Kreano 12 (1) (2021) : 130-149

KREANO

Jurnal Matematika Kreatif-Inovatif

http://journal.unnes.ac.id/nju/index.php/kreano

The Coherence of Group Scheme of the High Initial Ability Students Based on Cognitive Style

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History Article

Received: March, 2021 *Accepted: May, 2021* *Published: June, 2021*

Abstract

Teaching and learning deductive proof is one of the most important goals in mathematics education. According to the APOS theory, learning a concept is facilitated when students have constructed an adequate APOS mental structure for the concep. There are characteristics differences between field-dependent and field-independent students in responding to tasks to construct proofs. The purpose of this study was to analyze the coherence of the group scheme constructed by students with the high initial ability based on cognitive style to construct proofs. This study was a qualitative. The research subjects were determined by the purposive sampling. Data collection using test and in-depth interviews. The credibility of data was carried out using triangulation. Data analysis used Miles and Huberman's model. The results showed that the FI and FN Subjects had thematized the group scheme and were coherent, while the FD Subject had thematized the group scheme but was not coherent.

Abstrak

Pengajaran dan pembelajaran bukti deduktif dalam matematika merupakan salah satu tujuan terpenting dalam pendidikan matematika. Menurut teori APOS (Aksi, Proses, Objek, Skema), belajar suatu konsep terfasilitasi apabila siswa telah mengkonstruksi struktur mental APOS yang memadai untuk konsep tersebut. Ditinjau dari gaya kognitifnya, ada perbedaan karakteristik antara mahasiswa field-dependent dan field-independent dalam merespon tugas yang memerlukan kemampuan mengkonstruksi bukti. Tujuan penelitian ini adalah menganalisis koherensi Skema grup yang dikonstruksi mahasiswa dengan kemampuan awal mengkonstruksi bukti adalah tinggi dan gaya kognitif FI, FN, FD. Penelitian ini dirancang sebagai penelitian kualitatif. Subjek penelitian ditentukan dengan teknik purposive sampling. Teknik pengumpulan data menggunakan teknik tes dan wawancara mendalam. Derajat kepercayaan data dilakukan dengan teknik pemeriksaan triangulasi. Analisis data selama di lapangan mengggunakan model Miles dan Huberman. Hasil penelitian menunjukkan Subjek FI dan FN sudah mentematisasi Skema grup dan sudah koheren, sedangkan Subjek FD sudah mentematisasi Skema grup namun belum koheren.

***Keywords**: APOS; cognitive style; proof; initial ability.*

INTRODUCTION

Teaching and learning deductive proof in mathematics is one of the most important goals in mathematics education (Miyazaki, *et al.*, 2017; Anton & Rorres, 2015, Solow, 2014; David, Yopp, & Rob, 2015; Sarah, Bleiler, & Jeffrey, 2017; Mills, 2014).

p-ISSN: 2086-2334; e-ISSN: 2442-4218

Group theory is an abstract topic involving formal definitions, theorems, and proofs. Theorems are deduced from axioms. As an abstract subject, group theory is open to misconceptions at the more general levels of proof and logical reasoning (Alcock, et al., 2015). The characteristic of group theory emphasizes on the abstract thinking aspect (Putra & Kristanto, 2017). According to Wasserman (2017), the abstractness of group theory is useful for teachers because it helps them understand and interpret the mathematics they are going to teach. Therefore, prospective teachers need to learn this material because it can help later as a teacher to connect advanced mathematics with school mathematics in terms of strengthening and deepening their understanding of the mathematics they will teach. In group theory lectures, knowledge of the student's concept of group should include understanding various mathematical properties and constructing examples. (Dubinsky et al, 1994).

APOS theory is a constructivist theory of how learning mathematical concepts occurs. The hypothesis on learning according to the APOS Theory is facilitated learning if individuals have adequate mental structures for certain mathematical concepts. If there is no adequate mental structure then learning concepts is almost impossible. (Arnon, 2014). In the APOS Theory, there are four mental structures, namely Action, Process, Object and Scheme.

According to Piaget and adopted by APOS Theory (Arnon, et al, 2014), a concept is understood first of all as Action. The interiorized Action is a Process. Dubinsky, et al (2005) states that if a person becomes aware of the Process as a totality, realizes that transformation can act on that totality, and can construct the transformation (explicitly or in someone's image), then it is said that the individual has encapsulated the Process into the cognitive Object. A Schema is defined in APOS Theory as an individual's collection of mental structures Actions, Processes, Objects, and others Schema linked consciously or unconsciously in a coherent framework in the individual's mind (Arnon, et al., 2014). The coherence of the Schema is determined by the individual's ability to determine whether it can be used in a particular mathematical situation. The indicators of the coherent group Scheme constructed by students used in this study are (1) able to provide group example and its proof; (2) able to apply a variety of group properties to solve related problems; and (3) able to examine the various properties of the group.

The way that is done consistently in capturing, understanding, and processing new information, and solving problems is called cognitive style (Witkin & Goodenough as quoted by Cataloglu & Ates, 2014; Altun & Cakan, 2006). There are various cognitive styles, one of which is field-dependent/ independent. The characteristics of field-independent (FI) students are choosing the deductive method and excel in problem-solving tasks. In contrast, the characteristics of field-dependent (FD) students are that they prefer inductive methods and excel in the knowledge domain that focuses on social problems (Dowlatabadi & Mehraganfar, 2014; Ates & Cataloglu, 2007). The characteristics of the cognitive style of FD that prefer inductive methods are not in line with the characteristics of the group material, namely axiomatic deductive. The differences in characteristics between FD and FI students in responding to tasks that require the ability to construct proofs will have an impact on the differences in group Scheme constructed by students.

Mathematical material is systematic, the concepts to be studied have prerequisites that must be mastered by students. Students will connect the new knowledge they have acquired with the initial knowledge they have (Ruseffendi, 2006; Wahyudin 2012). This is in line with Soehakso in Hanifah & Abadi (2018), Rubowo (2017), and Dubinsky, et al. (1994) which states that the student's ability in algebra, including groups, is closely related to the provisions of students when studying at a previous level, especially set and functions. Melhuish, et al. (2019) argue that binary operations are one of the fundamental structures underlying our algebraic systems that rely on sets and functions. Cecco (in Jamaan et al., 2020) states that the initial ability is the knowledge and skills possessed by students before they move on to the next level. Initial ability in this study is the ability to construct proof on binary operations. Based on the academic guidelines at the university where this research took place, students with a score of 81 received an AB grade. Students with a minimum score of 81 are categorized as high. Students with high initial abilities should be able to construct a coherent group Scheme. However, students with high initial ability may have a cognitive style of FD who prefers an inductive method that is not in line with the characteristics of the group material. Therefore, it is necessary to do an analysis regarding the coherence of the group Scheme of students with high initial abilities. In order to students to achieve the mental structure of the group scheme, it is needed to pay attention to their initial ability to construct proof.

Various studies on the proof for group theory have been carried out. Thompson, et al., (2012); Stylianou & Blanton, (2011) conducted research on the importance of writing proof, while Moore (1994), Samkoff, et al. (2012) conducted research on students' difficulties in proving. The leveling on proof carried out by Sowder & Harel (1998), Weber (2004), and Isnarto (2014). However, research has not found the coherence of group schemes associated with cognitive style and students' initial ability to construct proof.

In a preliminary study of the FI/FD students difficulties in proving on group theory conducted by Wijayanti (2016), prospective teachers experience difficulties which indicate that students have not constructed adequate mental structures for the group concept. This condition has not fulfilled the hypothesis on learning so that learning has not been facilitated. According to the APOS theory, it is almost impossible for these students to learn the concepts of group. Some of the difficulties experienced by students include identifying elements in the set, misconceptions related to the use of mathematical notation, identifying known statements, and using definitions to prove. According to Dubinsky, et al (1994) students' conceptions of sets and functions play important role in learning the group concepts. The difficulty of students in identifying the elements in the set shows that there are problems related to their initial ability. Barriers to this initial ability have an impact on the formation of student's group Schemes. Brijlall & Bansilal (2010) argue that interiorization is characterized by the ability to apply symbols, language, images, and mental images to construct internal processes as a way of understanding perceived phenomena. David, et al (2016) stated that general representations such as mathematical symbols or quantified variables are often used in formal proof. This shows that the difficulty of using mathematical notation will have an impact on the ability to prove, including examining various properties of groups and applying them in solving related problems. In line with this, the difficulty of using mathematical notation indicates that students have not yet constructed a mental structure of Process, which means that the group Scheme has not been formed.

Furthermore, there is an effect of students' initial ability on students' ability in constructing proofs in a class using APOS-based learning (Wijayanti, et al., 2018). Students with low initial ability had not yet constructed the group Scheme, while students with high initial ability displayed the constructed group Scheme (Wijayanti, et al., 2019).

Since the constructed group Scheme plays a role in solving various problems related to the group, it is necessary to analyze the coherence of the group Scheme constructed by students. This study aims to analyze the coherence of the group Scheme constructed by students with high initial ability based on the cognitive style. According to Witkin (1977), FD students can be guided in handling problem solving such as constructing proof so that they can perform as well as FI students. Therefore, the results of this analysis can be used to design group learning materials so that each student can construct the expected group Scheme.

METHODOLOGY

This study was designed as a qualitative study. The findings of the group Schemes constructed by students were qualitatively revealed based on their cognitive styles. Research subjects were determined by purposive sampling technique and subject selection using the GEFT instrument (score 0-18) developed by Witkin and the test of students' initial ability to construct proofs (score 0-100) developed by the researcher. Students' cognitive styles in this study were classified into 3 categories, namely FI (score 15-18), FN/ field-neutral (score 9-14), and FD(score 0-8). Based on the Academic Guidelines at a university, student who gets a score of 81 received an AB grade. In this study, student was classified into the high initial abilities if the student get a score at minimum of 81 on the test of students' initial ability to construct proofs. The research subjects were 3 students, namely students with the high initial abilities and FI, FN, or FD cognitive style. The data source is students at a university who were taking courses that contain group material. The research variable is the ability to construct proofs. The instruments used were: (1) Group Embedded Figure Test (GEFT), (2) Initial ability test of constructing proof, (3) test of constructing proof on group material, (4) interview guidelines. The instrument of students' initial ability test was an essay test about binary operations, consisted of 4 item, was empirically tested with the result 4 item were valid, the test reliability was high (the reliability coefficient was 0,8018), the difficulty index was medium (2 item) and high (2 item), the discriminant index was good (3 item) and very good (1 item). While the instrument of the test of constructing proof on group material was an essay test, consisted of 3 item, was empirically tested with the result 3 item were valid, the test reliability was medium (the reliability coefficient was 0,5819), the difficulty index was low (1 item) and medium (2 item), the discriminant index was good (1 item), fair (1 item), and poor (1 item). Data collection techniques using test and in-depth interviews. The credibility of the research data was carried out using triangulation techniques. Data analysis while in the field used Miles and Huberman's model, namely data reduction, data display, and conclusion drawing/verification.

The limitation of this study was the number of the high initial ability subjects was 5 of 36 consists of 2 FI, 2 FN, and 1 FD. This condition causes researchers to be unable to choose subjects who can provide complete information. However, the results of this study can provide an overview of the mental structures constructed by the subject.

RESULT AND DISCUSSION

Result

At the beginning of the lecture, which contained group material, the topics of sets, mapping, and binary operations were presented. An overview of students' initial ability in constructing proof was obtained through an essay test of binary operation which consisted of 4 problems. Furthermore, the GEFT was carried out to determine the cognitive style of students. The research subjects were 3 students with the high initial abilities and cognitive styles of FI, FN, and FD.

The learning was carried out based on the APOS theory with the stages of Activities, Classroom Discussions, and Exercises. In the Activities stage, students work in groups working on assignments on the Student Task Sheet (LTM) with the aim that students construct mental structures of Processes and Objects. At the Classroom Discussion stage, the lecturer allowed students to reflect on their work during the Activities stage. In this activity, the lecturer may provide definitions, explanations, and present an overview to unify what students have thought and done during the Activities stage. At the Exercises stage, students work in groups working on exercise questions which consist of standard questions designed to strengthen activities in the Activities and Classroom Discussion stages. Through activities at the Classroom Discussion and Exercises stage, it is hoped that students can construct mental structures of Objects and Schemes. At the end of the lesson, students do the essay test on the ability to construct proofs. The test consists of 3 questions to measure the predetermined indicators, namely students can (1) give an example of a group and provide proof; (2) apply various group properties to solve related problems; and (3) examine the various properties of the group.

The following presents question number 1 and the results of the analysis of the schemes constructed by the FI, FN, and FD subjects. Problem 1 was intended to reveal whether the subjects have thematized the group Scheme.

Problem 1
a. Write a group definition.
b. Give an example of a group and prove it.

The work of Problem 1 of the FI Subject can be seen on Figure 1 (***See Appendix A** of this article*). FI Subject can write group definition correctly. Symbols and notations are presented appropriately and mathematical language is used correctly, but associative properties are only expressed in the mathematical language without using symbols.

The FI Subject can properly exemplify the group i.e $<\mathbb{Z},+>$. The FI Subject shows the applicability of the associative property of addition to the set of integers using symbols and mathematical language correctly. Group axioms have been thematized by the FI Subject which are demonstrated in examining the associative property, the existence of the identity element, and the inverse element. The FI Subject has constructed a mental structure Schema for the definition of a group, axioms fulfilling associative properties, the existence of identity elements, and the existence of inverse elements. This is shown by the ability of the FI Subject to proving the validity of the associative law, the existence of the identity element, and the existence of the element inverse with valid steps.

The FI Subject has constructed the mental structure of Object for the addition of binary operation on the set of integers with the indicator being able to check the addition of binary operations satisfies associative properties, the existence of identity element, and the existence of element inverse. In this section, FI Subject correctly states associative property and identity element with symbols and language. In contrast, FI Subject states the definition of a group in mathematics language correctly for associative axiom and uses symbols for the existence of identity element and the existence of an inverse element.

Based on the description above, the coordination between the set of integers, binary operation of addition, and group axioms has been carried out by the FI Subject properly and correctly. The FI Subject can give an example and prove an example is a group. The ability of the FI Subject to determine that the set of integers under addition is a group indicates that the group Scheme has been thematized.

The work of Problem 1 of the FN Subject can be seen on Figure 2 (***See Appendix A*** *of this article*). Based on Figure 2, the FN Subject can write the group definition with symbols and mathematical language appropriately. The FN Subject can properly exemplify the group, namely $< \mathbb{Z}, + >$, without describing the symbol. The FN Subject shows the associative law of addition on the set of integers using the associative property argument to be proved. The FN Subject can show the existence of the identity element and the existence of the inverse element for the addition of the set of integers accompanied by the correct use of symbols.

Based on the results of the work in Figure 2, the FN Subject can write a group definition with the correct language and symbols. This shows the FN Subject constructs a mental structure Schema for group axioms. Since the FN Subject shows associative law using the properties to be shown, it is said that the FN Subject constructs the mental structure of Process for associative axiom. Furthermore, the FN Subject succeeded in showing the axiom of the existence of the identity element and the existence of the inverse element with the correct steps. The FN Subject is said to have constructed a mental structure Schema for the axioms of the existence of an identity element and an inverse element. The FN Subject uses language and symbols correctly. Therefore, the FN Subject is said to construct the mental structure of Process for language and symbols.

The membership of the integer set can be recognized well by the FN Subject with an indicator that can identify the identity element and the inverse element for the addition of the set of integers. This shows that the FN Subject has constructed a mental structure for Action for the set of integers. The FN Subject can examine the addition satisfying properties of associative, identity element, and inverse element. In this section, the FN Subject states properties of associative, identity element, and inverse element with the correct symbols and language. This shows that the FN Subject has constructed the mental structure of Object for addition on the set of integers associated with properties of associative, identity element, and inverse element.

Based on the description above, the coordination between the Processes of the set of integer numbers, the binary operation of addition, and group axioms has been carried out by the FN Subject properly. The FN Subject can give an example and prove the example is a group. The ability of the FN Subject in determining that a set of integers under the addition is a group indicates that the group Scheme has been thematized.

The work of Problem 1 of the FD Subject on Figure 3 (***See Appendix A*** *of this article*). The FD Subject can state the group definition using the correct language but not accompanied by using symbols. Based on the results of the work in Figure 3 and the interview, the FD Subject wrote the group definition using the correct language. The meaning of the axioms stated in the group definition has not been explained. It can be said that the FD subject constructs the mental structure of the Process for the group definition.

The FD Subject can exemplify the group correctly, using the symbol:

$$< \mathbb{Z}, + >$$

but not mentioning the meaning of the $\mathbb{Z}$ symbol.

The FD subject succeeded in showing the associative law of addition on $\mathbb{Z}$ and concluded with symbols and mathematical language correctly. Furthermore, the FD Subject concluded the existence of identity element for the addition on $\mathbb{Z}$ with symbols. This complements the existence of an identity element in the definition expressed in language.

The FD Subject showed that each element in $\mathbb{Z}$ has an inverse for the addition on $\mathbb{Z}$. The FD Subject concluded with symbols. It also complements the existence of an inverse element in the language-represented definition. Since the FD Subject can show the axioms of associative, the existence of identity element, and the existence of an inverse element, it is said that the FD Subject has constructed a mental structure of Schema for the axioms of associative, the existence of identity element, and the existence of an inverse element.

The membership of the set of integers can be recognized well by the FD Subject which is characterized by being able to identify the identity element and the inverse element for the addition on the set of integers. This shows that the FD Subject has constructed a mental structure of Action of the set of integers.

The FD Subject has constructed the mental structure of Object for the addition on the set of integers with the indicator being able to check the addition satisfies the properties of associative, identity element, and inverse element. In this section, the FD Subject states the associative law with symbols and language but other axioms only use symbols. In contrast, the FD Subject stated the group definition in the mathematical language without symbols correctly. This shows that the FD Subject can express group axioms using language and symbols correctly. Thus, it is said that the FD Subject has constructed the mental structure of Process for language and symbols.

Based on the description above, the coordination between the set of integers, the binary operation of addition, and group axioms were carried out by the FD Subject. The Schema of the group has been thematized which is shown by the ability to determine the set of integers under addition is a group.

In Problem 1, the three Subjects gave the same example for the group, namely the set of integers under addition and the three Subjects were able to prove it. The result of the analysis of the work of this problem was the three Subjects of FI, FN, and FD have thematized the group Scheme.

Furthermore, the three Subjects' abilities were analyzed in applying the group characteristics to solve the problems at hand. Problem 2 was used to reveal this ability.

Problem 2
Write the Lagrange's Theorem
Let $\varphi: G \rightarrow G'$ be a group homomorphism. Show that if $|G'|$ is finite then $|\varphi(G)|$ is finite and divide $|G'|$!

The work of Problem 2 of FI Subjec can be seen on Figure 4 (***See Appendix A** of this article*). The FI Subject can write Lagrange's Theorem using language and symbols correctly. Based on Figure 4 and the results of the interview, the FI Subject can write down Lagrange's Theorem correctly. The language and symbols are used appropriately.

Based on Figure 4, the FI Subject can apply properties of finite groups, the map of a finite group, and Lagrange's Theorem. The proof begins by showing that if $\varphi: G \rightarrow G'$ is a homomorphism and K is a subgroup of G, then $\varphi(K)$ is a subgroup of G'. The proving step is carried out following the statement that must be proven, namely showing $\varphi(K)$ is a non-empty subset of G' and for every $x, y \in \varphi(K)$ satisfies $xy^{-1} \in \varphi(K)$. Then, the FI Subject applies the finite group property of G' to apply Lagrange's Theorem at the end of the proof. The results of the interview showed that the FI Subject could prove Problem 2 correctly. Based on Figure 4 and the results of the interview, the FI Subject can apply the properties of the group in proving Problem 2.

The work of Problem 2 of the FN Subject can be seen on Figure 5 (***See Appendix A** of this article*). Based on Figure 5, the FN Subject can write Lagrange's Theorem correctly. In the proof, the FN Subject has not written what is completely known. In the step of the proof, the FN Subject does not show a statement that if $\varphi: G \rightarrow G'$ is a homomorphism and K is a subgroup of G then $\varphi(K)$ is a subgroup of G' but directly uses this result. At the time of the interview, the FN Subject could indicate that $\varphi(K)$ is a subgroup of G'. Furthermore, using the property of G' to be a finite group, the FN Subject applies Lagrange's Theorem to end the proof. Based on Figure 5 and the results of the interview, the FN Subject can apply the properties of groups to prove Problem 2.

The work of Problem 2 of the FD Subject can be seen on Figure 6 (***See Appendix A** of this article*). Based on Figure 6, the FD Subject is not quite right in writing Lagrange's Theorem. The statement in the form of implication has not been stated correctly. The FD Subject can write a statement that must be proven. The initial step of the proof taken by the FD Subject was to show $\varphi(G)$ is a subgroup of G in the domain of φ. The FD Subject proved this statement by arguing that from the definition of $\varphi(G)$, it was clear that $\varphi(G)$ was a subgroup. This indicates that the FD Subject does not understand the proof of this statement. Furthermore, the FD Subject has not used finite group properties to apply Lagrange's Theorem.

Based on Figure 6 and the results of the interview, the FD Subject did not understand the statement that had to be proven and was unable to apply the properties of the group to solve Problem 2. Furthermore, the three Subjects' abilities were analyzed in proving group properties. Problem 3 is used to reveal this ability.

Problem 3
Write down the definition of a cyclic group.
Prove that if G is a cyclic group generated by a and N is a normal subgroup of G, then G/N is a cyclic group generated by Na.

The work of Problem 3 of the FI Subject can be seen on Figure 7 (***See Appendix A** of this article*). Based on Figure 7, the FI Subject can write the definition of cyclic group correctly. The FI Subject can show the G/N is a factor group to be cyclic in the correct steps. Based on Figure 7 and the results of the interview, the FI Subject can apply the property of the cyclic group G in the proof and the FI Subject can examine the property of the group.

The work of Problem 3 of the FN Subject can be seen on Figure 8 (***See***

***Appendix A** of this article*). Based on Figure 8, the FN Subject can write down the definition of a cyclic group correctly. The FN Subject can prove Problem 3 correctly, although there are still some steps that have not been explained with the argument, namely $Na^m = (Na)^m$. At the time of the interview, the FN Subject was able to explain this argument.

Based on Figure 8 and the results of the interview, it was obtained that the FN Subject could apply the properties of the cyclic group G in the proof and the FN Subject could examine the properties of the cyclic group of the G/N factor group.

The work of Problem 3 of the FD Subject can be seen on Figure 9 (***See Appendix A** of this article*). Based on Figure 9, the FD Subject can write down the definition of a cyclic group correctly. The FD Subject has not been able to apply the properties of the cyclic group G to examine the cyclic properties of the G/N factor group. In the interview, the FD Subject did not understand the elements of the factor group and the cyclic properties correctly.

Based on Figure 9 and the results of the interview, the FD Subject has not been able to apply the properties of the cyclic group G in proof and the FD Subject has not been able to examine the cyclic properties of the G/N factor group.

Discussion

GEFT was carried out to determine the cognitive style of students.. The characteristics of FI students are choosing the deductive method and excel in problem-solving tasks. In contrast, the characteristics of FD students are that they prefer inductive methods and excel in the knowledge domain that focuses on social problems (Dowlatabadi & Mehraganfar, 2014; Ates & Cataloglu, 2007). The characteristics of the cognitive style of FD that prefer inductive methods are not in line with the characteristics of the group material, namely axiomatic deductive. The differences in characteristics between FD and FI students in responding to tasks that require the ability to construct proofs will have an impact on the differences in group Scheme constructed by students.

Based on the Academic Guidelines (2018) at a university, students with a score of 81 received an AB grade that means more than good. In this study, student was classified into the high initial abilities if the student get a score at minimum of 81 on the test of students' initial ability to construct proofs. The student ability to construct proofs on group theory, include (1) able to provide group example and its proof; (2) able to apply a variety of group properties to solve related problems; and (3) able to examine the various properties of the group, is very related to student initial ability (Soehakso in Hanifah & Abadi, 2018; Rubowo, 2017, Dubinsky et. al, 1994; Melhuish, et al, 2019; Fraleigh, 2014; Hammack, 2013). Wijayanti, et al (2018) conducted research with result that there is an effect of students' initial abilities on students' ability to construct proofs in a class using APOS-based learning. Furthermore, Wijayanti, et al (2019) found that students with low initial abilities had not yet constructed the group Scheme, while students with high initial abilities displayed the constructed group Scheme. Zahid & Sujadi (2017) found that students with high ability can construct mental structures of algebraic factorization Schemes. Therefore, this study focuses on students with high initial ability in the learning of APOS-based learning.

Students in a class using APOS-based learning can construct proofs better than students in a class that use direct learning (Wijayanti, et al., 2018). Moreover, there is an effect of students' initial abilities on students' ability to construct proofs in a class using APOS-based learning (Wijayanti at al., 2019). Therefore, this study analyzes the coherence of the group Schemes constructed by students in the class using the learning based on the APOS theory.

Based on the results of the analysis on the work of Problem 1, 2, and 3, the three FI, FN, and FD Subjects can provide examples of groups and are equipped with the proof correctly. The FI Subject can apply group properties in proving and can examine group properties. Likewise, with the FN Subject, but the presentation of proof of the FN Subject still requires further explanation. The FD Subject has not been able to apply group properties in proving and also cannot examine group properties. The following is a discussion of group Schemes for each subject.

The FI Subject's group Schemes

The FI Subject can define sets and binary operations constitute groups, can instantiate and show that a set together with a binary operation is a group. The FI Subject can check all the properties of a binary operation. All group axioms are correctly understood. The group Scheme is thematized and coherent. The FI Subject can check the properties of the group and can apply the properties of the group when solving the problems faced so that it is said that the FI Subject group Scheme is coherent.

The FI Subject is a subject with a field-independent cognitive style and the high initial ability to prove. Witkin et al (Oh & Lim, 2005) stated that several characteristics of FI students are well-organized and structured in their learning. These characteristics support students to study group material. In the interview, it was revealed that the FI Subject had no difficulty in carrying out proving steps including the use of language and mathematical symbols. The FI Subject realizes that logic, set, and mapping are very important and exercises help to solve proving problems. This ability accompanies the characteristics of the FI Subject so that he succeeded in carrying out proving activities. The expression of the FI Subject is in line with Weber (2004: p.128) that students must have a basic understanding of logic to attend advanced mathematics courses. In accordance with the results of this interview, Blanton & Stylianou (2014) stated that students struggle with what supports proof.

The FN Subject's group Scheme

The FN Subject can define a set together with a binary operation is a group and the FN Subject can exemplify and show that a set that is equipped with a binary operation is a group. The FN Subject has already thematized the group Scheme. The FN Subject can examine the properties of the group and can apply the properties of the group when solving the problems faced so that it is said that the FN Subject group Scheme is coherent.

The FN Subject is a subject with a field-neutral cognitive style and the high initial ability to prove. The FN Subject can write the definition and use language and symbols appropriately. Learning how to use symbols correctly is a significant challenge for most students (Durand-Guerrier, et al, 2012). Basic symbolic skills must be mastered by students and sufficient experience to work with concepts at the symbolic level (Weber, 2004). According to Miyakawa (2017), mathematical symbols help readers to quickly grasp ideas in the proof. Proof and language are closely related, especially in mathematics (Balacheff, 2008). Language can contribute significantly to understanding mathematical reasoning and proving practice (Williams-Pierce, et al, 2017).

The FD Subject's group Scheme

The FD Subject can define a set and a binary operation is a group and can exemplify and show a set together with a binary operation as a group. The FD Subject can examine all the properties of the binary operation and group axioms correctly. Based on this fact, the FD Subject is said to have thematized the group Scheme. This thematized group scheme has not been followed by the ability of the FD Subject to check group properties and apply group properties to solve problems. The FD subject still has difficulty in proving it. This shows that the FD Subject group scheme is not coherent yet.

The FD Subject is a subject with a field-dependent cognitive style and the high initial ability to prove. The characteristics of students with a field-dependent cognitive style require more explicit instruction in problem-solving strategies (Witkin, et al. 1977). Constructing proof can be viewed as an advanced problem-solving task (Mamona-Downs & Downs, 2005; Cai, Mamona-Downs, & Weber, 2005; Selden & Selden, 2003). The FD Subject can write the definition of group correctly. To construct proofs requires the ability to dismantle and logically manipulate definitions (Weber, 2004). When constructing proof, one can start with definitions, known assumptions, and use logical inference including applying theorems (Mejía-Ramos, et al, 2015). Edward & Ward (2004) states that students know well the content of the definitions they use, but this is not enough. Students also need to understand the role and use of definitions in mathematics. Difficulties arise from students' understanding of the properties of mathematical definitions, not only from the content of definitions. Students experience difficulty when trying to write mathematical proofs in an introduction to abstract algebra, real analysis, or number theory. This difficulty was experienced by the FD Subject. The FD Subject's understanding of the prerequisite material for constructing proof is in the high category, but the understanding of the prerequisite material, namely logic, the set, and the necessary mapping, may not be sufficient. This understanding still has to be followed, one of which is the use of adequate mathematical language (Koichu & Leron, 2015). The FD Subject has not been able to use language and symbols correctly. Cañadas, et al (2018) state that students may have difficulty giving meaning to algebraic symbolism. Difficulties in the use of language and symbols are one of the triggers for the group Scheme thematized by the FD Subject which is not coherent yet. In line with Stylianou, et al (2015) the FD Subject still has difficulty in proving. One possible note is that students may not have a sufficient understanding of what constitutes/supports the proof (Inglis & Alcock, 2012). There are circumstances in which students are expected to state what the definition stipulates before applying theorems to draw conclusions (Dimmel, 2018).

At the Activities stage of learning, the FD Subject stated that the FD Subject always did the LTM well because it was very helpful. At the Classroom Discussion stage, the FD Subject could understand the explanation at the beginning, but at end the concentration had decreased. To overcome the understanding of the material at the end, the FD Subject asked a friend. At the Exercise stage in learning, the FD Subject in the interview revealed difficulties in working on exercises. Scusa (2008) states that mathematical reasoning is a complex skill so it requires a lot of practice. According to Musser (1998), the characteristics of the FD students must use repetitive learning strategies or exercises. The time dedicated by the FD Subject to do exercises in one week of fewer than 3 hours is not sufficient.

CLOSING

Conclusion

The coherence of the group Scheme constructed by students with the high initial ability and the cognitive style of FI, FN, FD to construct proofs as follows. The FI Subject has already thematized the group Scheme, can check group properties, and can apply group properties. The group Scheme of the FI Subject is coherent. The FN Subject has already thematized the Group Scheme, can check group properties, and can apply group properties. The FN Subject group Scheme is coherent. The FD Subject has already thematized the group Scheme, but cannot check the group properties yet, and cannot apply the group properties yet. The FD Subject group Scheme is not yet coherent.

Suggestion

The coherence of the Schema determines the individual's ability to use the Schema in certain mathematical situations. The coherence of group Scheme plays a role in solving various problems related to the group.

According to Witkin, students with FD cognitive style can be guided in handling problem solving such as constructing proof so that they can perform as well as FI students. Therefore, the results of this study can be used to design group learning materials so that each student can construct a coherent group Scheme.

For a coherent group Scheme to be achieved, one alternative to group material learning that can be done is to reinforce the method of proof, logic, set, and mapping. This strengthening can be carried out at several initial meetings integrated with introductory material. The reinforcement model in the effective method of proof, logic, set, and mapping can be studied further.

Besides that, it also provides assignments that support students in constructing the schemes that are given before lectures on related material. The FD students are given more assignments with guiding questions. Further research is needed to determine the form of tasks that encourage the formation of a coherent Scheme.

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Appendix A. The work result of subject

Wijayanti, K.[1] and Mulyono[2]. (2021). The Coherence of Group Scheme of the High Initial Ability Students Based on Cognitive Style
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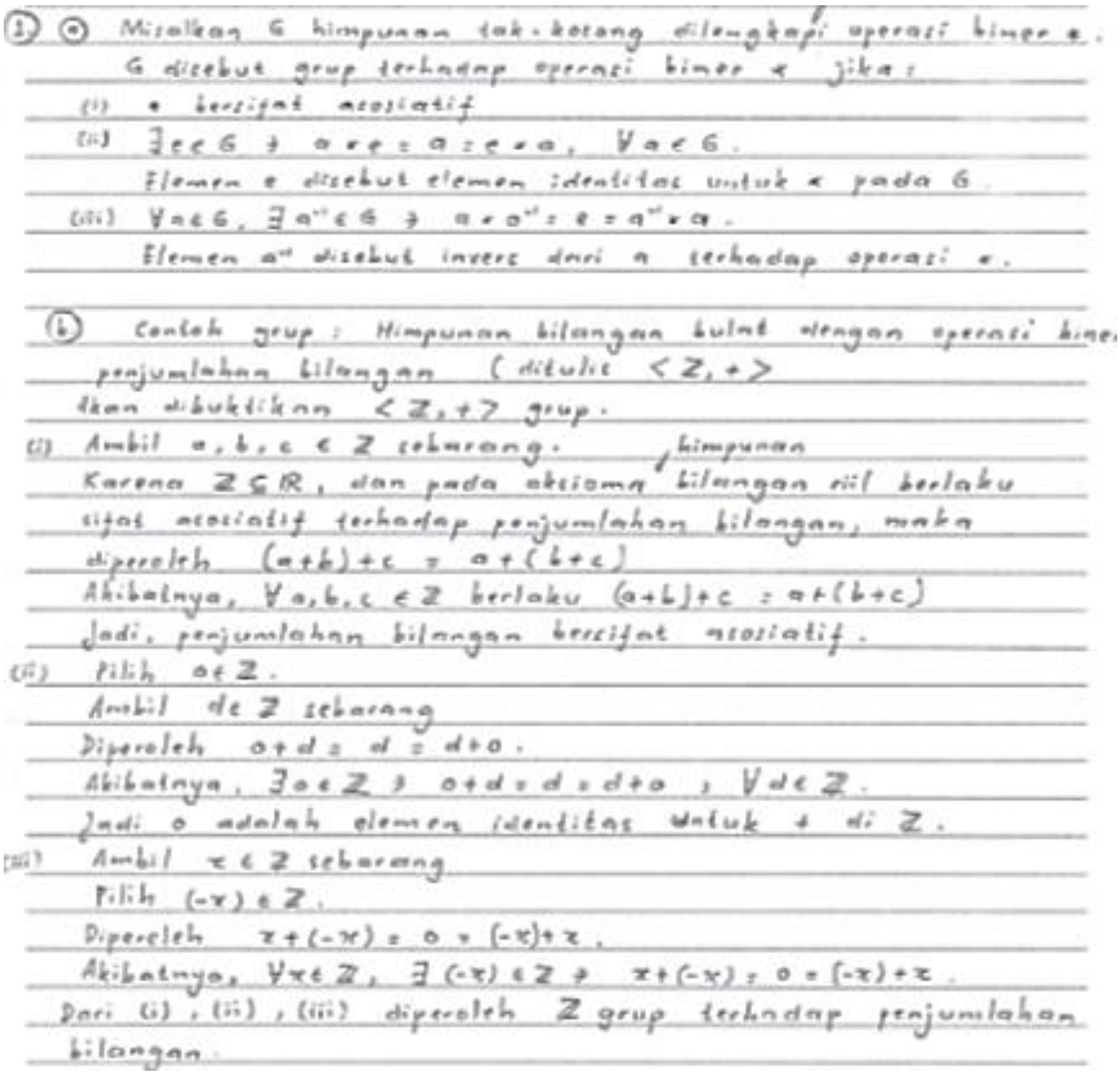
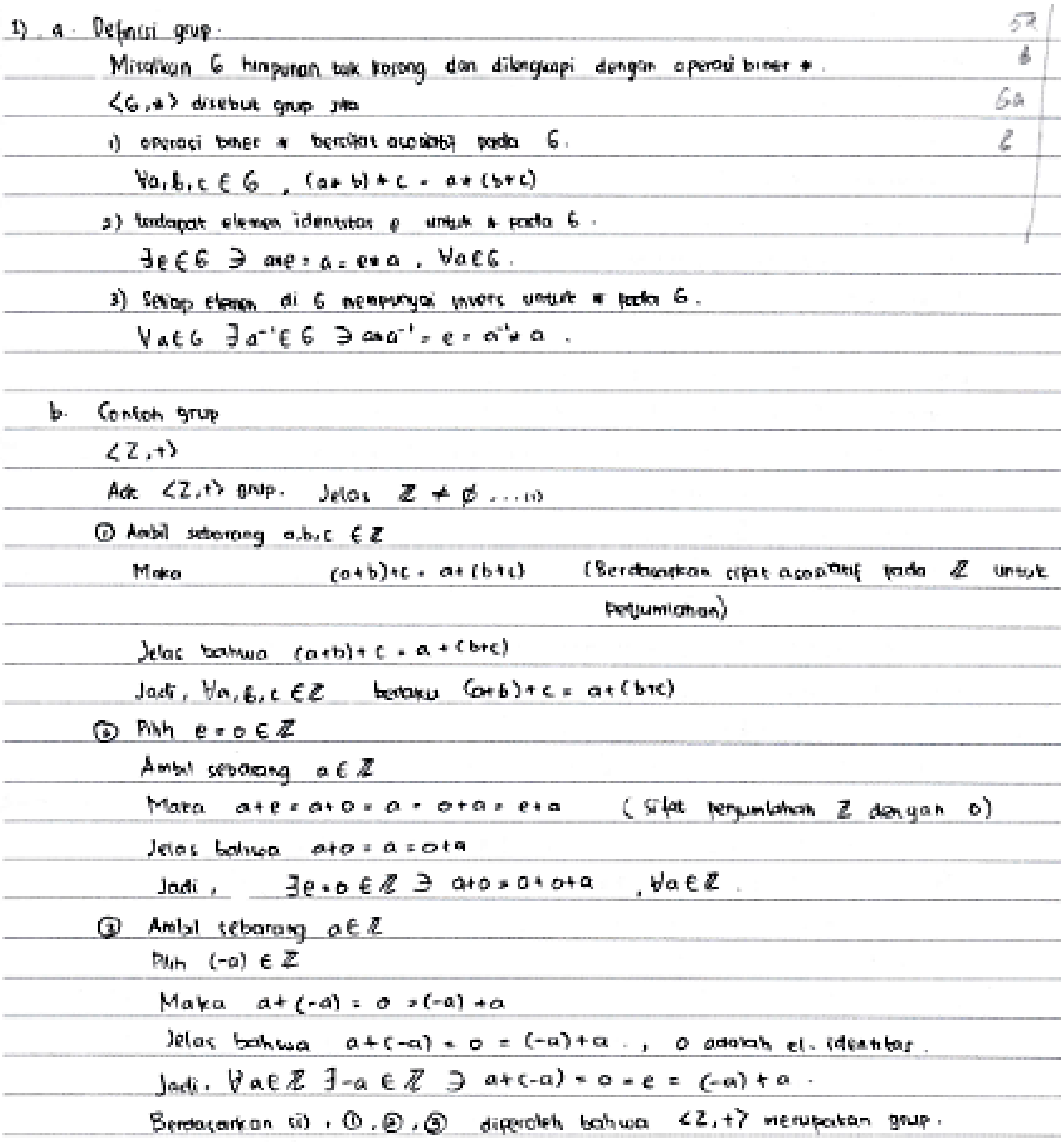
Editor Note: The results of the student's work are still in Bahasa Indonesia. For further details, you can contact the author at the corresponding author.



(1) (a) Misalkan G himpunan tak kosong dilengkapi operasi biner $*$.
G disebut grup terhadap operasi biner $*$ jika:
(i) $*$ bersifat asosiatif
(ii) $\exists e \in G \ni a * e = a = e * a, \forall a \in G$.
Elemen e disebut elemen identitas untuk $*$ pada G.
(iii) $\forall a \in G, \exists a^{-1} \in G \ni a * a^{-1} = e = a^{-1} * a$.
Elemen a^{-1} disebut invers dari a terhadap operasi $*$.

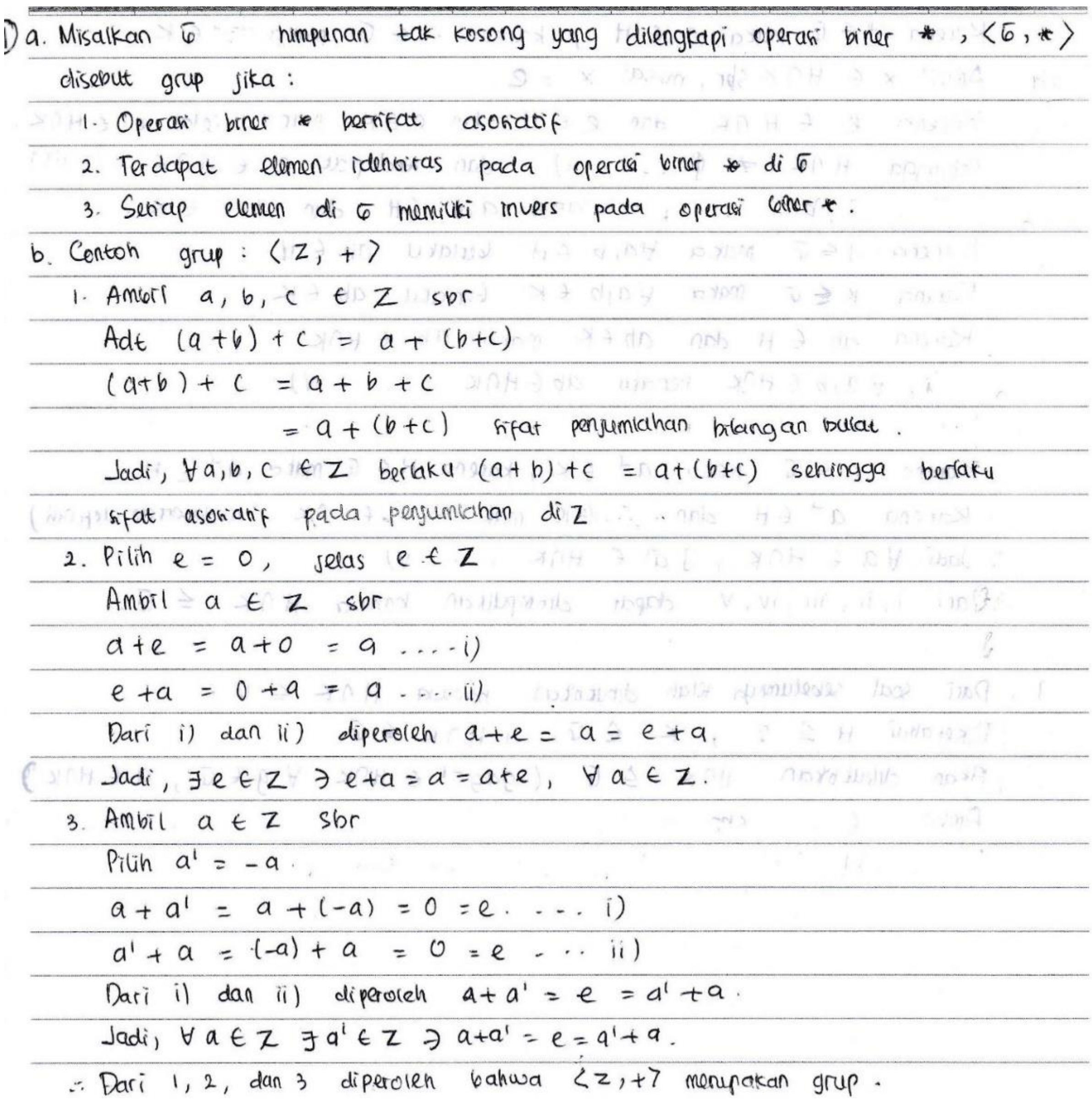
(b) Contoh grup: Himpunan bilangan bulat dengan operasi biner penjumlahan bilangan (ditulis $\langle Z, + \rangle$)
Akan dibuktikan $\langle Z, + \rangle$ grup.
(i) Ambil $a, b, c \in Z$ sebarang.
Karena $Z \subseteq R$, dan pada aksioma himpunan bilangan riil berlaku sifat asosiatif terhadap penjumlahan bilangan, maka diperoleh $(a+b)+c = a+(b+c)$
Akibatnya, $\forall a, b, c \in Z$ berlaku $(a+b)+c = a+(b+c)$
Jadi, penjumlahan bilangan bersifat asosiatif.
(ii) Pilih $0 \in Z$.
Ambil $d \in Z$ sebarang
Diperoleh $0+d = d = d+0$.
Akibatnya, $\exists 0 \in Z \ni 0+d = d = d+0, \forall d \in Z$.
Jadi 0 adalah elemen identitas untuk + di Z.
(iii) Ambil $x \in Z$ sebarang
Pilih $(-x) \in Z$.
Diperoleh $x+(-x) = 0 = (-x)+x$.
Akibatnya, $\forall x \in Z, \exists (-x) \in Z \ni x+(-x) = 0 = (-x)+x$.
Dari (i), (ii), (iii) diperoleh Z grup terhadap penjumlahan bilangan.

Figure 1 The Work of Problem 1 of the FI Subject


1) a. Definisi grup.
Misalkan G himpunan tak kosong dan dilengkapi dengan operasi biner $*$.
$\langle G, * \rangle$ disebut grup jika
1) operasi biner $*$ bersifat asosiatif pada G.
$\forall a,b,c \in G$, $(a*b)*c = a*(b*c)$
2) terdapat elemen identitas e untuk $*$ pada G.
$\exists e \in G \ni a*e = a = e*a$, $\forall a \in G$.
3) Setiap elemen di G mempunyai invers untuk $*$ pada G.
$\forall a \in G \; \exists a^{-1} \in G \ni a*a^{-1} = e = a^{-1}*a$.

b. Contoh grup
$\langle Z, + \rangle$
Adt $\langle Z, + \rangle$ grup. Jelas $\mathbb{Z} \neq \emptyset$ ... (i)
① Ambil sebarang $a,b,c \in \mathbb{Z}$
Maka $(a+b)+c = a+(b+c)$ (Berdasarkan sifat asosiatif pada $\mathbb{Z}$ untuk penjumlahan)
Jelas bahwa $(a+b)+c = a+(b+c)$
Jadi, $\forall a,b,c \in \mathbb{Z}$ berlaku $(a+b)+c = a+(b+c)$
② Pilih $e = 0 \in \mathbb{Z}$
Ambil sebarang $a \in \mathbb{Z}$
Maka $a+e = a+0 = a = 0+a = e+a$ (Sifat penjumlahan $\mathbb{Z}$ dengan 0)
Jelas bahwa $a+0 = a = 0+a$
Jadi, $\exists e = 0 \in \mathbb{Z} \ni a+0 = a = 0+a$, $\forall a \in \mathbb{Z}$.
③ Ambil sebarang $a \in \mathbb{Z}$
Pilih $(-a) \in \mathbb{Z}$
Maka $a+(-a) = 0 = (-a)+a$
Jelas bahwa $a+(-a) = 0 = (-a)+a$, 0 adalah el. identitas.
Jadi, $\forall a \in \mathbb{Z} \; \exists -a \in \mathbb{Z} \ni a+(-a) = 0 = e = (-a)+a$.
Berdasarkan (i), ①, ②, ③ diperoleh bahwa $\langle Z, + \rangle$ merupakan grup.

Figure 2 The Work of Problem 1 of the FN Subject


1) a. Misalkan G himpunan tak kosong yang dilengkapi operasi biner $*$, $\langle G, * \rangle$ disebut grup jika :

1. Operasi biner $*$ bersifat asosiatif
2. Terdapat elemen identitas pada operasi biner $*$ di G
3. Setiap elemen di G memiliki invers pada operasi biner $*$.

b. Contoh grup : $\langle Z, + \rangle$

1. Ambil $a, b, c \in Z$ sbr

Adt $(a+b)+c = a+(b+c)$

$(a+b)+c = a+b+c$

$= a+(b+c)$ sifat penjumlahan bilangan bulat.

Jadi, $\forall a,b,c \in Z$ berlaku $(a+b)+c = a+(b+c)$ sehingga berlaku sifat asosiatif pada penjumlahan di Z

2. Pilih $e = 0$, jelas $e \in Z$

Ambil $a \in Z$ sbr

$a+e = a+0 = a$ ....i)

$e+a = 0+a = a$ ....ii)

Dari i) dan ii) diperoleh $a+e = a = e+a$.

Jadi, $\exists e \in Z \ni e+a = a = a+e, \ \forall a \in Z$.

3. Ambil $a \in Z$ sbr

Pilih $a' = -a$.

$a + a' = a + (-a) = 0 = e$. ... i)

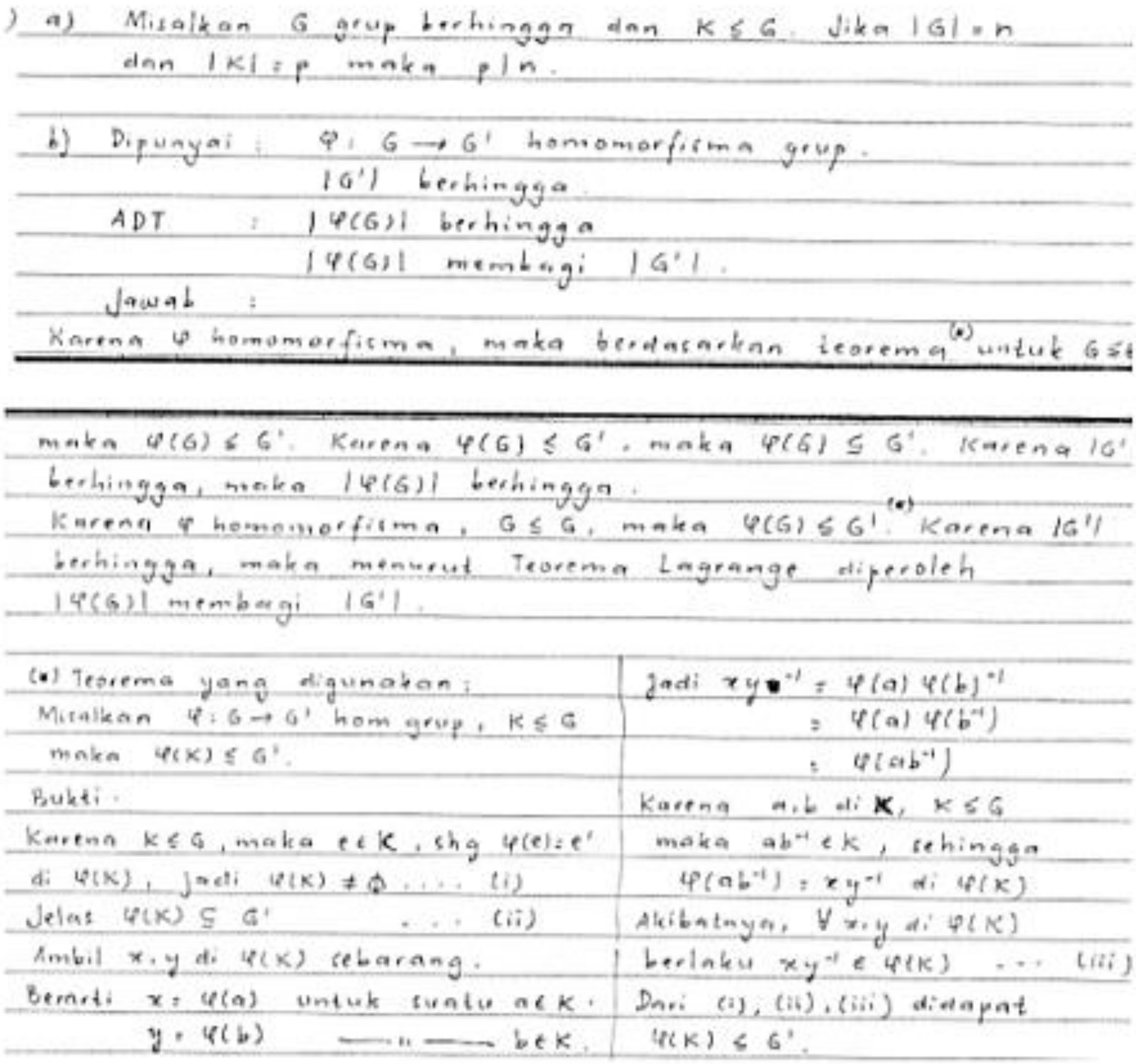
$a' + a = (-a) + a = 0 = e$ ... ii)

Dari i) dan ii) diperoleh $a + a' = e = a' + a$.

Jadi, $\forall a \in Z \ \exists a' \in Z \ni a+a' = e = a'+a$.

$\therefore$ Dari 1, 2, dan 3 diperoleh bahwa $\langle Z, + \rangle$ merupakan grup.

Figure 3 The Work of Problem 1 of the FD Subject



a) Misalkan G grup berhingga dan $K \leq G$. Jika $|G| = n$ dan $|K| = p$ maka $p \mid n$.

b) Dipunyai : $\varphi : G \to G'$ homomorfisma grup.
$|G'|$ berhingga

ADT : $|\varphi(G)|$ berhingga
$|\varphi(G)|$ membagi $|G'|$.

Jawab :

Karena φ homomorfisma, maka berdasarkan teorema(*) untuk $G \leq G$

maka $\varphi(G) \leq G'$. Karena $\varphi(G) \leq G'$, maka $\varphi(G) \subseteq G'$. Karena $|G'|$ berhingga, maka $|\varphi(G)|$ berhingga.

Karena φ homomorfisma, $G \leq G$, maka $\varphi(G) \leq G'$(*). Karena $|G'|$ berhingga, maka menurut Teorema Lagrange diperoleh $|\varphi(G)|$ membagi $|G'|$.

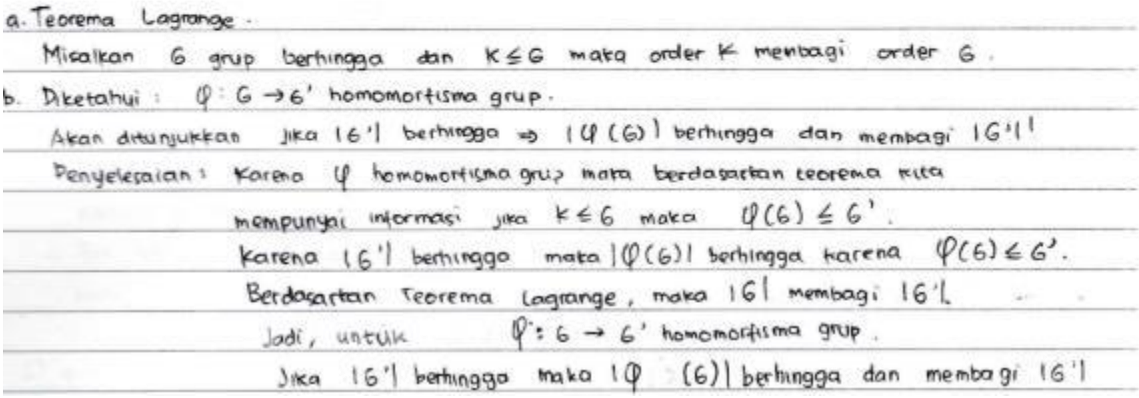
(*) Teorema yang digunakan :
Misalkan $\varphi : G \to G'$ hom grup, $K \leq G$ maka $\varphi(K) \leq G'$.

Bukti :
Karena $K \leq G$, maka $e \in K$, shg $\varphi(e) = e'$ di $\varphi(K)$, jadi $\varphi(K) \neq \emptyset$ .... (i)
Jelas $\varphi(K) \subseteq G'$ .... (ii)
Ambil x, y di $\varphi(K)$ sebarang.
Berarti $x = \varphi(a)$ untuk suatu $a \in K$.
$y = \varphi(b)$ —— " —— $b \in K$.

Jadi $xy^{-1} = \varphi(a)\varphi(b)^{-1}$
$= \varphi(a)\varphi(b^{-1})$
$= \varphi(ab^{-1})$

Karena a, b di K, $K \leq G$ maka $ab^{-1} \in K$, sehingga $\varphi(ab^{-1}) = xy^{-1}$ di $\varphi(K)$.
Akibatnya, $\forall x, y$ di $\varphi(K)$ berlaku $xy^{-1} \in \varphi(K)$ ... (iii)
Dari (i), (ii), (iii) didapat $\varphi(K) \leq G'$.

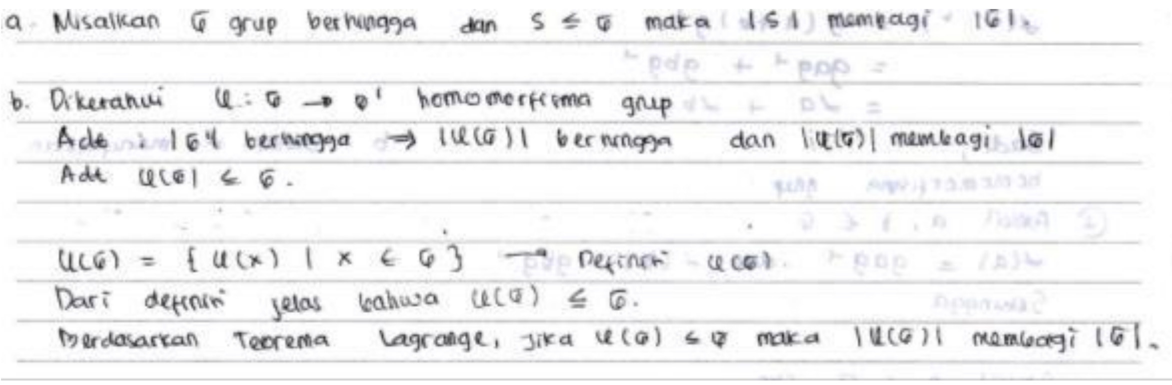
Figure 4 The Work of Problem 2 of the FI Subject



a. Teorema Lagrange.
Misalkan G grup berhingga dan $K \leq G$ maka order K membagi order G.

b. Diketahui : $\varphi : G \to G'$ homomorfisma grup.
Akan ditunjukkan jika $|G'|$ berhingga $\Rightarrow$ $|\varphi(G)|$ berhingga dan membagi $|G'|$!
Penyelesaian : Karena φ homomorfisma grup maka berdasarkan teorema kita mempunyai informasi jika $K \leq G$ maka $\varphi(G) \leq G'$.
Karena $|G'|$ berhingga maka $|\varphi(G)|$ berhingga karena $\varphi(G) \leq G'$.
Berdasarkan Teorema Lagrange, maka $|G|$ membagi $|G'|$.
Jadi, untuk $\varphi : G \to G'$ homomorfisma grup.
Jika $|G'|$ berhingga maka $|\varphi(G)|$ berhingga dan membagi $|G'|$

Figure 5 The Work of Problem 2 of the FN Subject



a. Misalkan G grup berhingga dan S ≤ G maka |S| membagi |G|.

b. Diketahui φ : G → G' homomorfisma grup

Ada |G| berhingga ⇒ |φ(G)| berhingga dan |φ(G)| membagi |G|

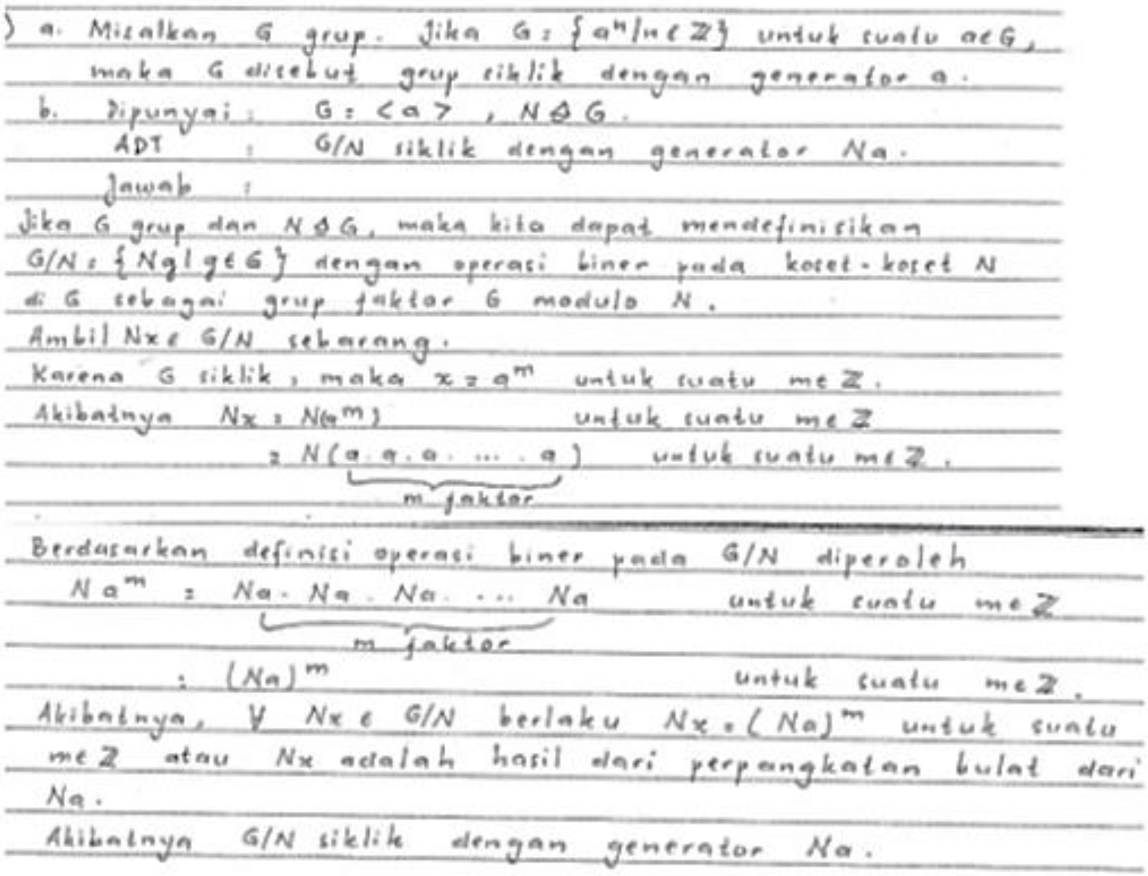
Adt φ(G) ≤ G.

φ(G) = { φ(x) | x ∈ G } → Definisi φ(G).

Dari definisi jelas bahwa φ(G) ≤ G.

Berdasarkan Teorema Lagrange, jika φ(G) ≤ G maka |φ(G)| membagi |G|.

Figure 6 The Work of Problem 2 of the FD Subject



a. Misalkan G grup. Jika G = {aⁿ | n ∈ Z} untuk suatu a ∈ G, maka G disebut grup siklik dengan generator a.

b. Dipunyai : G = <a> , N ⊴ G.

ADT : G/N siklik dengan generator Na.

Jawab :

Jika G grup dan N ⊴ G, maka kita dapat mendefinisikan G/N = {Ng | g ∈ G} dengan operasi biner pada koset-koset N di G sebagai grup faktor G modulo N.

Ambil Nx ∈ G/N sebarang.

Karena G siklik, maka x = a^m untuk suatu m ∈ Z.

Akibatnya Nx = N(a^m) untuk suatu m ∈ Z

= N(a . a . a . ... . a) untuk suatu m ∈ Z. (m faktor)

Berdasarkan definisi operasi biner pada G/N diperoleh

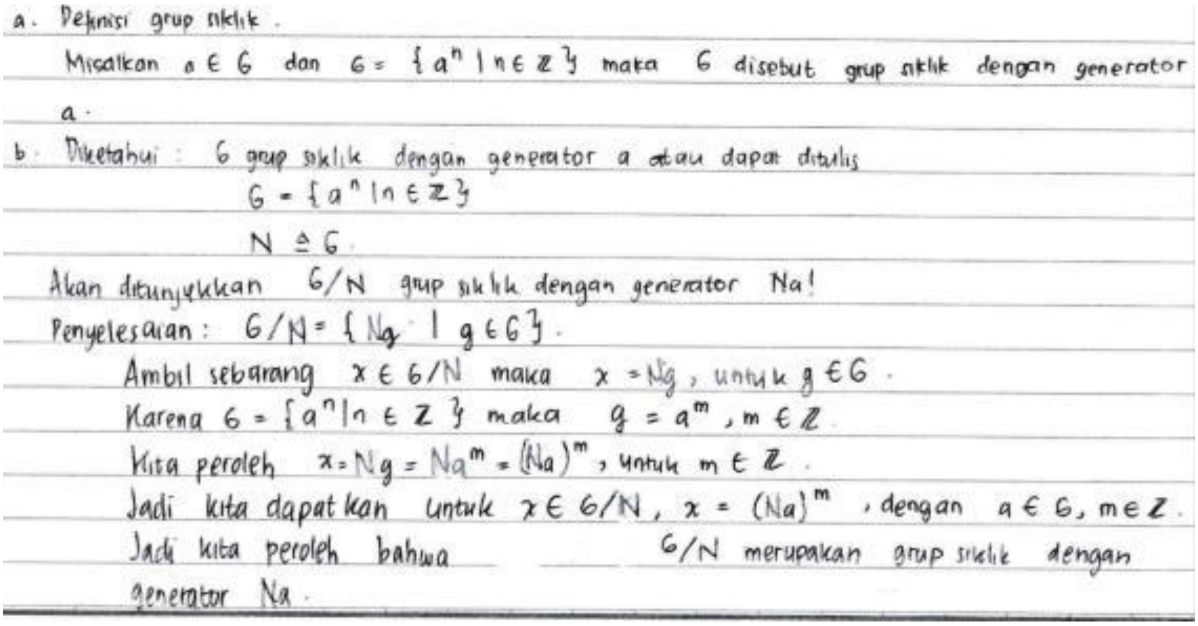
Na^m = Na . Na . Na . ... Na untuk suatu m ∈ Z (m faktor)

= (Na)^m untuk suatu m ∈ Z.

Akibatnya, ∀ Nx ∈ G/N berlaku Nx = (Na)^m untuk suatu m ∈ Z atau Nx adalah hasil dari perpangkatan bulat dari Na.

Akibatnya G/N siklik dengan generator Na.

Figure 7 The Work of Problem 3 of the FI Subject



a. Definisi grup siklik.

Misalkan $a \in G$ dan $G = \{a^n \mid n \in \mathbb{Z}\}$ maka G disebut grup siklik dengan generator a.

b. Diketahui: G grup siklik dengan generator a atau dapat ditulis

$G = \{a^n \mid n \in \mathbb{Z}\}$

$N \trianglelefteq G$.

Akan ditunjukkan G/N grup siklik dengan generator Na!

Penyelesaian: $G/N = \{Ng \mid g \in G\}$.

Ambil sebarang $x \in G/N$ maka $x = Ng$, untuk $g \in G$.

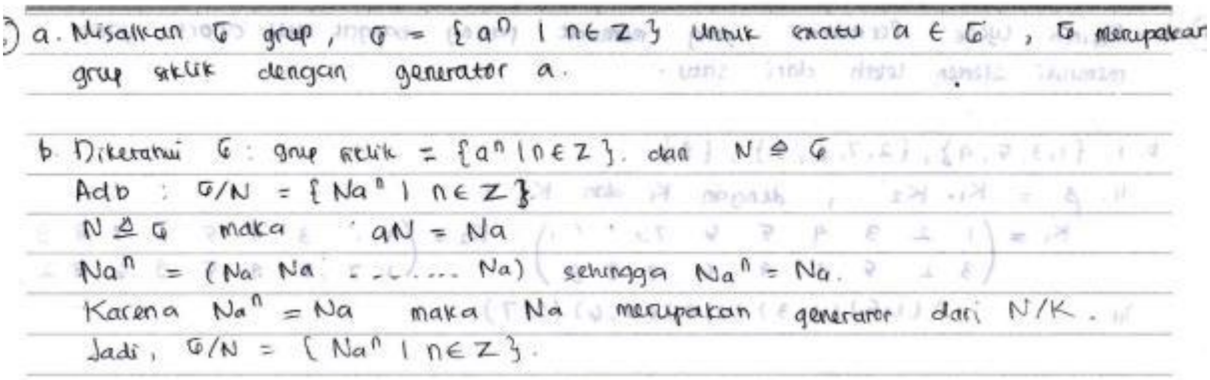
Karena $G = \{a^n \mid n \in \mathbb{Z}\}$ maka $g = a^m$, $m \in \mathbb{Z}$.

Kita peroleh $x = Ng = Na^m = (Na)^m$, untuk $m \in \mathbb{Z}$.

Jadi kita dapatkan untuk $x \in G/N$, $x = (Na)^m$, dengan $a \in G$, $m \in \mathbb{Z}$.

Jadi kita peroleh bahwa G/N merupakan grup siklik dengan generator Na.

Figure 8 The Work of Problem 3 of the FN Subject



a. Misalkan G grup, $G = \{a^n \mid n \in \mathbb{Z}\}$ untuk suatu $a \in G$, G merupakan grup siklik dengan generator a.

b. Diketahui G: grup siklik $= \{a^n \mid n \in \mathbb{Z}\}$. dan $N \trianglelefteq G$

Adb: $G/N = \{Na^n \mid n \in \mathbb{Z}\}$

$N \trianglelefteq G$ maka $aN = Na$

$Na^n = (Na\ Na \ldots Na)$ sehingga $Na^n = Na$.

Karena $Na^n = Na$ maka Na merupakan generator dari N/K.

Jadi, $G/N = \{Na^n \mid n \in \mathbb{Z}\}$.

Figure 9 The Work of Problem 3 of the FD Subject