creating generalizations of numbers and arithmetic begins at kindergarten and continues as students learn all aspects of numbers and calculations including basic knowledge and meaning of operations.

According Surajiyo, et al (2005) generalization is a reasoning that concludes a general conclusion of the premises in the form of empirical propositions. In contrast to Radford (2003) who views from a psychological point of view, generalization implies that something new has been made clear (for example, that the relationship between a particular concrete object applies to other concrete objects or even to a new object). According to Walle (2008) in creating generalizations need to use symbolism, therefore both generalization and understanding of variables and symbolism must be developed simultaneously.

Variables are very useful representation tools to perform expressions of generalizations. Caraher and Martinez (2008) state that children not only use notations / symbols but also must represent and give mathematical reasons, make conclusions and generalizations in their own way. So it can be concluded that generalization is a reasoning in making conclusions that contain symbolism and general. The generalization process can be applied to the matter of the number pattern. According to Siti (2015) that the process of pattern generalization is one form of algebraic thinking. In line with that Walle (2008), states that one component in algebra thinking is a generalization pattern, the child is able to describe the rules of a pattern.

Radford (2013) identifies generalizations into three stages: factual, contextual, and symbolic. Factual stage is the ability of students to capture the similarities seen in some elements of a sequence, so that at this stage students are able to find the regularity between patterns. This is in line with Siti (2015) who reveals that the generalization of the pattern in algebra thinking lies in the ability of students to capture the similarities and to look at some elements in a sequence of patterns and to realize that this similarity applies to the requirements of the sequence of patterns and is able to use them to give general expressions in abstract form. The generalization of algebraic patterns can be constructed from the understanding of similarities and differences between patterns. The second stage is the contextual stage in which the students' ability to recognize that the similarity applies to all similarities, so that students are asked to provide general rules on the pattern in the form of a sentence. In the generalization process is often done trial and error where the child gives a simple rule. This is in line with the research conducted by Raford (2007), stating that the heuristics of students in the generalization of the pattern is based on trial and error. In addition, Raford (2006), found that children in generalizing patterns see the common features of the given numbers then generalize these numbers in the next sequence. In generalizing the pattern of the child not only states in sentence form but also in algebraic form. Last is the symbolization stage in which at this stage the ability of students to use the direct expression of any term, so that students are able to give general rules on the pattern in the form of symbols.

This study uses an objective semiotic approach that focuses on gesture, words, and symbolic. When associated with the generalization process then gesture or gestures that accompany the child in generalizing the pattern can be the expression of the fingers or facial expression. At the factual stage, the process of determining the similarities and differences in a pattern can be observed from the expression of the fingers or expression shown by the child against the pictorial pattern. Words or words in the process of generalizing patterns are expressed by the child in the form of words or sentences that are not symbols. Symbolic or signs used by the child in generalizing patterns can be signs, pictures, or letters. Usually the sign used is a sign that is often known by the child. Berdasarkan paparan di atas, makalah ini bertujuan mendeskripsikan cara siswa SMP menggeneralisasikan pola berdasarkan perspektif semiotik. Subject dibedakan berdasarkan tingkat kemampuan matematika tinggi, sedang dan rendah.

## **METHODS**

This research uses qualitative approach. Data were collected in the form of student test result, interview transcript and observation result recorded. Subjects in this study were taken using purposive sampling technique from students of class VIII B SMP Kristen 2 Salatiga. Subject has studied the matter of the number pattern. Subjects were grouped on the basis of high, medium, and low mathematical abilities using the test scores of 23 students of class VIII B divided into 3 sections, ie high categories at intervals of 86-97, moderate categories at 75-85 intervals and low categories at intervals 65-74. Subjects picked up one from each interval.

The research instrument consists of the main instrument ie the research team and the auxiliary instrument that is the description test and the unstructured interview guide. Test description to find out the way students in solving problems related to the pattern of numbers in accordance with the stages of

generalization. Table 1 provides an indicator of the generalization stages of the pattern reviewed from a semiotic perspective.

**Table 1.** Indicators of Generalization Stage

Generalization Stage	Meaning of Generalization Stage	Semiotic Component	Meaning of Semiotic Component	Indicator
Factual	The ability of students to capture the apparent similarity in some elements of the sequence.	<i>Gesture</i>	<i>Gestures in factual stages contain: Component signs: markers and markers. Signature level: denotation</i>	Students are able to find regularities between patterns
		<i>Words</i>	<i>Words in factual stages contain: Mark component: marker and marker Signature level: connotation Axis sign: paradigm and sintagma</i>	
Contextual	The ability of students to realize that the similarity applies to all requirements	<i>Words</i>	<i>Words in the contextual stage contain: Mark component: marker and marker Signature level: connotation Axis sign: sintagma</i>	Students are able to give general rules to the pattern
Symbolization	The ability of students to use the direct expression of any term	<i>Words</i>	<i>Words in the symbolization phase contain: Mark component: marker and marker Axis sign: paradigm and sintagma Signature level: connotation</i>	Students are able to provide general rules on patterns in the form of symbols
		<i>Symbol</i>	<i>Symbol in the symbolization stage contains: Signature level: connotation Axis sign: sintagma The relation between the sign: metonymy</i>	

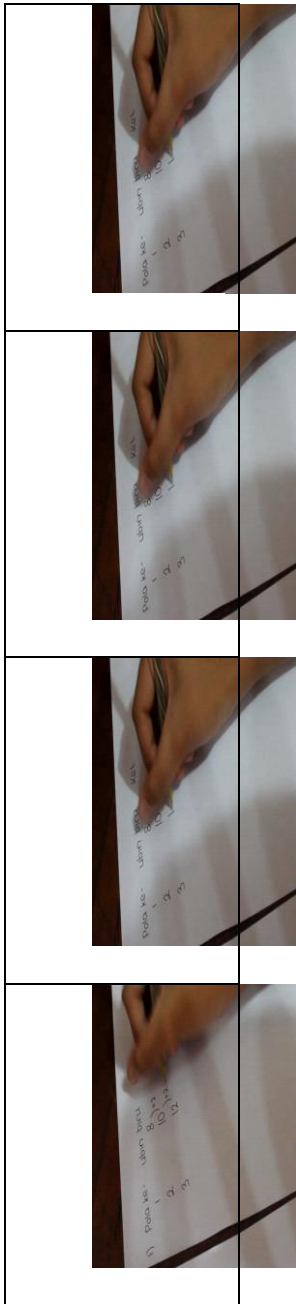
## RESULTS AND DISCUSSION

### Factual Stage

On tile-related problems it appears that subjects with high, medium, and low math skills have the same gesture. In calculating the number of tiles they do by pointing one by one the tile image on the question. They start counting from the first, second, and third diagrams in sequence.

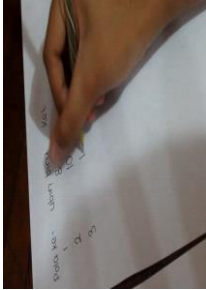
Unlike a tile-related problem, working on a gesture-brick problem that shows not only the expression of the fingers but the facial expressions also plays a role. It can be seen that subjects with high, medium, and low mathematical abilities in calculating the number of bricks in diagrams 1 and 2 use facial or eye movements, whereas in calculating the number of bricks in diagrams 3 and 4 using hand gestures by pointing at one image Bricks that are written on the matter.

Subjects with high, medium, and low math skills on tile-related problems have the same step of writing down the number of blue tiles that have been calculated on the provided worksheets. Writing the results is done alternately, in which the subject counts the number of blue tiles first and then write the results. This is done from the first to the last diagram in sequence. In the interview stage the subject states that the number of blue tiles each diagram is different. In addition, the number of blue tiles and the number of bricks between diagrams is always two, where two are the difference between the number of blue and brick tiles on each diagram.



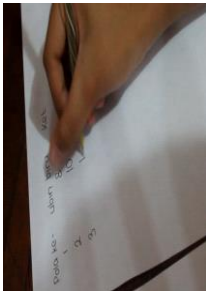
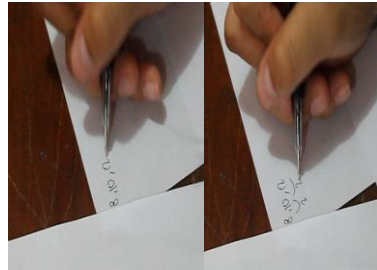
Researcher: Can you explain each step in doing this?

High Subject: If I think this is the first pattern of the blue tile there are eight, the pattern of the two tiles is ten, the pattern of the three tiles is twelve, of the eight tenths it adds two, from the twelfth ten plus two ...



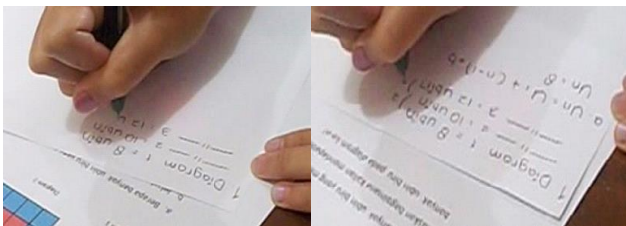
Researcher: Please explain by step how do you get that answer?

Medium Subject: First look for the formula first, then look for the difference of eight to ten, ten to twelve, it's U1 its eight



Researcher: Please explain step by step how you can do this problem?

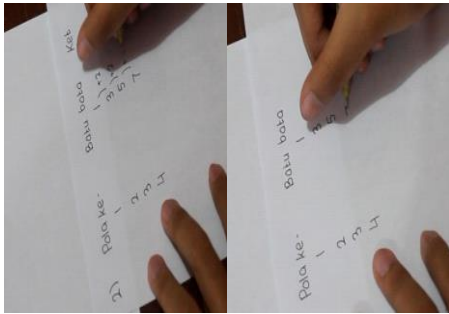
Low Subject: So first look for the formula to how, it's out of u1 means the first number of eight nuke .



Researcher: After counting, to find the difference you do how?

Low Subject: From 8 to 10 the difference is 2, it means 8 to 10 is added, it means in plus two.

**Figure 3.** The writing of the number and the difference between the blue tiles of each diagram and the transcript of subject interviews with high mathematical abilities (a), moderate (b), and low (c)



The workmanship step on bricks made by subjects with high, medium, and low math skills is not much different from the steps on the tiles. Each subject writes the number of bricks on the worksheet after the calculation, which is done intermittently on each diagram in sequence. There is a difference in the subject of moderate math in which the subject writes by adding a plus sign (+) between the number of each diagram or in the form of an arithmetic series. But when the interview stage and the subjects are asked to write it back it looks that the structure of writing is different where the subject write in the form of arithmetic sequence. After each subject to write the number of bricks, then the subject can determine the difference between the diagram is two. This can be seen from the steps of each subject and subject explanation when interviewing.

Researcher:

The question like this please explain how you do it?



High Subject:

If this is the first pattern of bricks there is a second one there are 3, the third there is 5, the fourth there is 7, now one to three two plus two, three to five plus two as well, five to seven plus two ...

Researcher:

Please explain per step how can you do this?

Medium

Subject:

This initially find the formula first, find the difference first, 1 to 3, 3 to 5, 5 to 7, the difference is 2, 2,2, trus kayak was  $2x 1$

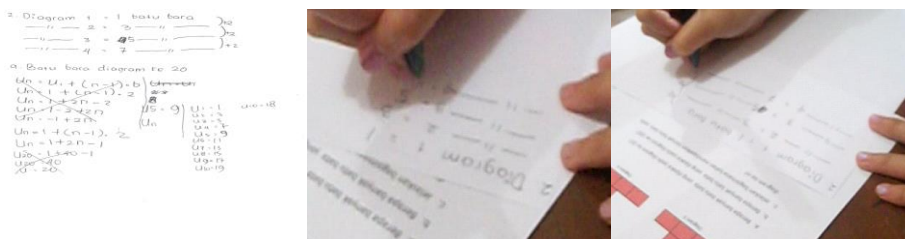
Researcher:

Please write again

Medium

1 to 3 to 5 to 7 difference 2, 2, 2 ...

Subject:



Researcher: Please explain the steps you are working on  
 Low Subject: So it's written first, which is ditanake diagram 20, so try this selisihe two two ..

**Figure 4.** The writing of the number and the difference of the bricks of each diagram as well as the transcript of interview subjects with high mathematics (a), moderate (b), and low (c)

**Contextual Stage**

The semiotic component of this stage is word, where the word here can be words and writings. Subjects with high, medium, and low math skills have the same way that they begin to experiment to find common ground and make equations as per their own understanding.

On matters related to subject tiles with high, medium, and low mathematical abilities both utilize the previously known gap to make a general rule. But the difference is their step in making the general rule. Subjects with high math skills begin to experiment with using a known difference, then input some numbers and mathematical operations to get results that match the number of blue tiles on each diagram.

Steps to work on a subject matter tile with math skills are using the term U or tribe, where U1 denotes the first term. The first step of a moderate-math subject is to use the arithmetic sequence formula, but the formula is not used. Then the mathematics-capable subject is beginning to take advantage of the known difference and multiplying it by the nth diagram. After that try to enter mathematical operations and some numbers so the result matches the number of blue tiles on each diagram.

In contrast to other subjects, a low mathematical subject directly uses the arithmetic sequence formula ( $U_n = a + (n-1) b$ ), where b is the difference and a is the sum of the first term. By using the formula it is seen that the difference and the first term applies to all diagrams.

Researcher: Can you explain each step in doing this?  
 High ..because it is always added two then both are inserted  
 Subject: kerumusnya multiplied by the pattern of kemapanya added inikan for example two at times one, two, continue to be added how the result of that eight plus six, it means all plus six, if the second pattern is twice Two, four, four plus six ten ...

1) Pola ke- ubin biru Ket.

1	8	$2n + 6 = 2 \cdot 1 + 6 = 8$
2	10	$2n + 6 = 2 \cdot 2 + 6 = 10$
3	12	$2n + 6 = 2 \cdot 3 + 6 = 12$

a) 50

$$2n + 6 = 2 \cdot 50 + 6 = 100 + 6 = 106 \text{ ubin}$$

b) Pola ke-1 = Ubin biru  
 Pola ke-2 = 8 } selalu ditambah 2  
 Pola ke-3 = 10 }  
 12 } maka,  
 $2 \cdot n + 6$



$8, 10, 12$   
 $8 + (2n-1)b$   
 $2.50 + b = 100$   
 $U_n = 2 + 2n + 6$

Researcher: U1 is for?  
 Medium Tribe, trus twice one plus six let be eight, then U2 samadengan  
 Subject: two multiplied two plus six equals ten

Researcher: Please explain how you can do this?  
 Low So first look for a formula that seberapa, out of it from u1  
 Subject: means the number of pertamanekan eight, (using the existing  
 formula) nah if the pertamane eight that (n-1), (pointing) n the  
 tribe tribe seberapa, times b, b That is the difference that  $u_n = 8$   
 $+ (n-1) 2$ ,  $8 + 2n$ , both times n times 1, the  $2n-2$  isni multiplied  
 by  $U_n$  equal to mean minus both displaced sinikan means eight  
 minus two plus  $2.n$ ,  $U_n$  Rumuse ketemune  $6 + 2n \dots$

1. Diagram 1 = 8 ubin )2  
 " 2 = 10 ubin )2  
 " 3 = 12 ubin )2  
 a.  $U_n = U_1 + (n-1) \cdot b$   
 $U_n = 8 + (n-1) \cdot 2$   
 $U_n = 8 + 2n - 2$   
 $U_n = 8 - 2 + 2n$   
 $U_n = \boxed{6 + 2n} \rightarrow \text{rumus}$   
 $U_{50} = 6 + 2n$   
 $U_{50} = 6 + 2 \cdot 50$   
 $U_{50} = 6 + 100$   
 $U_{50} = 106$   
 b. Menggunakan rumus  $U_1 + (n-1) \cdot b$

**Figure 5.** Interview transcripts and equations of each diagram are made by subjects with high mathematical abilities (a), moderate (b), and low (c)

In working on brick-related problems subjects with high and medium math skills have the same way of using the difference and trying to enter mathematical operations and some numbers so that the results match the number of bricks each diagram. But the subject is capable of mathematics while the interview goes on realizing that the subject is wrong in entering the mathematical operations that should be reduced (-) but the subject adds (+), since the subject assumes the number of bricks on the second diagram represents the number of first diagrams. So when subject interviews begin to improve the results of his work. In contrast to the other two subjects, subjects with low mathematics have their own way that the subject starts counting manually without using a known equation. Then after the subject searched the

number of bricks until the 10th diagram, the subject began to experiment to find the general rule.

2)	Pola ke-	Batu bata	Ket.
	1	1 ) +2	$2n - 1$
	2	3 ) +2	$2n - 1 = 2 \cdot 2 - 1 = 4 - 1 = 3$
	3	5 ) +2	
	4	7	

a) 20

$$2n - 1 = 2 \cdot 20 - 1 = 40 - 1 = 39 \text{ Batu bata}$$

b) 100

$$2n - 1 = 2 \cdot 100 - 1 = 200 - 1 = 199 \text{ batu bata}$$

c)

1	1	selalu ditambah 2
2	3	
3	5	
4	7	

Rumus =  $2n - 1$

Researcher: The question like this please explain how you do it?  
 High Subject: ... One to three plus two, three to five plus two as well, five to seven plus two, both of these are entered into the formula, multiplied by the pattern beberapa, if for example pattern to one, two multiplied one to two, to be able to This means less one, if the second pattern means two times multiplied by four to become three also lessen one, then so too.

Researcher: Please write again  
 Medium Subject: 1 to 3 to 5 to 7 difference 2, 2, 2, for example tribe 1, means 2 multiply 1 let be 3 so add 1, 2 multiply 2 equals 4 plus 1 so  $5, 3 \times 2 = 6 + 1 = 7$

Handwritten work showing the sequence  $1 + 3 + 5 + 7$  with arrows indicating the addition of 2 between terms.

Below this, there are several equations for the general term  $U_n$  and the sum  $S_n$ :

$$U_1 = 1$$

$$U_2 = 3$$

$$U_3 = 5$$

$$U_n = 2n - 1$$

On the right side, the sum  $S_n$  is calculated for  $n=2, 3, 4$ :

$$S_2 = 2 \cdot 2 + 1 = 5$$

$$S_3 = 2 \cdot 3 + 1 = 7$$

$$S_4 = 2 \cdot 4 + 1 = 9$$

At the bottom, the sum  $S_n$  is calculated for  $n=20$  and  $n=100$ :

$$S_{20} = 2 \cdot 20 + 1 = 41$$

$$S_{100} = 2 \cdot 100 + 1 = 201$$

Researcher: Means the first term starts from this (pointing to the 2nd diagram), this one (pointing diagram 1) is not considered..  
 Medium Subject: It means I am wrong  
 Subject: How should it be?  
 Researcher: How should it be?

Medium                    It should be  $2n-1$   
 Subject:  
 Researcher:             $2n$  can be retrieved from...  
 Medium                    difference of this and that, the sum of  $u_1 u_2 u_3 u_4$  from the  
 Subject:  
 Researcher:             $-1$  because  
 Medium                    Because this started from 1, for example, plus 1, the result is  
 Subject:                    like that, 3

$1, 3, 5, 7$   
 $2, 2, 2$   
 $u_1 = 2 \cdot 1 + 1 = 3$   
 $u_2 = 2 \cdot 2 + 1 = 5$   
 $u_3 = 2 \cdot 3 + 1 = 7$   
 $2n - 1$   
 $u_1 = 2 \cdot 1 - 1 = 1$   
 $u_2 = 2 \cdot 2 - 1 = 3$

2. Diagram 1 = 1 batu bara  
 " 2 = 3 " "  
 " 3 = 5 " "  
 " 4 = 7 " "

a. Batu bara diagram ke 20

~~$u_n = u_1 + (n-1) \cdot b$~~   
 ~~$u_n = 1 + (n-1) \cdot 2$~~   
 ~~$u_n = 1 + 2n - 2$~~   
 ~~$u_n = -1 + 2n$~~   
 ~~$u_n = 1 + (n-1) \cdot 2$~~   
 ~~$u_n = 1 + 2n - 1$~~   
 ~~$u_{20} = 1 + 40 - 1$~~   
 ~~$u_{20} = 40$~~   
 ~~$u = 20$~~

$u_n = 2n - 1$   
 $u_5 = 9$   
 $u_1 = 1$   
 $u_2 = 3$   
 $u_3 = 5$   
 $u_4 = 7$   
 $u_5 = 9$   
 $u_6 = 11$   
 $u_7 = 13$   
 $u_8 = 15$   
 $u_9 = 17$   
 $u_{10} = 19$

Researcher: Please write again  
 Low  
 Subject: write the difference here, if it is added with 1, it is not appropriate.  
 Researcher: You get two as the difference, where do you get

from?

Low One to three, the difference is 2  
Subject:

$2n$	0 1 3 5 7 9 11 13 15 17 19
$2 \cdot 5 = 10$	$2 \cdot 5 - 1$
$2n - 1$	$10 - 1$
$2n - 1$	$2 \cdot 10 - 1$
$2 \cdot 6 - 1$	$20 - 1$
$12 - 1$	
11	

**Figure 6.** Interview transcripts and equations of each diagram made by subjects with high (a), moderate (b1), and low (c), as well as new equations created by moderate math subjects (b2)

**Symbolization Stage**

At this symbolization stage there are two components of semiotic symbol and word. The symbol component used by the three subjects on the tile problem has the same formula that is  $2 \cdot n + 6$ . In the interview stage the three subjects have the same opinion that they explain that two is the difference between the diagrams, the variable "n" denotes the diagram or the nth term, and adds six so that the result matches the number of blue tiles of each diagram.

Researcher: Perhaps, from the two, how come you get 2-n?  
Subject: The two is because of this (showing how many blue floor  
tinggi: tile), it is always added by two, the n is the pattern of the  
upcoming number...

1) Pola ke-	ubin biru	Ket.
1	8 ) +2	$2n + 6 = 2 \cdot 1 + 6 = 8$
2	10 ) +2	$2n + 6 = 2 \cdot 2 + 6 = 10$
3	12 ) +2	$2n + 6 = 2 \cdot 3 + 6 = 12$
a) 50	$2n + 6$	
	$= 2 \cdot 50 + 6$	
	$= 100 + 6 = 106$ ubin	
b) Pola ke-1 =	ubin biru	
Pola ke-2 =	8 ) selalu ditambah 2	
Pola ke-3 =	10 )	
	12 )	maka,
		$2 \cdot n + 6$

Researcher So the formula is?

:  
 Medium Th formula is  $2n+6$   
 Subject:  
 Researcher Where do you get  $2n$ ?  
 :  
 Medium  $2n$  from the difference from previous to miss itukan  $U_n$ ,  
 Subject: means the difference of 2, "n" of this tribe earlier, eg this  
 tile in misalke  
 Researcher So the 'n' is the floor tile?  
 :  
 Medium Yes  
 Subject:  
 Researcher Means 2 in can from the difference and "n" of the tiles  
 : 1,2,3, then six from where?  
 Medium this is added by 8,  $2 \times 1$  then it is added by what number?  
 Subject: In order to get 8?

2). 106  
 b).  $2n+6$   
 → mencari selisih dari diagram → "4"  
 mengalikakan ubin merah. → "n"  
 menambahkan bilangan → 6

Researcher: We can say that the formula is already there, so you should find the existing formula, What is n here?  
 Low Subject: To find out, the  $2n$  here should not be written, the two is multiplied by n here.  
 Researcher: What is the meaning of 2 here?  
 Low Subject: Two I the difference.

1. Diagram 1 = 8 ubin )<sub>2</sub>  
 — " — 2 = 10 ubin )<sub>2</sub>  
 — " — 3 = 12 ubin )<sub>2</sub>

a.  $U_n = U_1 + (n-1) \cdot b$   
 $U_n = 8 + (n-1) \cdot 2$   
 $U_n = 8 + 2n - 2$   
 $U_n = 8 - 2 + 2n$   
 $U_n = \boxed{6 + 2n} \rightarrow \text{rumus}$

b. Menggunakan rumus  $U_1 + (n-1) \cdot b$

$U_{50} = 6 + 2n$   
 $U_{50} = 6 + 2 \cdot 50$   
 $U_{50} = 6 + 100$   
 $U_{50} = 106$

Figure 7. Transcription of interview with high, medium, and low achiever.

The formula obtained by the three subjects on the problem related to the same brick is  $2.n-1$ . Although at first the subject with mathematical ability was writing with the formula  $2.n + 1$ , but at the interview stage the subject realized that the formula was made wrong and began to replace it. During the interview stage the three subjects have the same opinion that two are the difference, the variable "n" denotes the nth diagram, and subtracts one so that the result matches the number of bricks per diagram.

Researcher: What is the two here?  
 High Subject: Telling the difference of each pattern.  
 Researcher: So, one is gained from?  
 High Subject: This is two, so two times one, so the result is one minus one.

Handwritten notes and calculations:

2) Pola ke-  
 $1$   
 $2$   
 $3$   
 $4$

Batu bata  
 $1$   
 $2$   
 $5$   
 $7$

Ket.  
 $2n - 1$   
 $2 \cdot 2 - 1 = 2 \cdot 2 - 1 = 4 - 1 = 3$

a) 20  
 $2n - 1$   
 $= 2 \cdot 20 - 1$   
 $= 40 - 1 = 39$  Batu bata

b) 100  
 $2n - 1$   
 $= 2 \cdot 100 - 1$   
 $= 200 - 1 = 199$  batu bata

c)  $2n - 1$   
 $1$   
 $3$   
 $5$   
 $7$   
 selalu ditambah 2  
 Rumus:  $2n - 1$

Researcher : So?  
 Medium Subject: It should be  $2n-1$   
 Researcher :  $2n$  is gained from?  
 Medium Subject: The difference from this and this, the sum of unit  $U_1 U_2 U_3 U_4$

Handwritten notes and calculations:

2. Diagram 1 = 1  
 " 2 = 3  
 " 3 = 5  
 " 4 = 7

Diagram 20 = ?  
 rumus =  $2n - 1$

Jawab =  
 $2n - 1$   
 $= 2 \cdot 20 - 1$  (a) }  
 $= 40 - 1$  }  
 $= 39$  }

$2n - 1$  (b) }  
 $= 2 \cdot 100 - 1$  }  
 $= 200 - 1$  }  
 $= 199$  }

mencoba rumu  $2n - 1$

Researcher : Where did you get n?  
 Low Subject: The two is from here, and n is questioned.  
 Researcher : Then lower than 1 from...  
 Low Subject: Just now when I try to minus one, the answer is correct.

**Figure 7.** Interview transcripts and formulas created by subjects with high mathematics (a), moderate (b2), and low (c), as well as new formulas created by moderate math subjects (b1)

### **Factual Stage**

At the factual stage students are able to capture the apparent similarities in some elements or sequences. This is in line with Siti (2015), arguing that the generalization of algebraic patterns can be constructed from the understanding of similarities and differences between patterns.

Based on the results of research at this stage subjects capable of high, medium, and low mathematics able to change the sequence of pictorial patterns to the sequence of pattern numbers. In changing the pattern the subject uses gestures in the form of hand movements and eye movements, as well as word in the form of handwriting and spoken words. Gestures and words used by the three subjects in both the tile and brick related issues are the same. From the sequence of number patterns that have been created, the subject can determine the similarities and differences between patterns. The similarity is the difference between diagrams is always the same and the difference is each diagram has a number of tiles or different bricks. This study is in line with the research that has been made by Siti (2015), where the process of finding similarities and differences in a pattern can be observed from the expression of the fingers or mimics shown by the child against the pictorial pattern. And children can express with words or sentences that are not symbols. Radford (2007), found that children in generalizing patterns were done through coordinating gestures, observations, and voices.

### **Contextual Stage**

The contextual stage is the stage where students are able to realize that the similarity applies to all requirements. At this stage the subjects with high, medium, and low ability to experiment either using the existing formula or experiment with mathematical operations and enter some numbers to determine the equations that match the desired pattern. This is in accordance with the findings of Radford (2007), which states that the heuristic students in generalizing the pattern based on the trial and error. Although at this stage students do the same way of experimenting, but their steps in trying to look different when the process of workmanship. Scribbling done by the subject when workmanship and interview is the way the subject understands. This is in accordance with the opinion of Oers (2010) which states that through symbols (eg words or numbers) that children give to their images is an attempt to understand children. In addition, Radford (2006), found that children in generalizing patterns see the common features of the given numbers then generalize these numbers in the next sequence.

- The process of generalizing patterns is one form of algebraic thinking. The generalization of patterns in algebra thinking lies in the ability of students to capture the similarity in looking at some elements in a sequence of patterns and to realize that this similarity applies to the ordering requirements of patterns and is able to use them to give general expressions in abstract form (Siti, 2015). In line with Siti (2015), Walle (2008), states that one component in algebra thinking is a generalized pattern, the child is able to describe the rules of a pattern.

### **Symbolization Stage**

The last stage of the pattern generalization process is the symbolization stage, in which at this stage students are able to give general rules to the pattern in the form of symbols. According to Walle (2008) that in creating generalizations need to use symbolism, therefore both generalization and understanding of variables and symbolism must be developed simultaneously.

Subjects in this final stage begin to use the variables in the general rules it creates. According to Walle (2008), the variable is a very useful representation tool for expression and generalization. Based on

the result of research done component of symbol and word done by third subject is same. But the subject not only writes the formula, but also explains how the formula has been made. This is in line with Caraher and Martinez (2008), stating that children not only use notations / symbols but also must represent and give mathematical reasons, make conclusions and generalizations according to them.

## CONCLUSION

Semiotics have an important role in the process of generalizing patterns in which to generalize the pattern not only seen from the work of students, but based on the process of students in understanding and making something or called semiotic objectification. Semiotic components that appear in the generalization process are gesture, word, and symbol. The generalization stage also consists of three stages: factual, contextual, and symbolization. Based on research conducted at the factual stage, the gesture and word of the three subjects are the same. At the contextual stage each subject has different steps of workmanship or manner according to the understanding of each subject. As well as in the final stages of symbolization, the three subjects can create the same formula.

This paper is expected to be a reference for teachers to not only assess the outcome but can see the ability of students in the process of generalizing a pattern, so that teachers can know the understanding of each student. For other researchers this paper can be used as a reference to examine the generalization of patterns primarily based on semiotic perspectives.

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