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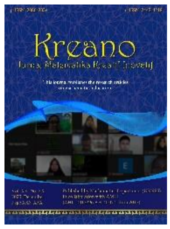

Students' Understanding of The Equal Sign Based on Their Learning Experience in Arithmetic

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Abstract

The equal sign is an important concept in learning mathematics karena it is used in almost all branches of mathematics, but not many studies in Indonesia have made the equal sign the focus of research. This study aims to explore how students from elementary school to college students understand the equal sign in the context of arithmetic and algebra at school. A qualitative comparative analysis can be used to analyze several cases in complex situations so that it fits the purpose of this study. Participants consisted of 6 elementary school students, 14 junior high school students, 7 high school students and 3 college students in Bandung. The results of the study indicate that the equal sign is still interpreted narrowly as "result" or a sign to put the Answer and has dependence on computational methods in solving problems and drawing conclusions. Thus, it can be concluded that students' understanding of the equal sign is still at the basic level. The results of this study show evidence that the operational meaning of the equal sign that students have when learning arithmetic will not change by itself without the stimulus provided by the teacher, and even tends to cause obstacles when students learn equations in algebra.

Keywords: *algebra; arithmetic; the equal sign; qualitative comparative analysis*

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Abstrak

Tanda sama dengan merupakan konsep penting dalam pembelajaran matematika karena digunakan pada hampir seluruh cabang matematika, namun belum banyak penelitian di Indonesia yang menjadikan tanda sama dengan sebagai fokus penelitian. Penelitian ini bertujuan untuk mengeksplorasi bagaimana para siswa dari mulai sekolah dasar hingga mahasiswa memahami tanda sama dengan dalam konteks aritmatika dan aljabar di sekolah. A qualitative comparative analysis dapat digunakan untuk menganalisis beberapa kasus dalam situasi yang kompleks sehingga sesuai dengan tujuan penelitian ini. Partisipan terdiri dari 6 siswa SD, 14 siswa SMP, 7 siswa SMA dan 3 Mahasiswa di Bandung. Hasil penelitian menunjukkan bahwa tanda sama dengan masih dimaknai secara sempit yaitu sebagai "menghasilkan" atau tanda untuk meletakkan jawaban, serta memiliki kebergantungan terhadap metode komputasi dalam menyelesaikan masalah dan mengambil kesimpulan. Dengan demikian, dapat disimpulkan bahwa pemahaman siswa tentang tanda sama dengan masih berada di tingkat dasar. Hasil penelitian ini menunjukkan bukti bahwa makna operasional dari tanda sama dengan yang dimiliki siswa pada saat belajar aritmatika tidak akan berubah dengan sendirinya tanpa adanya stimulus yang diberikan oleh guru, bahkan cenderung menimbulkan hambatan saat siswa belajar persamaan pada aljabar.

INTRODUCTION

Students' understanding of the equal sign, which is still in the operational category, namely as a sign to carry out a series of computational processes, is one of the main stumbling blocks for students to succeed in algebra. This is because almost all manipulations in algebraic equations require the understanding that the equal sign is a symbol denoting a relational equivalence of both sides of the equal sign rather than simply the result of calculations. Many thoughts and studies in various countries have explored students' understanding of the equal sign, starting from the elementary school level (such as Vermeulen & Meyer, 2017), high school (such as Kindrat & Osana, 2018) to tertiary institutions (Baiduri, 2015) stating that the meaning 'same' is a complex and difficult notion to grasp. The researchers observed that students still tend to regard the equal sign in arithmetic sentences as an operational symbol, namely the equal sign is interpreted because of calculations or an answer to a problem, so they tend to read and perform calculations on algebraic equations from left to right.

Preliminary studies that have been conducted on 2 elementary school students, 3 seventh grade junior high school students, and 1 student show conditions in the field where there is a similar tendency, namely giving responses based on a result-oriented process so that they have a "must to calculate" which is categorized as an operational view of the equal sign. When students encounter expressions like 5 + n, for example, most students tend to write 5n as 'answer'. In addition, elementary school students also often react negatively when sentence numbers do not match the conception they have so far, such as in sentences of the form c = a + b then change them to a + b = c. In sentences of the form a + b = _____+ d, students tend to write down the sum of a + b in the ___ section and when students are given math sentences 3 + 5 = _____+1, students tend to fill in 8 in the ____ section. This is in accordance with Alibali's (2007) statement, namely that although the equal sign is a very important concept for understanding mathematics at all levels of school mathematics, the meaning of 'same' is a complex and difficult idea for most students to understand.

The meaning of the equal sign (=) in mathematics expresses an identity relationship that is, equal (is equal to / is the same as) or identical (is identical to) where all refer to the specific relationship of the equivalence relation. That is, equality has a different meaning from equivalence because there are other equivalent relations which do not refer to the same relationship. Even though there are differences, equivalency and identity do share some of the same properties, namely identity (equality) is a certain equality relationship in the sense of being reflective, transitive, and symmetrical (Mirin, 2019). The equal sign is very important to be interpreted as a relational sign that tends to be non-computational in order to understand rules in an algebraic context, because in the process there are many cognitive demands that must be met by students to operate on algebraic symbols, for example students need to accept that a sign of arithmetic operations such as '+' , '-' and '=' have multiple meanings. It is intended that students can understand the meaning of different symbols according to the context, choose one that is appropriate to the situation, and recognize that symbols represent results and processes.

A formal understanding of mathematical equivalence involves students' understanding of the equal sign as a relational symbol such as encoding and identifying two sides of an equation, the equal sign not only as a process but also as a mathematical object that can be manipulated or acted on, numbers and expressions can be represented in a variety of different ways. interchangeably claimed by some researchers such as (Baiduri, 2015; Knuth et al., 2006; McNeil, et al, 2017) is important for students because it becomes a basic concept in mathematics which makes it easier for them to acquire new mathematical information, facilitates conceptual understanding of arithmetic, improve math achievement, and promote algebra readiness such as solving algebraic equations in high school. Baiduri (2015) reports that students need a relational perspective in learning to solve algebraic equations using operations on both sides, for example, $5x - 5 = 2x + 1$ and understand that the transformations made in the process of solving equations must maintain equality.

The National Council of Mathematics Teachers Principles and Standards for School Mathematics (NCTM) in 2000 recommended that algebraic thinking be emphasized not only at the secondary school level but also at the elementary level. This is an appeal to teachers in elementary schools to cultivate students' ability to find patterns, generalizations, and other skills to develop basic knowledge in children that will support algebra learning in the next class. The equal sign is a very important symbol for students in primary and secondary schools to learn because it is the basis for understanding equations where equations are the essence of understanding algebra. An understanding of equality is inseparable from the concept of an equal sign so that the concept of an equal sign and the concept of equality cannot be separated (Baiduri, 2015). Students' understanding of the equal sign in an equation is the basis for supporting the transition from arithmetic to algebra (Leavy, Hourigan, & McMahon, 2013). The equal sign can cause many difficulties for students in understanding algebra if it is not understood correctly (Banerjee, 2011). Therefore, students' understanding of the equal sign is an important foundation for their success in learning algebra (Stephens, et al., 2013).

Some of the studies previously mentioned (such as Baiduri, 2015; Kindrat & Osana, 2018; Knuth et al., 2006; McNeil, et al, 2017; Vermeulen & Meyer; 2017) have not used a qualitative comparative analysis methodology in conducting an analysis for seek and find similarities and differences in phenomena. This study aims to explore various information about learning experiences and the meaning of the equal sign resulting from these learning experiences. The various kinds of experiences found are then sought for similarities and differences in the meaning of the equal sign which are then used to find learning experiences that are often experienced but do not support a strong understanding of the equal sign as well as learning experiences that can support a strong understanding of the equal sign so that students can relate meaning to the procedures normally used to solve algebraic equations.

METHOD

This study uses a qualitative comparative analysis methodology, a case-based approach that is regularly used to investigate situations in specific contexts and settings. A qualitative approach was chosen because this study aims to investigate things that exist in natural settings and try to interpret these phenomena. The comparative perspective is considered the most suitable for the purpose of this research because this research was conducted on three groups of students with different grades. The reliability techniques used in this study include field notes, audio recorders for accuracy, and coding techniques. The themes and codes were derived from qualitative data which were informed by the theoretical framework that supported the research and the literature review. The identified validation techniques commonly used in this study included triangulation of data sources to corroborate evidence, and member checks were performed to determine the credibility of findings and interpretations.

The selection of the sample used in this study used an incidental sampling technique which involved selecting individuals who happened to be available and accessible at that time. Researchers take samples at random (whenever and wherever they find) as long as they meet the requirements as samples from the desired population, so that bias can be minimized. The population in this study is homogeneous, namely students who have learned about the equal sign in arithmetic and/or school algebra. In this case, the researcher does not attempt to generalize about the wider population but uncovers existing phenomena. There were 30 participants in this study. All participants are elementary school students aged 12 to graduate students aged 45, have learned about the equal sign at school arithmetic / algebra, and are willing to voluntarily become participants. The respondents of this study consisted of 30% men and 70% women. There are 20% elementary students, 70% junior and senior high school students, 10% university students in Bandung, Indonesia. All participants in this study were individuals who were different from participants in the preliminary study.

The problem of trustworthiness in this study is improved through checklists in each stage of the research, namely from the preparation stage, the organizational stage, to the reporting stage as described by Elo, et.al., (2014). The stages of this research are shown in Figure 1.

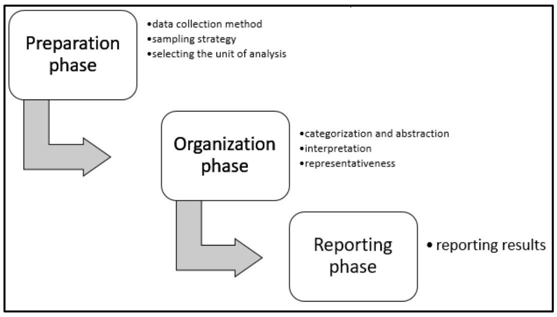

Figure 1. Research Stages

Checking the validity of the data in this study was carried out through credibility, transferability, dependability, and confirmability tests. The credibility test is carried out by extending observations,

Table 1. Stages of Relational Thinking about the Equal sign

Rigid operational	Students are asked to solve problems in the standard "a + b = c" format
Flexible operational	Students are asked to solve equations in several non-standard formats for example "c = a + b"
Relational with computational support	Students are asked to solve equations with operations on both sides, for example "a + b + c = a + _____"
Relational without computational support or fully relational	The relational view dominates, and students demonstrate an understanding for the nature of equality

persistence of observations, and triangulation to ensure that the data obtained previously and those obtained from the results of extended observations do not experience significant changes or differences. The extension of the observation was carried out by means of repeated interviews; persistence in making observations by means of document analysis, reading various references, and comparing with the results of previous studies; while triangulation was carried out using several different methods in collecting data in the field, namely written tests, interviews, and document analysis in the form of student notebooks and textbooks used to study the equal sign at school.

The transferability of this research is enhanced through detailed, clear, systematic, and reliable descriptions. Dependability testing is carried out by always actively discussing research progress and showing every "trace" of activity in the field with the supervisor. Meanwhile, confirmability is done by associating research results with the entire research process that has been carried out. The confirmability test is carried out simultaneously with the dependability test.

Before being asked for personal consent by the researcher, students had been given an understanding that they would be given the task of answering several questions about the equal sign, answers had to be honest according to their personal thoughts and any answers they gave were not related to their school grades, and these answers would be documented in a research report with guaranteed anonymity.

The researcher assured the participants that the data collected would be treated confidentially and that no punitive action would be taken against them if they decided to withdraw. All participants have filled out written statements willing to participate in this research voluntarily and without coercion from any party. Students who are willing to complete the task are participants of this study. The instrument used in this study was a task-based interview referring to the four stages of relational thinking proposed by Matthews, et.al., (2010) to increase difficulty. All questions in tests and interviews use Indonesian. The tasks given consist of two types, namely open number sentence questions for tasks 1, 2 and 3; as well as true or false number sentence questions for task 4.

There are four main types of questions used in several arithmetic and symbolic formats, namely: type 1 (arithmetic identity) which aims to interpret the equal sign and provide an answer; type 2 (algebraic equations) aims to determine solutions to algebraic equations; type 3 (mathematical statement) to interpret the equal sign by stating whether the number sentence is true or false and to write the mathematical sentence into a regular sentence; and type 4 (symbol interpretation) to interpret the meaning of the symbols "=,_____, , and n". After the respondent completes the written task then proceed with the interview process. Investigations

Table 2. Student Performance on All Tasks

Level	Tasks (T: True, F: False)							
	1		2		3		4	
	T (%)	F (%)	T (%)	F (%)	T (%)	F (%)	T (%)	F (%)
Elementary School (SD)	17	83	0	100	0	100	0	100
Junior High School (SMP & SMA)	79	21	50	50	54	46	53	47
College	90	10	86	14	85	15	82	18

and follow-up questions were added as needed to encourage elaboration and clarification of responses. Specific questions were added as the interview process progressed according to the responses provided by the informants.

After they filled out the written assignment, the researcher then conducted an interview session with a duration of between 10 and 15 minutes. Each question asked aims to confirm their answers, the difficulties, or obstacles they may experience, what experiences, why and how so that they understand the meaning of the equal sign, contextual questions, and additional questions according to the responses given. The documentation included students' written answers along with every stroke they wrote on the answer sheets provided, audio recordings during the interview process, and analysis of textbooks used at school.

Data collected from answers to written assignments were analyzed using the following categories: (a) correct and incorrect responses; and (b) operational and non-operational arguments. Incorrect responses are given a value of zero (0) and correct responses are given a value of one (1). Both incorrect and correct responses related to understanding of the equal sign were followed up during the interview. Responses were coded as operational if a student expressed the general idea that the equal sign means 'add a number' or 'answer', and 'signal to compute' (Alibali, et al., 2007) and coded as non-operational if with no calculation or with calculation only to justify written relational responses (Kindat & Osana, 2018).

The findings from this study are presented and discussed according to the theme of the research question, namely: student performance, strategies used, and learning experiences applied in completing written tasks.

RESULTS AND DISCUSSION

Student Performance

The performance of all respondents in completing the task is presented in Table 2 below. The number of students who have correct (B) and incorrect (S) answers is written as a percentage (%). Table 2 shows that respondents performed better on tasks that involved calculations than those that did not involve calculations. This became one of the important results of this study, namely that respondents, both several students or a number of adults, responded to open number sentences and true/false number sentences based on a result-oriented process so that they have a 'necessity to calculate' which often encourages the view operational of the equal sign. Students are more likely not to regard the equal sign as a symbol indicating a relationship but as 'finding a result or doing something', 'a sign for calculating', and 'an operation symbol or symbol-syntax indicator used before an answer' where these findings are in harmony with several studies such as Barlow & Harmon (2012); Kiziltoprak & Kose (2017); Machaba (2017); Matthews, et.al., (2010); Stephens et al., (2013). The operational view of the equal sign suggests limited knowledge about the meaning of the equal sign which can hinder students' performance in algebra (Matthews, et.al., 2010). The operational view of the same sign is mostly claimed by some researchers as a 'side effect' of students' experiences with symbols in elementary school mathematics (Knuth, et al., 2008; McNeil and Alibali, 2005; Vermeulen & Meyer, 2017).

In task 1, even though it was only oriented towards arithmetic operations in a standard format, the performance of all students was not optimal because they were used to the right side of the equal sign showing results. Students tend to interpret expressions in the same way as reading numbers, namely sequentially from left to right. For example, in the first question, namely filling ____ in 8- ______ = 10 with a value that is considered correct, students who fill in 2 as an answer think that the subtraction sign is the difference of two numbers without paying attention to the position of the equal sign or answering 6 voluntarily reformulates the syntax such as reading 19 = 6 + 25 as '6 plus 25 equals 19'.

Task 2 has a flexible operating difficulty level where students are asked to solve equations in several non-standard formats. The types of questions posed in task 2 allow the respondent to induce equivalent additive pairs based on transitive relationships. This ability is a fundamental arithmetic skill that allows writing number sentences with mathematical symbols, understanding the basic features of operations and conceptualizing numbers in various forms (Koziltoprak & Kose, 2017). Question 2.2, namely filling in the box at $4 \times 3 = 7 + \square$ with a value that is considered correct, is the question that has the worst performance in task 2. The four respondents who answered 14 saw an equal sign as a kind of command to complete the calculation '4+3+7' and make answers in the box without paying attention to the operation sign and the equal sign in the given mathematical sentence. Meanwhile, the other three respondents add up 4×3=12 and then add 7 for a result of 19. They assume that 'equivalent' is something to indicate that it is necessary to add an answer (Darr, 2003). It seems that the equal sign is often presented in contexts that support operational interpretations, such as problem structure 'operations equal answers' (e.g., 38 + 27 = ?), 'find the total' or 'put the answer'. Although the 'operational' view may be sufficient when solving typical elementary school arithmetic problems, it can become problematic when students encounter more complex equations in later grades e.g., 3x + 5 = 11.2x - 3 = 4x + 5 (McNeil, et al., 2017). High school students who are advanced in arithmetic don't seem to have a sophisticated interpretation of the equal sign either. Their performance decreases as the difficulty level of the questions increases.

Task 3 involves more than two operations in one sentence which is a special problem that can only be solved if students have a broad understanding of the equal sign (Molina & Ambrose, 2006). There are still many high school students who experience parsing obstacles as reported by (Gunnarson, Sonnerhed & Hernell, 2015) to be the cause of these errors. Students have performed fairly well in all task 4 questions except in the question of determining true or false of 4.3, namely 8 + (3 × 8) = (5 × 8)- 8 and 4.4, namely 42:16 = 84:32. Question 4.3 generally students stated "false" because they saw that the math sentence has a different form on the left and right sides of the equal sign where they believed that 8 + (3 × 8) would not have the same result as (5 × 8)- 8. In question 4.4, all students believe that the quotient of the left and right sides of the equal sign will not be the same because the right side has a larger number than the left side so it is certain that the right side will have a greater quotient than the left side of the equal sign.

Student Strategies in Completing Tasks

Task 1

Task 1 involved students with four standard equations in an arithmetic context to assess students' knowledge of the equal sign with a rigid operational level of difficulty i.e., only succeeding on equations in the standard format 'a + b = c' and thinking of an equal sign operationally to test whether the respondent understood that variables represent specific and constant numerical values (Matthews, et al., 2010). Even though task 1 was at the easiest level of difficulty, there were around 38% of respondents who had difficulty answering each question item correctly. All respondents used a computational strategy in answering questions. They see the equal sign as a kind of command to complete calculations and create answers. So that many students, even adults, end up being 'stuck' in calculations that don't make sense. When traced in the interview session, they only focused on "doing something to get results" without paying attention to the equal sign as a "relation" and the operations used in the equation. This shows evidence that the concept of equal sign gets less attention in the teaching and learning process in the early grades. This kind of situation is a barrier that prevents individuals from internalizing the properties and meaning of arithmetic operations, from establishing relationships and even from generating deep mathematical thinking (Kiziltoprak & Kose, 2017).

Task 2

The questions in task 2 have a flexible operational difficulty level where students are asked to solve equations in several non-standard formats such as 'a=b+c'. This ability is a fundamental arithmetic skill that allows writing number sentences with mathematical symbols, understanding the basic features of operations and conceptualizing numbers in various forms (Koziltoprak & Kose, 2017). Respondents involved in this study admitted that they almost always saw operations in traditional arithmetic practices, namely to the left of the equal sign and 'answer' to the right (McNeil, et al., 2017). From Table 2 it is known that the respondents did not perform better in task 2 compared to task 1. This situation can be attributed to the fact that students learn arithmetic to be result oriented and that they focus on computation rather than the relationship between numbers and operations.

Task 3

The questions in Task 3 are designed to investigate students' ability to solve equations with operations on both sides. Through computing support, it is hoped that it will be able to bring up a relational view with an operational view. These problems are designed to test students' knowledge of the properties of arithmetic equivalence, for example the distributive property, which has been cited as the rationale underlying formal transformational algebra (Matthews, et al., 2010). The square and triangle symbols are given to ensure students do not have a dependency on certain symbols to indicate unknown. This is expected to be a good introduction to students' understanding of variables. According to the nomological network of changes resistance accounts defined by McNeil, et al, (2017) that solving equations, encoding equations, and defining the equal sign are three different, but theoretically related, constructs involved in children's understanding of mathematical equality.

Task 4

Task 4 provides eight statements varying from arithmetic to algebra and in the form of true/false number sentences. Statements are given in the form of true and false number sentences that can be used to help develop conceptions about the equal sign (Kindrat & Osana, 2018). Respondents were asked to indicate whether the sentence was true or false by circling the symbols '(√) or (B)' for statements they considered correct and '(×) or (S)' for statements they considered incorrect on the test paper. Respondents were then asked to provide reasons for the answers given as well as written justifications for their responses in the blank space provided for the questions.

All respondents (except elementary school students) used computational-relational arguments, namely using calculations to justify their responses. They understand that both sides of the equal sign have the same value and confirm it with calculations (Kindrat & Osana, 2018; Stephens, et al., 2013). This idea was confirmed during the interview. They still have a very high dependence on computing to answer every question. Even adults who are very proficient in algebra respond 'true' to each question with the argument 'the equal sign guarantees that the left and right sides must have the same value' but the strategy used is still computational to ensure that the left side of the sign equals has the same value as the right hand side. Even though they are expected to be able to provide arguments for numerical relationships, for example in a mathematical sentence 10 + 16 = 15 + 11 is true 'because 11 is 1 greater than 10 and so are 16 and 15'.

Learning Experience

Completing arithmetic arithmetic operations in the format a+b=c is the most common learning experience throughout elementary school mathematics learning. So that in general, respondents do not see the equal sign as a 'relationship' but as an operational sign to perform calculations from left to right where the numbers to the right of the equal sign are the result of calculations from a series of arithmetic operations to the left of the equal sign. The equal sign and blank answers in arithmetic problems are usually presented regularly at the end of the problem (e.g., 4 + 7 = ...; 4 - 3 + 6 = ...). However, many higher-level problems don't fit a pattern like "2 × ... + 6 = 20" doesn't have a blank answer to the right of the equal sign. This kind of learning experience has also been reported by several researchers (e.g., Fuchs, L.S. et al., (2014); Kiziltoprak & Kose (2017)).

Understanding of the equal sign as "complete answer" is so strong that Darr (2003) also found that out of 300 students more than half wrote 9 or 12 in the box in equation 4 + 5 = ____+3 as the answer in his study. Likewise with McNeil, et al. (2017) found that when elementary school students from grades one to six were asked sentences such as 8 + 4 = ____ + 5, less than 10% of them were able to give the correct answer. Molina & Ambrose (2008) reported that elementary school students tend to consider the equal sign in arithmetic sentences as operational symbols and react negatively when arithmetic sentences do not match the conception they have so far, such as in sentences of the form c = a + b then change them to a + b = c.

Learning experiences such as completing arithmetic operations in the format "a+b=c" when taught in a very procedural way with little or no reference to the relational concept of the equal sign can form a student's concept image including: the equal sign as "result" " so it can lead to misconceptions about closure types, such as writing ____ with 1 in the equation

____+3=4+5; the equal sign as a marker to put the answer so that the format is like a=a; a=b+c or a+b=⋯+d is hard to understand; the equal sign as the answer marker so that it will tend to fill in 2 at 8-____=10 where the number 10 is "answer"; and an equal sign as a marker to put the answer filling in _____ with 10 at 4 + 6 = _____ + 5.

There are three approaches that apply to introducing a more sophisticated the equal sign to children to provide a more meaningful arithmetic learning experience according to McNeil, et al (2017), namely: 1) focusing on explicit conceptual instructions; 2) focuses on practice with basic arithmetic facts; and 3) focusing on the role of existing knowledge in children's difficulties. The first approach aims to help children examine and reflect on the relationships between the fundamental properties of operations and numbers and express them as generalizations. The second approach aims to increase proficiency with basic facts, higher order thinking and problem solving. Whereas the third approach aims to build a better understanding of mathematical equivalence through non-traditional problem formats, activating students' relational thinking and understanding of equivalence through relational words such as "is the number equal to" instead of using the "=" symbol used for represent equivalences, as well as organize arithmetic problems into exercise sets that allow students to induce equivalent additive pairs based on transitive relationships (eg if 3 + 4 = 7 and 5 + 2 = 7, then 3 + 4 = 5 + 2).

Implications

It is hoped that the results of this study can provide some information to mathematics curriculum developers, mathematics book authors and mathematics educators so that they can provide more opportunities for elementary, secondary and university students to develop concepts correctly about the equal sign. Instead of waiting to introduce the concept during the middle school years, teachers should help students in elementary schools to recognize the equal sign as a symbol representing equality relationships.

Limitations

This research has limitations, including the small number of respondents in one of the big cities in Indonesia, which also limits the variety of responses, and the study has not been conducted longitudinally to see whether the students' understanding that has been found will change or last from time to time. Nonetheless, the respondents involved came from various school levels. So, the results can be used as evidence that students' understanding of the equal sign will not change by itself along with the high school level.

CONCLUSION

Based on the findings obtained in this study indicate that many students at all grade levels have not developed an adequate understanding of the meaning of the equal sign. Students' understanding of the equal sign is evidently not a type of problem that can be considered trivial. Adequate understanding of students about the equal sign does not happen instantly because the equal sign has been introduced to students since they were in elementary school when they studied mathematics at school and they have little time to learn this symbol in the next class. Students' misunderstanding about the equal sign still occurs in higher education. The findings of this study support this claim.

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