Skewed Normal Distribution Of Return Assets In Call European Option Pricing

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Abstract
Option is one of security derivates. In financial market, option is a contract that gives a right (not the obligation) for its owner to buy or sell a particular asset for a certain price at a certain time. Option can give a guarantee for a risk that can be faced in a market.This paper studies about the use of Skewed Normal Distribution (SN) in call europeanoption pricing. The SN provides a flexible framework that captures the skewness of log return. We obtain aclosed form solution for the european call option pricing when log return follow the SN. Then, we will compare option prices that is obtained by the SN and the Black-Scholes model with the option prices of market.

Keywords*:* skewed normaldistribution, log return, options.

1. INTRODUCTION

Option is a contract that gives the right (not obligation) to the contract holder (option buyer) to buy (call option) or sell (put option) a particular stock at a certain price within a specified period.

In practice, there are many different types of option. Based on the form of rights that occurs, the option can be divided into two, namely a call option and put option. Whether buying or selling option, they can type in europe and america.

In the pricing of stock option, Black-Scholes model is the first model used in option pricing. This model limited the problem by making an assumption that stock returns follow a Normal Distribution while stock prices follow Lognormal Distribution

In fact, the return data are often not normally distributed. In addition, the return data are often found in non zero skewness, see Hsieh [1], Nelson [3] and Theodossiou [4]. Thus, the Normal Distribution for the return and Lognormal Distribution for stock data which is the assumption in Black-Scholes model is less able to explain the existence of this skewness. This has encouraged researchers to perform calculations of the option price based on distribution approach. They expect that the characteristics of data is used can be modeled well by the approach of the distribution (other than Normal and Lognormal Distribution).One such distribution is Skewed Normal Distribution (SN). Therefore, this paper will discuss the use of SN for stock return on assets in determining the price of european call option.

The remainder of this paper begins with Ito's Lemma, Brownian Motion and Black-Scholes Model in section 2, while section 3 derives SN approach pricing model for europeancall options and the last section offers the conclusions.

2. ITO's LEMMA, BROWNIAN MOTION, ANDBLACK-SCHOLES MODEL

2.1 Itô's Lemma

The price of each derivative security is a function of the security stochastic variables underlying the derivative and time. Therefore, it is need an understanding about the functions of stochastic variables. An important result in understanding this area was discovered by the mathematician, K. Ito in 1951, later known as Itô'sLemma.

Itô's Lemma.*If F(x,t)be a continuous function, twice differentiable function ofx andt, or*

$$\frac{\partial F}{\partial t},\frac{\partial F}{\partial x},\frac{\partial^2 F}{\partial x^2}$$

and

$$\{x(t), t \geq 0\}.$$

Defined a stochastic differential equation with driftrate $a(x,t)$ *and variance rate* $b^2(x,t)$,

$$dx = a(x,t)\ dt + b(x,t)\ dW\ ,\quad x(0) = x_0,\ 0 \leq t \leq T \tag{2.1}$$

where dW is Wiener Process, a and b *are function of* x *and* t, *then a function F of* x *and* t *follow the process:*

$$dF = \left\{\frac{\partial F}{\partial x}a(x,t) + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}b^2(x,t)\right\}dt + \frac{\partial F}{\partial x}b(x,t)\,dW. \tag{2.2}$$

2.2 Brownian Motion of Stock Price Process

Defined a Geometric Brownian Motion of stock price as follow

$$dS = \mu S dt + \sigma S dW \tag{2.3}$$

where μ and σ are constants. Equation (2.3) had been known as a model of stock price behavior.

Using Itô's Lemma, the solution of equation (2.3) is

$$S_T = S_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma W_T\right\}. \tag{2.4}$$

2.3 Black-Scholes Model

Black-Scholes model is the first model used in option pricing.

Hull [2] states that european call option with the corresponding payoff $f_T = (S_T - K, 0)^+$, rational price C_{BS} set by the Black-Scholes formula is

$$C_{BS} = S_0 N(d_1) - Ke^{-r\tau} N(d_2) \tag{2.5}$$

where S_0 is stock price at time 0, S_T is stock price at a future time T, K is strike price,

$$d_1 = \frac{\ln(S_0/K) + \tau(r + \sigma^2/2)}{\sigma\sqrt{\tau}}, \tag{2.6}$$

$$d_2 = \frac{\ln(S_0/K) + \tau(r - \sigma^2/2)}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}, \tag{2.7}$$

and

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}dy$$

is the cumulative standard normal distribution.

3. SNAPPROACH FOR DETERMINING THE PRICE OF EUROPEAN CALL OPTION

This section describes the calculation of the european call option price using SN approach for return assets. However, the previously disclosed little about the Skewed Generalized Error Distribution (SGED) which is a generalization of SN.

3.1 Density Function ofSGED andSN

A continuous random variable Y followsSGEDwith parameter μ, σ, k, λ, if it has probability density function which can be expressed as follows:

$$f\left(y|\mu,\sigma,k,\lambda\right)=\frac{C}{\sigma}\exp\left(-\frac{1}{\left(1+sign\left(y-\mu+\delta\sigma\right)\lambda\right)^{k}\theta^{k}\sigma^{k}}\left|y-\mu+\delta\sigma\right|^{k}\right)(3.1)$$

where

$$C=\frac{k}{2\theta}\Gamma\left(\frac{1}{k}\right)^{-1},\ (3.2)$$

$$\theta=\Gamma\left(\frac{1}{k}\right)^{\frac{1}{2}}\Gamma\left(\frac{3}{k}\right)^{-\frac{1}{2}}S\left(\lambda\right)^{-1},\quad(3.3)$$

$$\delta=2\lambda AS\left(\lambda\right)^{-1},(3.4)$$

$$S\left(\lambda\right)=\sqrt{1+3\lambda^{2}-4A^{2}\lambda^{2}}\text{ , and}(3.5)$$

$$A=\Gamma\left(\frac{2}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-\frac{1}{2}}\Gamma\left(\frac{3}{k}\right)^{-\frac{1}{2}}.\ (3.6)$$

Notation of μ and σ represent the expected value and standard deviation of random variable Y.

In the equation above, the sign states Signum Function, while $\Gamma(.)$ stating Gamma Function. Parameter k with constraints $k>0$ to control the height and the tails of density function, while λ which obey the following constraint $-1<\lambda<1$ is a skewness parameter, as in Theodossiou [4].

In the case of positive skewness $\lambda>0$, the density function is skewed to the right, whereas for the case of negative skewness $\lambda<0$, the density function is skewed to the left.Theodossiou and Trigeorgis [5] state that the probabilitydensity function of SN is a special case of the probabilitydensity function of SGED for parameter values $k=2$.

Through substitution $X=Y-\mu+\delta\sigma$ is obtained that the probability density function of SN is

$$f\left(x|\mu,\sigma,\lambda\right)=\frac{C}{\sigma}\times\exp\left(-\frac{|x|^{2}}{\left[1+sign(x)\lambda\right]^{2}\theta^{2}\sigma^{2}}\right).(3.7)$$

where $C=\frac{1}{\theta}\Gamma\left(\frac{1}{2}\right)^{-1}$,

$\theta=\Gamma\left(\frac{1}{2}\right)^{\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)^{-\frac{1}{2}}S(\lambda)^{-1},\ \delta=2\lambda AS\left(\lambda\right)^{-1}$,

$S\left(\lambda\right)=\sqrt{1+3\lambda^{2}-4\lambda^{2}\Gamma\left(\frac{1}{2}\right)^{-1}\Gamma\left(\frac{3}{2}\right)^{-1}}$, and

$A=\Gamma\left(1\right)\Gamma\left(\frac{1}{2}\right)^{-\frac{1}{2}}\Gamma\left(\frac{3}{2}\right)^{-\frac{1}{2}}$.

Probability of SN density function illustrated by this figure

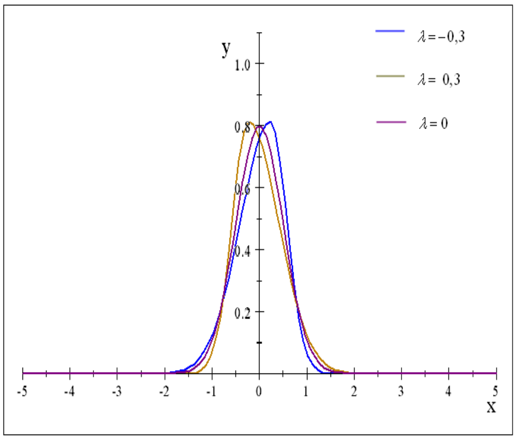

Figure 2 Probability of SN density function

Note that, figure 2 only differ in the value of λ, while for value of μ,σ and k are the same.

Can be calculated directly that

$E\left(X\right)=\delta\sigma$, $\mathrm{Var}(X)=\sigma^2$, $\Psi_1=0$,

$$\Psi_2=\sigma^2\left[(1+3\lambda^2)\Gamma\left(\frac{3}{k}\right)\Gamma\left(\frac{1}{k}\right)^{-1}\theta^2-\delta^2\right],$$

and $\Psi_3=\left[A_3-3\delta-\delta^3\right]\sigma^3$.

3.2 Estimation of SN Parameter by Moment Method

This section derives estimating of SN parameters by using moment method.

Let the unknown mean and variance of the SN respectively are μ and σ^2

If skewness is also known, it can be easily calculated that $E(Y^2)=\mu^2+\sigma^2$ and $E(Y^3)=SK\left(\sigma^2\right)^{\frac{3}{2}}+3\sigma^2\mu+\mu^3$.

Based on the definition of sample moment, it was known that sample moments of SN are $m_1=\frac{\sum_{t=1}^{n}Y_t}{n}=\overline{Y}$,

$m_2=\frac{\sum_{t=1}^{n}Y_t^2}{n}$, and $m_3=\frac{\sum_{t=1}^{n}Y_t^3}{n}$.

Furthermore, the calculation shows that estimates of μ, σ^2 and λ respectively are $\hat{\mu}=\frac{\sum_{t=1}^{n}Y_t}{n}=\overline{Y}$, $\hat{\sigma}^2=\frac{\sum_{t=1}^{n}\left(Y_t-\overline{Y}\right)^2}{n}$, and $\hat{\lambda}=\left(0.626657068656\right)\widehat{SK}$.The details of derivation of $\hat{\mu}$, $\hat{\sigma}^2$ and $\hat{\lambda}$ can be found in appendix 1, below.

Appendix 1-Estimation of SN Parameter by Moment Method

Using Moment Method, mean and variance of SN respectively are

$E(Y)=\mu$

$$Var(Y)=\sigma^2 \Leftrightarrow E[(Y-E(Y))^2]=E(Y^2)-[E(Y)]^2=E(Y^2)-\mu^2$$

Then, let*skewness* of SN is

$$SK=\frac{E[(Y-E(Y))^3]}{[E[(Y-E(Y))^2]]^{\frac{3}{2}}}=\left[A_3-3\delta-\delta^3\right]$$

On the other hand, we know

$$SK\left(\sigma^2\right)^{\frac{3}{2}}=E(Y^3)-3\sigma^2\mu-\mu^3,$$

so

$$E(Y^3)=SK\left(\sigma^2\right)^{\frac{3}{2}}+3\sigma^2\mu+\mu^3$$

Consider that moment population of SN are

$$\mu_1=E(Y)=\mu,\mu_2=E(Y^2)=\mu^2+\sigma^2,\mu_3=E(Y^3)=SK\left(\sigma^2\right)^{\frac{3}{2}}+3\sigma^2\mu+\mu^3$$

Based on definition of sample moment of moment method, sample moment of SN are

$$m_1=\frac{\sum_{t=1}^{n}Y_t}{n}=\overline{Y},m_2=\frac{\sum_{t=1}^{n}Y_t^2}{n},m_3=\frac{\sum_{t=1}^{n}Y_t^3}{n}.$$

Furthermore, using moment method we conclude that

$$\mu_1=m_1 \Leftrightarrow \hat{\mu}=\frac{\sum_{t=1}^{n}Y_t}{n}=\overline{Y}. \quad (A.2.1)$$

$$\mu_2=m_2 \Leftrightarrow \mu^2+\sigma^2=\frac{\sum_{t=1}^{n}Y_t^2}{n}. \quad (A.2.2)$$

$$\mu_3=m_3 \Leftrightarrow SK\left(\sigma^2\right)^{\frac{3}{2}}+3\sigma^2\mu+\mu^3=\frac{\sum_{t=1}^{n}Y_t^3}{n}. \quad (A.2.3)$$

Using equation (A.2.3), we get

$$\widehat{SK}=\frac{\frac{\sum_{t=1}^{n}Y_t^3}{n}-3\left(\frac{\sum_{t=1}^{n}Y_t^2}{n}-\overline{Y}^2\right)\overline{Y}-\overline{Y}^3}{\left(\hat{\sigma}^2\right)^{\frac{3}{2}}}=\frac{1}{n}\frac{\sum_{t=1}^{n}\left(Y_t-\overline{Y}\right)^3}{\hat{\sigma}^3}.$$

Based on the above results, we get $\hat{\mu},\hat{\sigma}^2$ and $\widehat{SK}$ are

$$\hat{\mu}=\overline{Y} \quad (A.2.4)$$

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n}\left(Y_t - \overline{\mathrm{Y}}\right)^2}{n} \Leftrightarrow \hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}\left(Y_t - \overline{\mathrm{Y}}\right)^2}{n}} \quad \text{(A.2.5)}$$

$$\widehat{SK} = \frac{1}{n}\frac{\sum_{t=1}^{n}\left(Y_t - \overline{Y}\right)^3}{\hat{\sigma}^3} \cdot \quad \text{(A.2.6)}$$

Using equation (A.2.4), (A.2.5) and (A.2.6), we will find the parameter estimator for λ as follows

$$SK = \left[A_3 - 3\delta - \delta^3\right] = \left\{\begin{array}{l} 4\lambda\left(1+\lambda^2\right)\Gamma(2)\Gamma\left(\frac{1}{2}\right)^{-1}\left[\Gamma\left(\frac{1}{2}\right)^{1/2}\Gamma\left(\frac{3}{2}\right)^{-1/2}\left(\sqrt{1+3\lambda^2-4\left(\Gamma\left(\frac{1}{2}\right)^{-1}\Gamma\left(\frac{3}{2}\right)^{-1/2}\right)^2\lambda^2}\right)^{-1}\right]^3 \\ -3\left[2\lambda\left(\Gamma\left(\frac{1}{2}\right)^{-1/2}\Gamma\left(\frac{3}{2}\right)^{-1/2}\right)\left(\sqrt{1+3\lambda^2-4\left(\Gamma\left(\frac{1}{2}\right)^{-1}\Gamma\left(\frac{3}{2}\right)^{-1/2}\right)^2\lambda^2}\right)^{-1}\right] \\ -\left[2\lambda\left(\Gamma\left(\frac{1}{2}\right)^{-1/2}\Gamma\left(\frac{3}{2}\right)^{-1/2}\right)\left(\sqrt{1+3\lambda^2-4\left(\Gamma\left(\frac{1}{2}\right)^{-1}\Gamma\left(\frac{3}{2}\right)^{-1/2}\right)^2\lambda^2}\right)^{-1}\right]^3 \end{array}\right\},$$

So

$$\left\{\begin{array}{l} 8\lambda\left(1+\lambda^2\right)\frac{\sqrt{2}}{\sqrt{\pi}}\left[1+3\lambda^2-\frac{8\lambda^2}{\pi}\right]^{-3/2} - \\ 6\lambda\frac{\sqrt{2}}{\sqrt{\pi}}\left[1+3\lambda^2-\frac{8\lambda^2}{\pi}\right]^{-1/2} - \\ 8\lambda^3\frac{2}{\pi}\frac{\sqrt{2}}{\sqrt{\pi}}\left(1+3\lambda^2-\frac{8\lambda^2}{\pi}\right)^{-3/2} \end{array}\right\} - \frac{\frac{\sum_{t=1}^{n}Y_t^3}{n} - \mu^3 - 3\sigma^2\mu}{\left(\sigma^2\right)^{\frac{3}{2}}} = 0. \quad \text{(A.2.7)}$$

Estimator for λ can be found by completing equation (A.2.7) through the use of Taylor Series approach. Founding $\hat{\lambda}$ is analog with searching of root from equation (A.2.7). This process is done by Scientific Work Place (SWP) with the following stages:

1. For purposes of simplification, the first term on the left side of equation (A.2.7) simplified by utilizing the simplify command with the following results

$$2\sqrt{2}\frac{\lambda}{\left(\pi+3\pi\lambda^2-8\lambda^2\right)^{\frac{3}{2}}}\left(\pi-5\pi\lambda^2+16\lambda^2\right)$$

2. Notice the second term on the left side of equation (A.2.7). Based on previous results, the term that value is determined by equation (A.2.7). Thus, equation (A.2.7) can be written as

$$2\sqrt{2}\frac{\lambda}{\left(\pi+3\pi\lambda^2-8\lambda^2\right)^{\frac{3}{2}}}\left(\pi-5\pi\lambda^2+16\lambda^2\right)-\widehat{SK}$$

3. Then, the equation obtained in stage 2 above was approached by third-order of Taylor Series through Power Series command.
4. By cutting the error term in the Taylor Series generated in stage 3, then using Polynomial Roots command, we will find some roots of it. At this stage will produced three types of roots that will determine the value of $\hat{\lambda}$.

   In this case will be chosen real-valued root.
5. Furthermore, since at the fourth stage is still obtained real root whose form is still quite complicated, the roots obtained were approached by a second-order Taylor Series.

   Through some stages above, we obtained estimator for parameter λ as follows

$\hat{\lambda}=\left(0.626657068656\right)\widehat{SK}$.

□ *End of Appendix 1.*

3.3 SN Approach for Call European Option Pricing

Call option pricing using SN approach is as follows:

$$C_0=e^{-r\tau}S_0\int_{-d_2}^{\infty}e^{\left(r-0.5\sigma^2\right)\tau+\sigma\sqrt{\tau}z}f\left(z;k=2,\lambda\right)dz-e^{-r\tau}K\,p$$

$$=e^{-0.5\sigma^2\tau}S_0\int_{-d_2}^{\infty}e^{\sigma\sqrt{\tau}z}f\left(z;k=2,\lambda\right)dz-e^{-r\tau}Kp.\quad(3.8)$$

For purposes of simplification calculation of SN, conducted reparameterization equation (3.7)becomes

$$f\left(z;k=2,\lambda\right)=\begin{cases}B\exp\left(-\frac{1}{2}\left(b_1\left(z+m\right)\right)^2\right), & z\le -m\\ B\exp\left(-\frac{1}{2}\left(b_2\left(z+m\right)\right)^2\right), & z>-m\end{cases}\quad(3.9)$$

where $B=\frac{S(\lambda)}{\sqrt{2\pi}},b_1=\frac{S(\lambda)}{(1-\lambda)},b_2=\frac{S(\lambda)}{(1+\lambda)}$, ,

$S\left(\lambda\right)=\sqrt{1+\left(3-4D^2\right)\lambda^2}$, $\quad D=\sqrt{\frac{2}{\pi}}$ and $m=2\lambda DS\left(\lambda\right)^{-1}$.

Calculation of *p*

This section describes about result from the first stage of theexplicit formula calculation process of the call european option pricing by SN approach.

This stage considered the two cases as follows:

1. for $d_2-m<0$,

   $F\left(-d_2\right)=1-\left(1+\lambda\right)N\left[b_2\left(d_2-m\right)\right]$ so

   $p=\left(1+\lambda\right)N\left[b_2\left(d_2-m\right)\right]$.
2. for $d_2-m>0$,

   $F(-d_2)=\left(1-\lambda\right)-\left(1-\lambda\right)N\left(b_1\left(d_2-m\right)\right)$ so

   $p=\lambda+(1-\lambda)N\left[b_1\left(d_2-m\right)\right]$.

Calculation of *q*

This section describes about result from the second stage of theexplicit formula calculation process of the call european option pricing by SN approach.

Analogue with the first stage, this stage is also looking at two cases as follows:

1. for $d_2 - m < 0$,

$$q = S_0 J(b_2) N\left[b_2(d_2 - m) + (b_2)^{-1} \sigma\sqrt{\tau}\right]$$

where $J(b_2) = (1+\lambda) e^{-m\sigma\sqrt{\tau} - 0.5\left(1-(b_2)^{-2}\right)\sigma^2\tau}$.

2. for $d_2 - m > 0$,

$$q = S_0 J(b_1) \left\{ \begin{array}{l} N\left[\left(b_1(d_2 - m) + (b_1)^{-1}\sigma\sqrt{\tau}\right)\right] - \\ N\left[(b_1)^{-1}\sigma\sqrt{\tau}\right] \end{array} \right\} + S_0 J(b_2) N\left[(b_2)^{-1}\sigma\sqrt{\tau}\right]$$

where

$J(b_1) = (1-\lambda) e^{-m\sigma\sqrt{\tau} - 0.5\left(1-(b_1)^{-2}\right)\sigma^2\tau}$ and

$J(b_2) = (1+\lambda) e^{-m\sigma\sqrt{\tau} - 0.5\left(1-(b_2)^{-2}\right)\sigma^2\tau}$.

The details of *p* and *q* calculation of can be found in appendix 2, below.

Appendix 2-Calculation of *p* and *q*

Calculation of *p*

1. For $d_2 - m < 0$,

$$F(-d_2) = \int_{-\infty}^{-m} f(z; k=2; \lambda)\, dz + \int_{-m}^{-d_2} f(z; k=2; \lambda) dz$$

$$= \int_{-\infty}^{-m} B e^{-\frac{1}{2}(b_1(z+m))^2} dz + \int_{-m}^{-d_2} B e^{-\frac{1}{2}(b_2(z+m))^2} dz$$

$$= 1 - (1+\lambda) N\left[b_2(d_2 - m)\right].$$

So, we can conclude that

$$p = 1 - F(-d_2) = 1 - \left[1 - (1+\lambda) N\left[b_2(d_2 - m)\right]\right] = (1+\lambda) N\left[b_2(d_2 - m)\right].$$

2. For $d_2 - m > 0$

$$F(-d_2) = \int_{-\infty}^{-d_2} f(z; k=2, \lambda)\, dz$$

$$= \int_{-\infty}^{-d_2} B e^{-\frac{1}{2}(b_1(z+m))^2} dz$$

$$= \int_{-\infty}^{-d_2} B e^{-\frac{1}{2}(b_1(z+m))^2} dz$$

$$= \frac{B}{b_1}\sqrt{2\pi} \int_{-\infty}^{-b_1(d_2 - m)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\, dx$$

$$= \frac{B}{b_1}\sqrt{2\pi} N\left(-b_1(d_2 - m)\right)$$

$$=(1-\lambda)-(1-\lambda)N\left(b_1\left(d_2-m\right)\right).$$

So, we get

$$p = 1 - F(-d_2)$$
$$= \lambda + (1-\lambda)N\left[b_1\left(d_2 - m\right)\right].$$

Calculation of *q*

1. For $d_2 - m < 0$,

$$q = S_0 e^{-0.5\sigma^2\tau}\int_{-d_2}^{\infty} e^{\sigma\sqrt{\tau}\,z} B\exp\left[-\frac{1}{2}\left(b_2\left(z+m\right)\right)^2\right]dz$$

$$= S_0 e^{-0.5\left(1-b_2^{-2}\right)\sigma^2\tau - m\sigma\sqrt{\tau}}\int_{-d_2}^{\infty} B\exp\left[-\frac{1}{2}\left[\left(b_2\left(z+m\right)\right)-\left(b_2^{-1}\sigma\sqrt{\tau}\right)\right]^2\right]dz$$

$$= S_0 e^{-m\sigma\sqrt{\tau}-0.5\left(1-b_2^{-2}\right)\sigma^2\tau}\ \left(1+\lambda\right)N\left(b_2(d_2-m)+b_2^{-1}\sigma\sqrt{\tau}\right)$$

$$= S_0 J\left(b_2\right)N\left[b_2(d_2-m)+\left(b_2\right)^{-1}\sigma\sqrt{\tau}\right]$$

where $J\left(b_2\right)=\left(1+\lambda\right)e^{-m\sigma\sqrt{\tau}-0.5\left(1-\left(b_2\right)^{-2}\right)\sigma^2\tau}$.

2. For $d_2 - m > 0$

$$q = S_0 e^{-0.5\sigma^2\tau}\int_{-d_2}^{\infty} e^{\sigma\sqrt{\tau}z} f\left(z; k=2, \lambda\right)dz$$

$$= S_0 e^{-0.5\sigma^2\tau}\left[\int_{-d_2}^{-m} e^{\sigma\sqrt{\tau}z} B e^{-\frac{1}{2}\left(b_1\left(z+m\right)\right)^2}dz + \int_{-m}^{\infty} e^{\sigma\sqrt{\tau}z} B e^{-\frac{1}{2}\left(b_2\left(z+m\right)\right)^2}dz\right]$$

$$= S_0 J\left(b_1\right)\left\{N\left[\left(b_1\left(d_2-m\right)+\left(b_1\right)^{-1}\sigma\sqrt{\tau}\right)\right]-N\left[\left(b_1\right)^{-1}\sigma\sqrt{\tau}\right]\right\}$$
$$+ S_0 J\left(b_2\right)N\left[\left(b_2\right)^{-1}\sigma\sqrt{\tau}\right]$$

where

$J\left(b_1\right)=\left(1-\lambda\right)e^{-m\sigma\sqrt{\tau}-0.5\left(1-\left(b_1\right)^{-2}\right)\sigma^2\tau}$ and $J\left(b_2\right)=\left(1+\lambda\right)e^{-m\sigma\sqrt{\tau}-0.5\left(1-\left(b_2\right)^{-2}\right)\sigma^2\tau}$.

□ *end of appendix 2*

Call Option Price with SN Approach

Based on the result of p and q calculation, call option price with SN approach is

$$C_0 = \begin{cases} S_0 J(b_2) N\left[b_2(d_2-m)+(b_2)^{-1}\sigma\sqrt{\tau}\right] - e^{-r\tau} K\left\{(1+\lambda) N\left[b_2(d_2-m)\right]\right\}, d_2-m<0 & (3.10) \\ S_0 J(b_1)\left\{N\left[\left(b_1(d_2-m)+(b_1)^{-1}\sigma\sqrt{\tau}\right)\right] - N\left[(b_1)^{-1}\sigma\sqrt{\tau}\right]\right\} + S_0 J(b_2) N\left[(b_2)^{-1}\sigma\sqrt{\tau}\right] - \\ e^{-r\tau} K\left\{\lambda + (1-\lambda) N\left[b_1(d_2-m)\right]\right\}, d_2-m>0 & (3.11) \end{cases}$$

3.4 Performance of SN Approach of Return Assets and Black-Scholes on European Call Option Pricing

This section describes the application of european call option pricing with SN approach and Black-Scholes model for return assets.

Based on the discussion in previous section,the calculation process of european call option pricing with SN approachand Black-Scholes model needed some informations that the stock price at the beginning of the contract (S_0), strike price (K), risk free interest rates (r), maturity time(τ)and volatility(σ).Especially for the SN model is required to estimate the value of skewness parameter $\hat{\lambda}$.

The details of the option price obtained by using SN approach and Black-Scholes modelfor log return assets of Dominion Resources, Inc. (D), Tyco International Ltd. (TYC), Chesapeake Energy Corporation (CHK), Home Depot, Inc. (HD), Johnson & Johnson (JNJ), Loews Corporation (L), Qwest Communications International Inc. (Q), Time Warner Inc (TWX), Weatherford International, Ltd (WFT), dan Yahoo! Inc. (YHOO) are compared to option prices in the marketpresented in appendix 3, below.

Appendix 3

Table Comparison of 10 call option prices of stocks with r = 0,25% and maturity timeMarch 19, 2010

No	Stocks	S_0	K	Price of Market Option (X)	BS Price (X_B)	SN Price (X_{SN})	Error of BS $(X-X_B)^2$	Error of SN $(X-X_{SN})^2$
1	D	39,11	35	4	41,152	41,151	0,013271	0,013248
2	TYC	37,51	31	4,4	65,143	62,138	44,702,645	32,898,704
3	HD	31,8	34	0,02	0,063258	0,048302	0,0018713	0,000801
4	L	37,5	30	5,3	75,042	66,834	48,584,976	19,137,956
5	Q	4,66	5	0,03	0,036271	0,034518	3,93E-05	2,04E-05
6	TWX	30,54	33	0,05	0,082985	0,051699	0,001088	2,89E-06
7	WFT	17,38	19	0,11	0,14228	0,11163	0,001042	2,66E-06
8	CHK	26,31	17.5	9,37	8,81E+00	8,90E+00	0,3109178	0,2179956
9	M	20,45	12	4,8	8,45E+00	7,91E+00	13,334,913	96,814,323

No	Stocks	S_0	K	Price of Market Option (X)	BS Price (X_B)	SN Price (X_{SN})	Error of BS $(X-X_B)^2$	Error of SN $(X-X_{SN})^2$
10	JNJ	64,04	65	0,22	0,3206	0,27984	0,0101204	0,0035808
						SSE	23	151,207
						MSE	23,002	15,121

Notes:
X_B= Option price of Black Scholes
X_{SN} = Option price of SN Approach
 end of appendix 3

From these results, by taking 10 exercise prices (K) for $\tau = 14$ days, it seems that the price of the option with SN approach gives results that are relatively closer to the option price in the market when compared
to the option price obtained by the approach of the Black-Scholes model. In addition, the SNapproachmodel also gives the value of mean of square error (MSE) is relatively lower than the Black-Scholes model.

Based on the results obtained, SN approach was better able to explain the existence of skewness in the logreturn data than the Black-Scholes model.

As noted previously, the Black-Scholes model is less able to capture the skewness of logreturn data. This is caused by the distribution used in the model does not have skewness parameter $\hat{\lambda}$ as in the SN which is able to capture the skewness in the data.

4. CONCLUDING REMARKS

Based on result of writing that has been stated previously, we can conclude that Skewed Normal Distribution (SN) is a special case of Skewed Generalized Error Distribution for parameter value *k =2.*

Empirical results show that option pricing with SN approach results option price which is closer to the market price when compared to the option price obtained by Black-Scholes model.This caused by the distribution is used in the model does not have skewness parameter $\hat{\lambda}$ such as in SN which is able to capture skewness in the data.

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