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DIAGNOSING MENTAL MODELS OF UNDERGRADUATE STUDENT AND PHYSICS TEACHERS: STUDY CASE IN THE MOMENTUM AND ENERGY CONSERVATION PRINCIPLES USING NEWTON'S CRADLE

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ABSTRACT

The Newton's Cradle comprises a series of pendulums often used in physics classrooms to demonstrate the principles of conservation of momentum and kinetic energy in elastic collisions. By utilizing open-ended questions and several Newton's cradle scenarios, this study aims to clarify how the Newton's cradle can be used for assessing the mental models of the principles of conservation of momentum and energy. Interviews with 18 college students and five physics professors each lasted 30 to 45 minutes. Firstly, they were asked to explain how the Newton's cradle works. Next, a scenario started with three balls with equal masses where one collides with the other two, they were asked to explain if the possible outcomes for a collision with initial momentum = $mv_1 + 0 + 0$ and the final momentum = $(-mv_1/3) + (2mv_1/3) + (2mv_1/3)$. With a five-ball Newton's Cradle, students were asked to explain the outcomes when 1) one collides with the other four, 2) two collide with the other three, 3) three collide with the other two, and 4) four collide with the last one. Their drawings and explanations dealing with the Newton's cradle during the interviews were analyzed for their mental models and categorized into patterns. The results showed that both university students and physics teachers hold misinformed conceptions about conservation of momentum and energy and have unsound mental models in the context of Newton's Cradle. We also found that they did not recognize that sound and heat or lack of exact alignment of the balls are factors accountable for loss of energy causing the Newton's Cradle to depart from idealized situation. Based on the interview data, teaching principles of the conservation of momentum and energy through use of Newton's cradle are deficient and curriculum review is suggested.

Keywords: conservation of momentum and energy; mental models; Newton's cradle

INTRODUCTION

Conservation of momentum and energy are important topics of learning physics because they are fundamental concepts that can be applied in mechanics (Suprapto, 2020). These concepts are useful for students' daily work and lives. However, not only high school and undergraduate students but also physics teachers had grave misconceptions and did not fully understand the principles of conservation of movement and energy, (Singh & Rosengrant, 2003; Halim et al., 2014; Saifullah et al., 2017) especially when faced with new problem sets (Daud et al., 2015). Students have succeeded at multiple choice tests simply by recalling what they have previously seen, sometimes without necessarily understanding concepts (Marek, 1986; Samsudin et al., 2019), but they were not able to think beyond the classroom or apply their knowledge to solve a new problem.

Students' mental models regarding scientific phenomenon contain misconceptions, close to 70% of High School students were reported to have mental models that could not correctly describe abstract physics concepts (Chiou, 2013: Kurnaz & Eksi, 2015; Liu & Fang, 2017). We expected a similar result with our under-graduate

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participants and a more refined understanding from the 5 Physics teachers. Our research has shown that most of the students and teachers had flawed mental models.

t is not easy to comprehend the concepts and apply them to new situations scientifically. Thus, most teachers tried to facilitate students with several learning resources both in classroom and in the laboratory. However, students still hold misconceptions caused by various factors such as instructional processes, learning media, and learning atmospheres. Many studies on the relationship between momentum and energy found that students had difficulties in clarifying the differences between momentum and energy (Grimellini-Tomasini et al., 1993; Singh & Rosengrant, 2003, Gilbert & Watts, 2013; Dalaklioğlu et al., 2015; Hanson & Seheri-Jele, 2018; Samsudin et al., 2021; Bao & Fritchman, 2021). They were able to deal with collisions as single events but incapable of understanding a complex series of events in a question. Similar to work done by Singh & Rosengrant (2003), they also pointed out that many students lacked a coherent understanding of energy and momentum concepts and had difficulties implementing theory in different situations. These difficulties are often caused by a tendency to focus on surface features and distraction by irrelevant details. They found that university students had conceptual problems in the application of conservation of momentum and energy in new situations because: 1) students had difficulties understanding and applying the idea that momentum is a vector quantity. Instruction was largely ineffectual in remedying these difficulties, 2) many students operated on the premise that momentum was conserved for each object in a system rather than being conserved by the system of objects as a whole, 3) a large proportion of students thought that the size of the force that is exerted by an object hitting a surface was related only to the initial velocity of that object, rather than to its change in velocity and hence its change in momentum. Students focused on momentum and its conservation and thought that momentum is not conserved in collisions with ''immovable'' objects (Bryce & MacMillan, 2009; Xu et al., 2020; Halilović et al., 2021; 2022). Linking the concept of the change in momentum and impulse of an object was a problem faced in classroom learning (Lawson & McDermott, 1987; Pride et al., 1998; Alonzo & Mistades, 2021).

To enhance student understanding of conservation of momentum and energy, students' mental model is considered as a main factor of building concept. Mental models have been outlined as internal representations of concepts and ideas (Hegarty et al., 2013; Dankenbring & Capobianco, 2016; Haili et al., 2017). They are memory structures that lead students to build deeper understanding from their surface or prior understanding. Then they could be used as a tool to know the ways in which students learn and apply knowledge, how information gets into and is accessed from memory. Then they could provide us with the valuable information about the students' conceptual framework (Vosniadou, 1994).

Mental models are internal representations of information and experiences from the outside world. The mental model theory is believed to be an attempt model to explain human understanding of objects and phenomenon (Taher et al., 2017; Volfson et al., 2021). It evolves in the mind of a user as he or she learns and the mental model will represent the structure and internal relationships of a system (Özcan & Bezen, 2016; Sing & Rosengran, 2016; Burkholder et al., 2020; Dinçer & Özcan, 2021). Mental models can be defined as the internal, conceptual and operational representations that humans develop while interacting with complex systems (Jonassen & Henning, 1999; (Didiş et al., 2014). When a new situation or only an event are/is presented to students, networks of knowledge or memory dealing with that situation are used. As has been revealed by Jih & Reeves (1992) that "Since our understanding of human perception does (or should) play a crucial role in the design of interfaces. Research on mental models is a promising approach to analyzing human-computer interaction and improving interface designs". After students have provided understanding of the concept or domain, they need an engagement of building network of that understanding with new presented context, to develop students' application ability. Similar to Ackermann & Tauber (1990) who mentioned that mental model is based on the assumption that knowledge of how users represent systems and how users should represent systems will lead to a better understanding of usable systems.

Using the mental model theory in learning physics, especially the conservation of momentum and energy, student background and possible mental model need to be investigated for providing further conceptual framework of a study (Mason et al., 2013). The Newton's cradle, which is often used in physics textbooks to demonstrate the operation of the principles of conservation of momentum and kinetic energy in elastic collisions. Most students learned how Newton's cradle works as a device consisting of a row of steel balls, when the ball at one end is pulled aside and released it collides with the remaining stationary balls and the ball at the other end of the row moves off to reach what appears to be the same height from which the first ball was released. All the other balls are apparently at rest. From this operating, it seems like the Newton's cradle could be used as effectively demonstrating of the conservation of momentum (Freudenthal et al., 2014). This study aims to investigate for the possible mental models dealing with conservation of momentum and energy by assessing how physics teachers and undergraduate students build and apply these concepts representations to solve problems involving Newton's cradle. The research sought to address the following research questions: (1) What mental models do undergraduate students and physics teachers hold for conservation of momentum and energy?; (2) What scientific conceptions do undergraduate students and physics teachers hold for conservation of momentum and energy?; (3) How do students apply them in novel settings with Newton's cradle problems?

METHODS

This study attempts to explain how the Newton's cradle can be used for diagnosing the mental models of the principles of conservation of momentum and energy by using open-ended questions along with different Newton's cradle scenarios. 18 university students and five physics teachers were interviewed individually for 30-45 minutes. The following interview questions were asked of the participants: (1) Scenario 1 (SC1): With a five-ball Newton's Cradle, when one collides with the other four, how does the New-ton's cradle work?; (2) Scenario 2 (SC2): With a three-ball Newton's cradle, where one collides with the other two, if the possible out-comes for a collision with initial momentum = mv1 + 0 + 0 and the final momentum = (-mv1/3) + (2mv1/3) + (2mv1/3)?; (3) Scenario 3: With a five-ball Newton's Cradle, students were asked to explain the outcomes when : - SC31: Two collide with the other three; - SC32: Three collide with the other two; - SC33: Four collide with the last one.

Students' drawings and explanations dealing with the New-ton's cradle during the interviews were analyzed for their mental models and categorized into patterns.

RESULTS AND DISCUSSION

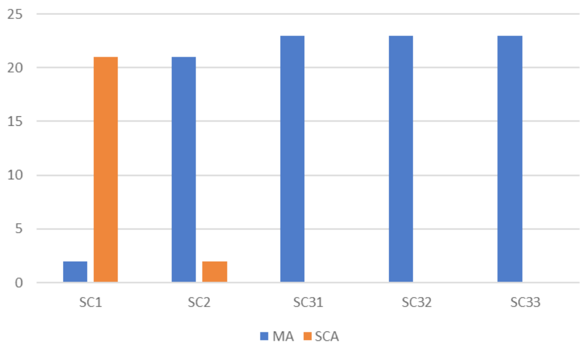
From Scenario 1, most of the undergraduate students and the physics teachers (16, 5) have seen Newtons' cradle and they know how it works when one ball is released initially. See Table 1.

Table 1. Participants' Answers of How Newton's Cradle works

Scenario	Answer type1	Answer type2	Answer type3	Scientific answer	GAP
SC1	PhS 16	2		16	2
	PhT 5			5	0
SC2	PhS =18			0	18
	PhT=4	2*		2*	16*
SC31	PhS 13	3	2	0	18
	PhT 5			0	5
SC32	PhS 13	3	2	0	18
	PhT 5			0	5
SC33	PhS 13	3	2	0	18

[Physics students (PhS) (n=18) attended fundamental phys-ics class.Physics teachers (PhT) (n=5) have been teaching for 5-10 years.
* The participants were not able to explain with principles of conservation of momentum and energy
GAP: misconception's answers- Scientific answers]

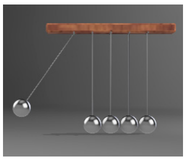
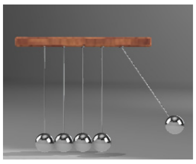
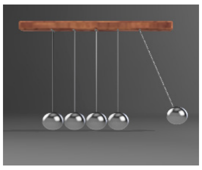
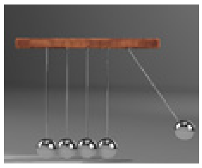
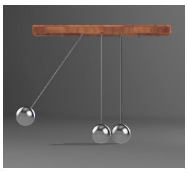
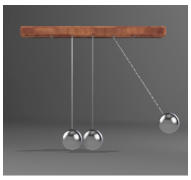
They explained that it deals with the conservation of momentum. The first ball (m_1) is initially moving and then comes to rest when the collision occurs; the balls in the middle (m_2, m_3, m_4) never move during the collision; and a last ball (m_5) which began at rest, then moves off after the collision. They also identified that the velocity of the first ball before the collision equals the velocity of the last ball after the collision using formula $m_1v_1=m_2v_2$, thus for the Newton's cradle $m_1=m_2$, so v_1 is supposed to equal v_2. From the interview, it seems like the undergraduate students and the physics teachers have considerable knowledge structure concerning the conservation of momentum. Figure 1 shows the participant gap scores distribution in answering how Newton's cradle works.

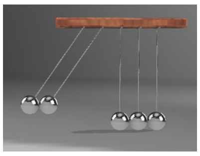
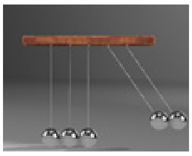
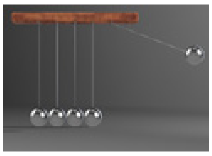
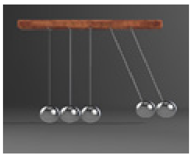
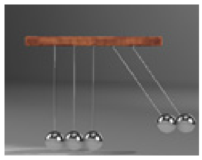
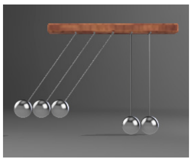
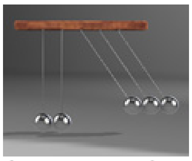
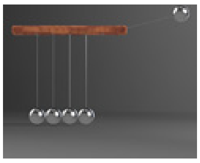
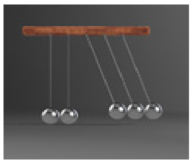
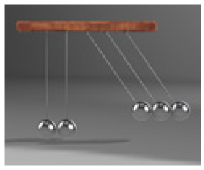
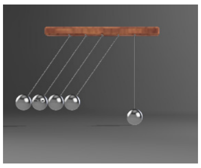
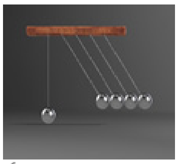
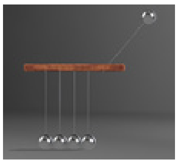
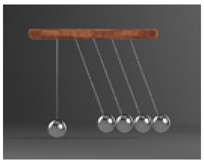
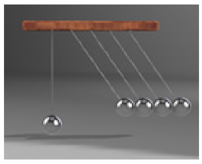


[MA: Misconception Answers, SCA: Scientific Answer]
Figure 1. Graphical Distribution of Gap Scores for Participants' Answers of How Newton's Cradle Works

In fact, the outcome of scenario 2 would observe the two right hand balls were combined to form one ball with mass 2m. So according to scenario 2, it is an impossible outcome for a collision with initial momentum = $mv_1 + 0 + 0$ and the final momentum = $(-mv_1/3) + (2mv_1/3) + (2mv_1/3)$. Actually, undergraduate students and the physics teachers always observed conservation of momentum and kinetic energy is conditions which the behavior of Newton's Cradle that a collision with initial momentum = $mv_1 + 0 + 0$ and the final momentum = $0 + 0 + mv_1$. Then when they were asked about this scenario, they were confused and were not able to explain it by using their understanding of the conservation of momentum. They were not able to explain why other outcomes, consistent with principles of conservation of momentum and energy, do not occur. Through this set of scenarios, undergraduate students' and the physics teachers' mental models of Newton's cradle are categorized into three groups as shown in Table 2.

Table 2. Mental Models of Newton's Cradle

Before Collision	**Mental Models After Collision**			**Scientific Models After Collision**
	Type 1	**Type 2**	**Type 3**	
 SC1	 $m1v1 = m5v2$ $v1 = v2$	 $m1v1 = m5v2$ $v1 > v2$		 $m1v1 = m5v2$ $v1 = v2$
 SC2	 $mv_1 + 0 + 0 = \frac{-mv_1}{3} + \frac{2mv_1}{3} + \frac{2mv_1}{3}$	 $mv_1 + 0 + 0 = \frac{-mv_1}{3} + \frac{2mv_1}{3} + \frac{2mv_1}{3}$		 $mv_1 + 0 + 0 = \frac{-mv_1}{3} + \frac{2mv_1}{3} + \frac{2mv_1}{3}$

Before Collision	Mental Models After Collision			Scientific Models After Collision
	Type 1	Type 2	Type 3	
 SC3.1	 $(m1+m2)v1 = (m4+m5)v2$ $v1 = v2$	 $(m1+m2)v1 = m5v2$ $v1 = 2v2$	 $(m1+m2)v1 = (m4+m5)v2$ $v1 > v2$	 $(m1+m2)v1 = (m4+m5)v2$ $v1 = v2$
 SC3.2	 $(m1+m2+m3)v1 = (m3+m4+m5)v2$ $v1 = v2$	 $(m1+m2+m3)v1 = m5v2$ $v1 = 3v2$	 $(m1+m2+m3)v1 = (m3+m4+m5)v2$ $v1 > v2$	 $m1v1 = m5v2$ $v1 = v2$
 SC3.3	 $(m1+m2+m3+m4)v1 = (m2+m3+m4+m5)v2$ $v1 = v2$	 $(m1+m2+m3+m4)v1 = m5v2$ $v1 = 4v2$	 $(m1+m2+m3+m4)v1 = (m2+m3+m4+m5)v2$ $v1 > v2$	 $m1v1 = m5v2$ $v1 = v2$

Category 1: The situation is a little more complicated when there are more incoming balls, the number entering and leaving is the same. The people who expressed this model could tell us correctly what happened for each sub-scenario 3 and they also tried to explain their answer using the principle of the conservation of momentum by keeping in mind that velocity of the ball before and after the collision are equal. So, if they want to make the formula balance, the mass or number of the leaving ball need to be equal to mass or number of the entering ball. Their mental model of how Newton's cradle works looks like the picture (figure2) below that means only one time collision has occurred.

Figure 2. Metal Model Category 1

Category 2: The people who expressed this model gave us the wrong answer of what happened for each sub-scenario 3, while they try to explain their answer using the principle of the conservation of momentum as well. They viewed incoming balls were combined to form one ball with mass 2m, 3m or 4m., after they collision, only the last ball is moving. However, people still tried to balance the formula; $m_1v_1=m_2v_2$. Then less mass of one moving ball after collision, its velocity needed to be higher than the velocity of the coming balls before the collision. Their mental model of how Newton's cradle works looks like the picture (Figure 3) Metal model category 2 below that means only a one time collision has occurred.

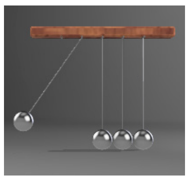


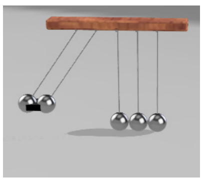
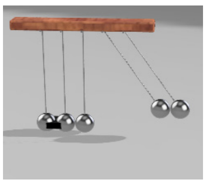
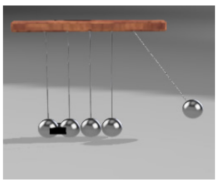
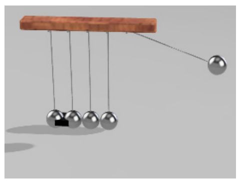
Figure 3. Metal Model Category 2

Category 3: This group thought similarly to the first group but had a different understanding of the speed of the ball after collision would be less than the speed of the first ball because energy will be lost during collision.

From the fundamental ideas, it is recommended that momentum and energy should be taught along with mechanical situations and students' real-world situations, rather than only using a simple scientific model in the classroom. We would like to propose that students building knowledge correspondence to Newton's cradle problems by setting a conflict situation as shown in Table 3.

However, understanding of students' prior know- ledge and possible mental models (Didiş et al., 2014) are useful in providing appropriate situations and effective pedagogies as well. Multi-scenarios should be used to introduce and explain the concepts of the conservation of momentum and energy so students can better understand the concept and see how the concepts are transferred and applied. The new approach should address students' prior knowledge and possible mental models in teaching and learning processes. Hence, teaching should give more emphases on 1) developing the student's conceptual understanding before solving theoretical or mathematical problems and 2) encourage students to express their ideas by using the technical terms and concepts with an event consistently in order to improve student's transfer of heat and thermodynamics concepts. If one would like to use Newton's cradle to demonstrate to the students, the explanation in mass and speed is needed, during collision each ball collides together. Although we will release the ball at the same time, do not forget, the balls were not fixed together, but when they are fixed together (the first ball and second ball), see the last instant scientific model in table 3. and released to collide with the others shows that only the last ball moves at twice the speed of the first collision. It is the misunderstanding of most students and professors that Type 1 and Type 2 scenario depicted in table 3. are the result expected when two fixed balls collide with three other balls. The task of educators will be to first understand the fundamentals of conservation of momentum and then to adequately convey that knowledge to their students. Using the example of newtons cradle, one can visualize the transfer of energy from one fixed object to the other (Gauld & Cross, 2020). Although the perceived concept of conservation of energy in the newtons cradle example is the balls that appear to remain stationary are in fact moving, although this movement is not easily detectable. Understanding this concept allows educators and learners to create mental models where they can understand that the energy of the whole mass of the impacting ball/s will be transferred through each body individually. The result will be the movement of the final body in the chain will be almost equal to the initial impacting body (Brunt & Brunt, 2013). The findings reveal information on the gap between the students' prior knowledge and the scientific answer, which is crucial for effective educational intervention. Practically speaking, the findings suggest that rather than concentrating on how to correct misconceptions, which is the aim of many science education programs, we should change the way we think about how to handle students' conceptions that, while false, appear to represent steps required in the learning process. Students must receive both of the following in science classes: information, scientific methodologies, and the necessary deductions to link their original hypotheses with paradoxical scientific explanations.

Table 3. A Settle of Newton's Cradle

Before Collision	**Mental Models after Collision**			**Scientific Model after Collision**
	Type 1	**Type 2**	**Type 3**	
				
$m1$ and $m2$ are fixed	$(m1+m2)v1 = (m4+m5)v2$ $v1 = v2$	$(m1+m2)v1 = m5v2$ $v1 = v2$		$(m1+m2)v1 = m5v2$ $v2 = 2v1$

CONCLUSION

In brief, individuals often develop and rely on mental models that are faulty or inaccurate dealing with the conservation of momentum. These faulty models can lead to flawed reasoning. For example, learners often possess faulty mental models for one collision occurred in Newton's cradle which leads to inaccurate expectations and/or predictions about other scenarios of entering balls. These two mental models represented the structure and internal relationships of a Newton's cradle. However, it is not easy to comprehend the concepts and be able to apply to new situations scientifically. 18 University students and five physics teachers were able to deal with or think of collisions as single events only, but they cannot work out with continuous event of collisions, such as in Newton's cradle. Learners across all academic levels prefer simple, realistic-looking mental models, it varied as to what extent to which they were able to use them to explain common physical and chemical properties of a substance. They also pointed out that many students lacked a coherent understanding of energy and momentum concepts and some difficulties applying in different situations. This study recognized the fact that Thai undergraduate students and physics teachers had difficulties in applying understanding of conservation of momentum with unfamiliar events. The learner also had problems with descriptions to produce detailed and consistent predictions about all the features of the system of Newton's cradle. This study indicated that in spite of the apparent simplicity of the demonstration, the behavior of Newton's Cradle is not adequately explained to the learners. Then they needed more complicated events dealing with momentum and energy in order to find a way to apply their knowledge. The learners needed a better understanding of the concept or domain, they also need an engagement of building network of that understanding with new presented context, in order to develop their own students' application ability.

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