Worked-Example Method on Mathematical Problem-Solving Ability in term of Students' Initial Ability

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Abstract
The research aims to understanding the phenomena of differences in mathematical problem-solving ability between students who obtained worked-example and expository learning method when reviewed from students’ initial ability. This study was a quasi-experimental with treatment by level 2×2 research design. We analyze the data used two-way analysis of variance. We found that (1) There were differences in mathematical problem-solving ability between students who obtained worked-example and expository learning method. (2) There were differences in mathematical problem-solving ability between students who had high and low initial ability. (3) In high initial ability, students who obtained expository and worked-example learning method were relatively the same. (4) In low initial ability, students who obtained worked-example learning method were better than expository. (5) There was an interaction between learning method and initial ability in mathematical problem-solving ability.

Keywords: Worked-Example; Problem-Solving Ability; Initial Ability.
INTRODUCTION

Mathematics is a subject that has an important role in the scope of education. In practice, mathematics exists at all levels of education, from elementary school, middle school, to college. In addition, mathematics also dubbed as Queen of Science which means the queen or mother of knowledge, or we could say mathematics is a basic science that forms the basis for other sciences, so that mathematics is interrelated with other sciences. From this, teachers are expected to have mathematical skills, deep understanding, and effective teaching skills so that students can successfully learn mathematics. NCTM (2000) defines that there are several abilities that should be achieved by students in understanding mathematics, namely: (1) problem-solving; (2) reasoning and proof; (3) communication; (4) connections; and (5) representation. Someone can master mathematics by developing mathematical thinking skills. In general, mathematical thinking can be categorized into two levels are low-level and high-level mathematical thinking (Tall, 2004). One of the high-order thinking ability is mathematical problem-solving ability. Mathematical problem-solving ability is a person’s ability to solve mathematical problems with specific characteristics that can’t be solved directly by ways or procedures which are available (Santosa, 2018). Problem-solving ability is considered important, because mathematical problem-solving ability is a skill that student have in order to use mathematical activities to solve problems in mathematics, problems in other sciences, and problem in everyday life (Soedjadi, 1994).

But students’ ability in solving mathematical problems is still low. This supported by the result of Daeka, Budiyono, & Sujadi (2014) study which states that students’ low mathematical problem-solving ability is caused by students not being used to practicing their ability to solve problems. Harahap & Surya (2017) also states that students were still not familiar with problem-solving questions, and they were less able to write down the solutions. When students are given mathematical problems, students can only do routine questions that have never been obtained by students can’t be solve.

Students’ ability in solving mathematical problems is related to having knowledge of mathematical concepts in the form of schemes (Santosa, 2018). Paas, Renkl, & Sweller (2004) argued that understanding occurs when students can construct new knowledge structures by linking the knowledge being learned with previous knowledge. For students who have gained the knowledge to solve these problems, students tend to find it easier to solve problems. It is different with students who have limited prior knowledge and get new problems, students will certainly find it more difficult to solve them. This means that in solving mathematical problems, student with high initial ability supported by their initial knowledge and ability to apply heuristic strategy.

Solving mathematical problems can’t be separated from cognitive load that processed by students. Cognitive Load Theory (CLT) is the latest theory about learning design which developed based on human cognitive system (Sweller, Ayres, & Kalyuga, 2011). Cognitive Load Theory divided cognitive load into three types, namely extraneous cognitive load, intrinsic cognitive load, and germane cognitive load (van Gog, Kester, & Paas, 2011). Cognitive load that hinders the process of understanding or learning something is called extraneous cognitive load, so the source of this load needs to be minimized (Mwangi & Sweller, 1998;
Sweller et al., 2011). If students have high cognitive load, especially extraneous cognitive load, then students will have difficulty in receiving learning material (Retnowati, 2008). Effective learning is learning that combines problem-solving by providing working examples known as worked-example learning (Renkl, Atkinson, Maier, & Staley, 2002).

Worked-example is solving problem by showing a step-by-step solution that contains formulation of problem, steps for solving, and final solution of the problem (Hoogerheide, Loyens, & Van Gog, 2014). The existence of steps in each problem aim to make it easier for students to learn and understand how to find solutions to existing problems. According to CLT, worked-example facilitates students with examples of how to solve new problems for students. Because the subject matter is new, students don’t have initial knowledge that is relevant and strong enough. The existence of examples helps students to build initial knowledge (schema acquisition), so that it can facilitate students to understand problem-solving more effectively (Nuraini, 2016). This is in line with van Gog & Kester (2012), Effective and efficient problem solving skills can be obtained through working examples, which means that solving problems efficiently helped by worked-example. Worked-example are designed for students with insufficient prior knowledge, or often called novice learners.

Based on the description above, this study aims to determine: (1) differences in students’ mathematical problem-solving ability based on learning methods; (2) differences students’ mathematical problem-solving ability based on their initial ability; (3) interaction between learning methods and initial ability of students’ mathematical problem-solving ability.

**METHOD**

Population in this study are students of tenth grade at one of Madrasah Aliyah Negeri (MAN) in the city of Serang in academic year 2019/2020 of even semester. Sampling in this study using purposive sampling technique. The selected subjects were experimental group and control group, totalling 57 students. In experimental group by applying worked-example learning method and control group by applying expository learning method. The value of one group does not affect the value of the other group because each sample has its own score, and each is tested. This means that the test results of each group first are then compared. The research method used was a quasi-experimental with treatment by level 2×2 design. The research design pattern is presented in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Research Design</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial ability (B)</strong></td>
</tr>
<tr>
<td>High (B₁)</td>
</tr>
<tr>
<td>Low (B₂)</td>
</tr>
</tbody>
</table>

**Instruments**

The instruments used in this study included test instruments for initial ability and mathematical problem-solving ability. Test instrument of initial ability was used to determine the initial ability of students in experimental class and control class. Its used was in the form of multiple-choice questions. The score given on multiple choice test is 1 point for each true item and 0 point for each false item.

Meanwhile, test instrument of problem-solving ability was used to measure students’ mathematical problem-solving ability after being given different treatments in experimental class and control class. The problem-solving ability test was in the form of essay. The score given is
based on scoring guidelines according to each indicator of mathematical problem-solving ability, with scale from 0 to 4 points.

Before being given to research sample, both of instruments were tested first for validity and reliability. Based on the result of validity test of initial ability instrument, it was obtained 13 items that were stated valid from 15 items tested. Furthermore, the result of reliability coefficient of initial ability instrument amounted .80 and by the method of internal consistency (Cronbach Alpha) is .82. It means that instrument of initial ability is reliable to use.

Meanwhile, validity of mathematical problem-solving ability instrument, it was obtained 6 items that were stated valid from 7 items tested. Also reliability coefficient of mathematical problem-solving ability amounted .40 and by the method of internal consistency (Cronbach Alpha) is .65. It means that instrument of mathematical problem-solving ability is reliable to use.

**Procedures**

The research begins by indentifying problems related to students’ mathematical problem-solving ability and collecting literature which relevant to the problem was found as well as previous studies. Then, compile the riset instruments and verify its validity and reliability. Furthermore, learning by different methods in each group and applying riset instruments to the research subjects. Next, analyze data and take conclusion.

**Data Analysis**

Data analysis carried out in this research are descriptive and inferential statistics. Descriptively, data described based on number of sample (N), minimum score (min), maximum score (max), average (mean), and standard deviation (SD). Inferentially, data analysis technique includes independent sample t-test (for initial ability data) and two-way ANOVA (for mathematical problem-solving ability data). In addition, Tukey-Kramer test was also used to determine which group differs significantly (Glass & Hopkins, 1970).

**RESULT AND DISCUSSION**

**Result**

*Initial ability*

The data analyzed was data of initial ability that have been given both experimental group and control group before learning process. The arithmetic mean, standard deviation, and independent sample t-test was used as shown as in Table 2.

From Table 2, average of experimental group is higher than control group, which is a difference of .2. Also, the result of mean difference test of initial ability in experimental group and control group shows that t-stat (55) = .06 is less than 2.00, so it can be concluded that students’ initial ability in both groups are not significantly different or relatively same.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Variance</th>
<th>Cohen’s d</th>
<th>t-stat</th>
<th>H0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>26</td>
<td>76.63</td>
<td>158.49</td>
<td>.02</td>
<td>.06</td>
<td>Accepted</td>
</tr>
<tr>
<td>Control</td>
<td>31</td>
<td>76.43</td>
<td>137.81</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mathematical problem-solving ability data

The data analyzed was data post-test of mathematical problem-solving ability that have been given to both experimental group and control group after students were given different treatments. As for post-test data on mathematical problem-solving ability are presented descriptively in the following table:

Table 3 Mathematical problem-solving ability based on group of learning method

<table>
<thead>
<tr>
<th>Method</th>
<th>Level of initial ability</th>
<th>N</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worked-example</td>
<td>High</td>
<td>7</td>
<td>64</td>
<td>86</td>
<td>80.29</td>
<td>8.83</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>7</td>
<td>59</td>
<td>91</td>
<td>81.00</td>
<td>11.66</td>
</tr>
<tr>
<td>Expository</td>
<td>High</td>
<td>8</td>
<td>73</td>
<td>86</td>
<td>80.50</td>
<td>5.25</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>8</td>
<td>59</td>
<td>73</td>
<td>63.37</td>
<td>4.84</td>
</tr>
</tbody>
</table>

From Table 3, worked-example group had a higher average than expository group, with difference of 1.41. The standard deviation of worked-example group was higher than expository group, which indicates that data on the result of students’ mathematical problem-solving ability test in worked-example group was more spread out than expository group.

Table 4 Mathematical problem-solving ability based on level of students’ initial ability

<table>
<thead>
<tr>
<th>Level</th>
<th>N</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>7</td>
<td>64</td>
<td>86</td>
<td>80.00</td>
<td>6.83</td>
</tr>
<tr>
<td>Low</td>
<td>8</td>
<td>59</td>
<td>91</td>
<td>73.73</td>
<td>10.91</td>
</tr>
</tbody>
</table>

From Table 4, students who have high initial ability had a higher average than low initial ability, with difference of 6.27. Meanwhile, the standard deviation of students who have low initial ability was higher than high initial ability, which indicates that data on the result of students’ mathematical problem-solving ability test with low initial ability was more spread out than high initial ability.

In Table 5, group of students who have low initial ability by applying worked-example method had higher average than high initial ability, with difference of .71. Meanwhile, group of students who have high initial ability by applying expository method had higher average than low initial ability, with difference of 13.13. Furthermore, standard deviation of students who have low initial ability by applying worked-example method was higher than high initial ability, which indicated that data on the result of mathematical problem-solving ability test of students who have low initial ability by applying worked-example method was more spread out than students who have high initial ability. The standard deviation of students who have high initial ability by applying expository method was higher than low initial ability, which indicated that data on the result of mathematical problem-solving ability test of students who have high initial ability by applying expository method were more spread out than students who have low initial ability by applying expository method.

Furthermore, data analysis of inferential statistics was carried out which included prerequisite test (normality and homogeneity), hypothesis testing, and post two-way ANOVA test. The inferential statistics analysis result is presented in Table 6.

From Table 6, the value of L-stat (Liliefors) is less than L-crit for each group so that the test decision at significance
level (.05), then all Ho are accepted. Thus, it can be concluded that data for each sample, both learning method group, initial ability group, and initial ability in learning method came from a normally distributed population.

Table 7. Summary of homogeneity data test of mathematical problem-solving ability

<table>
<thead>
<tr>
<th>Group</th>
<th>W</th>
<th>F-crit</th>
<th>H0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learning method</td>
<td>1.68</td>
<td>4.02</td>
<td>Accepted</td>
</tr>
<tr>
<td>Initial ability</td>
<td>3.64</td>
<td>4.19</td>
<td>Accepted</td>
</tr>
<tr>
<td>High initial ability</td>
<td>2.17</td>
<td>4.67</td>
<td>Accepted</td>
</tr>
<tr>
<td>Low initial ability</td>
<td>3.95</td>
<td>4.67</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

From Table 7, all of value Levene’ statistics (W) is less than F-crit so that the test decision at significance level (.05), then all Ho are accepted. Thus, it can conclude that all samples come from population that have same variance (homogeneous).

After prerequisite test of data analysis is fulfilled, then hypothesis testing is carried out. The result of hypothesis testing are presented in Table 8. From Table 8, it can be concluded that: (a) for learning method effect, the value of F-stat (5.34) is more than F-crit which show that null hypothesis (H01) is rejected, it means that there was a significant difference in mathematical problem-solving ability between students who obtained worked-example method and expository method; (b) for initial ability effect, the value F-stat (4.58) is more than F-crit which show that null hypothesis (H02) is rejected, it means that there was a significant difference in mathematical problem-solving ability between students who have high initial ability and low initial ability; (c) for interaction effect, the value of F-stat (5.69) is more than F-crit which show that null hypothesis (H05) is rejected, it means that there was an interaction between learning method and initial ability in mathematical problem-solving ability.

To see whether there is an interaction between learning method and initial ability in mathematical problem-solving ability, it can also be depicted clearly in the plot in Figure 1.
In Figure 1, it shows that both lines of worked-example method (blue line) and expository method (orange line) intersect each other. It means that there is interaction between learning method and initial ability in mathematical problem-solving ability.

Because the null hypotheses are rejected, it is necessary to do a post-ANOVA test. Data analysis using Tukey-Kramer method can be done by looking at the mean of each cell and marginal mean which is presented in Table 9.

<table>
<thead>
<tr>
<th>Initial ability</th>
<th>Learning method</th>
<th>Marginal means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Worked-example</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>80.29</td>
<td>80.39</td>
</tr>
<tr>
<td>Low</td>
<td>81</td>
<td>74.19</td>
</tr>
<tr>
<td>Marginal means</td>
<td>80.64</td>
<td>77.29</td>
</tr>
</tbody>
</table>

From Table 9, the marginal mean group of students who obtained worked-example method (80.64) was higher than marginal mean group of students who obtained expository method (73.94). Thus, it can be concluded that worked-example learning method is better than expository learning method. Based on Table 9, then the result of multiple comparison test between columns can be seen in Table 10.

<table>
<thead>
<tr>
<th>Comparison group</th>
<th>Mean</th>
<th>Standard error</th>
<th>q stat</th>
<th>p-value</th>
<th>Mean-crit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{A_1 B_1} - \mu_{A_2 B_1} )</td>
<td>.21</td>
<td>2.90</td>
<td>.07</td>
<td>.99</td>
<td>11.25</td>
</tr>
<tr>
<td>( \mu_{A_1 B_2} - \mu_{A_2 B_2} )</td>
<td>13.62</td>
<td>2.90</td>
<td>4.69</td>
<td>.01</td>
<td>11.25</td>
</tr>
</tbody>
</table>

From Table 10, it can be concluded that: (a) for comparison \( A_1 B_1 - A_2 B_1 \), mean difference (.21) is smaller than mean-crit which shows that null hypothesis (H_0) is accepted, so it can be concluded that at high initial ability, there is no significant difference between students who obtained worked-example method and expository method for their mathematical problem-solving ability; (b) for comparison \( A_1 B_2 - A_2 B_2 \), mean difference (13.62) is higher than mean-crit which shows that null hypothesis (H_0) is rejected, so it can be concluded that at low initial ability, there is significant difference between students who obtained worked-example method for their mathematical problem-solving ability. Furthermore, from Table 9, at low initial ability, mean of cell students who obtained worked-example method was higher than expository method. This indicates that at low initial ability, students who obtained worked-example method were better than students who obtained expository method for their mathematical problem-solving ability.

**Discussion**

*Mathematical problem-solving ability based on learning method*

The factor that causes students who obtained worked-example learning method has better mathematical problem-solving ability than students who obtained expository learning method is there are difference treatment between experimental group and control group. In the experimental group, students were given worked-example method. Worked-
example method is a learning method by showing a step-by-step solution to solve a problem. In the experimental group, students were facilitated with assignment sheet which contained example of question with worked-example steps that made easier for students to understand concepts or procedures to solve mathematical problems (Santosa, Suryadi, Prabawanto, & Syamsuri, 2018). By applying worked-example, students try to understand the problems that presented. While students did reflection activities to understand mathematical concepts and correct misunderstanding.

Whereas in the control group, students were given expository method. Expository method is one-way learning which students only focus on material that provided by the teacher. In the control group, assignment sheets were given aren’t presented with example of questions that were similar to practice questions that contained completion steps.

Based on the description above, this research is in line with previous study conducted by Santosa (2018), namely the worked-example method has a positive effect on mathematical problem-solving ability.

In addition, in learning mathematics, the worked-example can be related to the Theory of Didactical Situation (Brousseau, 2002), that students learn by adapting to milieu. The independence of students in the learning process without teacher intervention in gaining knowledge is very important for students to adapt to milieu without the help of teachers. In fact, according to Brousseau (2002) that students cannot solve some adidactic situations directly. This means that at the time of learning students also need guidance and explanation from the teacher to achieve learning objectives. This means that in the learning process, there is a need for direct intervention in learning to acculturate higher knowledge, which students have never known before.

Mathematical problem-solving ability based on initial ability

Mathematical initial ability has an influence on the achievement of subsequent mathematics skills. This is supported by Greeno, et al. (Jonassen, 2009) which states that “cognitive researchers agree that the learner’s prior domain knowledge is among the most important determination of problem-solving ability”, which means that researchers agree on the cognitive aspects that students’ prior knowledge is one of the most important determining factors for problem-solving ability. Good mathematical initial ability will enable students to understand mathematics material effectively. Thus, high ability will affect mathematical problem-solving ability.

Learning method at high initial ability

The expository method allows students to learn solving problems optimally. The expository method is considered very effective if the subject matter mastered by students is quite extensive. In the expository group, process of obtaining knowledge is not process of seeking and constructing knowledge. Meanwhile, based on Cognitive Load Theory, which is the basis for worked-example, students with high initial ability have sufficient schemes to solve problem (van Gog et al., 2011). Thus, students who have high initial ability feeling bored when given worked-example learning method. This is because students who have high initial abilities have understood the concept of solving problems quickly and precisely with their own way of solving them based on a schema that has been formed, so that when presented with a worksheet with systematic worked-example steps, it feels long and inef-
efficient. Thus, students with high initial abilities are more suitable for expository learning and discovery learning.

**Learning method at low initial ability**

Worked-example was design for students with insufficient prior knowledge. Because the material is still new, students don't yet have relevant and strong initial knowledge. Implementation of worked-example in students with low initial ability is an effort to reduce cognitive load and help students in forming knowledge schemes for long-term memory, thereby facilitating students to be able to understand and deal with mathematical problems (Santosa, 2019). Assignments designed with worked-examples steps make it easier for students to understand concepts or procedures in solving problems. Learning by example through worked-example method can easily and quickly improve students’ knowledge schemes. When the knowledge scheme is formed, students will be successful in solving mathematical problems. Continuous practice carried out by students will make problem-solving activities automatically which reduces workload of students' brains, so that worked-example is good for students who have low initial ability. This is in line with Baruda’s learning theory, where students learn through the examples provided to learn the completion process gradually to form a scheme through these stages. Based on cognitive load theory and effects of worked-example, it shows that learning from worked-example is better than solving equivalent problems (Irwansyah & Retnowati, 2019).

**Interaction between learning method and initial ability of mathematical problem-solving ability**

Interaction in this study can be seen from learning method and students’ initial ability. The learning method used are worked-example method and expository method, while the initial ability is divided into two categories, namely high level and low level. According to Irawan, Suharta, & Suparta (2016), there are things that affect mathematical problem-solving ability, namely learning method and mathematical initial ability. The use of worked-example method is more suitable for students who don’t have an initial scheme that is strong enough to solve problems (Hillen, Gog, & Gruwel, 2012). While the expository method has a passive impression of students in learning process, namely students only listen and record what the teacher says. Based on this, the expository method will be easier to understand by students with high initial ability, while for students who have low initial ability, it will be difficult to adapt in expository learning method.

**Limitation of Study**

This research was conducted online via Google Classroom. This is due to pandemic of Covid-19 that occurred in Indonesia when the research was conducted, so it wasn’t possible to do it directly in the classroom.

**CONCLUSION**

Based on the result of data analysis and discussion, also referring to the problem formulations that have been described, it can concluded that: (1) there are significant differences in mathematical problem-solving ability between students who obtained worked-example method and students who obtained expository method; (2) there are significant differences in mathematical problem-solving ability between students who have high initial ability and students who have low initial ability; (3) at high initial ability, mathematical problem-solving ability of
students who obtained expository method were relatively same as those who obtained worked-example method; (4) at low initial ability, mathematical problem-solving ability of students who obtained worked-example method were better that those who obtained expository method; and (5) there is interaction between learning method and initial ability in mathematical problem-solving ability.

For further researchers, it is recommended to see an increase in every indicator of mathematical problem solving and other abilities that can be applied through the worked-example learning method.

REFERENCES


