



Analysis of Elementary School Students' Difficulties in Fraction Addition

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Abstract

This study aims to analyze the fraction addition difficulties of fourth-grade students at SDN Candinegoro No. 484. This study utilizes a qualitative approach and case study method. This study's data collection methods included tests and interviews. Data analysis consists of data reduction, data presentation, and data verification. The results of the analysis indicate that students have difficulty learning fraction addition. Students' difficulties with applying concepts, applying principles, and solving verbal problems. Through this study, it is anticipated that teachers will be able to identify the types of difficulties encountered by students when learning fraction addition. Hence, teachers can overcome the errors made by students when working on fraction addition problems. Moreover, teachers can use these guidelines when creating lesson plans to anticipate that there will be no fraction-related difficulties.

Abstrak

Penelitian ini bertujuan untuk menganalisis kesulitan penjumlahan pecahan siswa kelas IV SDN Candinegoro No. 484. Penelitian ini menggunakan pendekatan kualitatif dan metode studi kasus. Metode pengumpulan data penelitian ini meliputi tes dan wawancara. Analisis data terdiri dari reduksi data, penyajian data, dan verifikasi data. Hasil analisis menunjukkan bahwa siswa mengalami kesulitan dalam mempelajari penjumlahan pecahan. Kesulitan yang dialami siswa pada saat menggunakan konsep, prinsip, dan menyelesaikan masalah verbal. Melalui penelitian ini diharapkan guru dapat mengidentifikasi jenis-jenis kesulitan yang dihadapi siswa ketika mempelajari penjumlahan pecahan. Dengan demikian, guru dapat mengatasi kesalahan yang dilakukan siswa saat mengerjakan soal penjumlahan pecahan. Selain itu, guru dapat menggunakan pedoman tersebut saat membuat rencana pelaksanaan pembelajaran untuk mengantisipasi agar tidak ada kesulitan terkait pecahan.

Keywords: Difficulty; The Addition of Fractions; Elementary School

INTRODUCTION

Fractions are important in elementary school students learning mathematics and everyday life (Braithwaite et al., 2017; Siegler et al., 2011). The importance of fractions in elementary school mathematics education is a prerequisite for students' success with subsequent material, including algebra (Fennell & Karp, 2016; Flores et al., 2020; Karamarkovich & Rutherford, 2019). Meanwhile, fractions in everyday life are often used to divide food into equal sizes or understand the quantity of packages that are divided into multiple parts (Muharram et al., 2019). Fractions are divided into three concepts: part to whole, division, and comparison (Bennett et al., 2012). During elementary school, part to whole is the most frequently used concept in learning fractions (Purnomo et al., 2017; Rahmawati et al., 2020).

According to the findings of previous studies, elementary school students have difficulty applying the concept of fractions. Students in elementary school have difficulty representing fractions using the part-to-whole relationship. Students with limited fraction concepts will mostly have difficulty in advanced fraction concepts (Simon et al., 2018). The difficulty of using the concept of fractions impacts the low learning outcomes of fractions (Istiqomah et al., 2018; Suarjana et al., 2018)

The fractions addition is typically difficult for fourth-grade elementary school students. According to a previous study, students have difficulty solving problems involving fraction addition operations, such as calculating (Mukminah et al., 2021). Students' difficulties in solving the addition of fractions are caused by difficulties in understanding questions, concepts, formulas, and symbol notation

(Rahmawati et al., 2021). Thus, the addition of fractions is different from other fractional operations. The addition of fractions plays an essential role in solving fraction problems. The addition operation is basic or the basis for solving other operations.

Based on a preliminary study on November 17, 2021, with 19 students in grade fourth at SDN Candinegoro No. 484 that: (1) Students cannot represent a fraction image to a fraction value; (2) Students cannot distinguish between denominator and numerator; (3) Students do not understand the meaning of the image's numerator and denominator; (4) Students consider fractional material to be a difficult subject. Many students do not like math (Ricks, 2009), because this lesson is considered difficult to understand by students and considers mathematics to be a scary subject (Dewi et al., 2020; Forgues et al., 2015; Tian & Siegler, 2016; Trivena et al., 2017).

Based on Cooney's theory, students' learning difficulties are grouped into three types: Students' difficulties in utilizing concepts, applying principles, and solving verbal problems are examined (Cooney et al., 1975). The importance of understanding concepts in fractional material and the existence of learning difficulties students face is necessary to assess students' learning difficulties in adding fractions (Hansen et al., 2017).

According to previous studies, many elementary school students continue to struggle with fraction addition problems. Therefore, a deeper analysis of students' difficulties with fraction addition is required. Several analyses of the difficulties elementary school students face when solving fraction addition problems have been conducted (Izsak et al., 2008; Kara & Incikabi, 2018; Putri et al., 2018). However, this study does not analyze learning

difficulties based on Cooney's theory (Understanding concepts, applying principles and solving verbal problems). The importance of understanding concepts, principles, and verbal problems needs to be studied about learning difficulties, especially in the addition of fractions material, to overcome errors made by students in solving fraction addition problems (Abadi & Amir, 2022; Waluyo & Nuraini, 2021).

Thus, this study implies that it is hoped that teachers will be able to identify the types of difficulties students encounter when adding fractions. So that teachers can overcome the difficulties encountered by students when working on fractional addition problems and so that they can use these strategies as guidelines when developing lesson plans. Learning plans must be made according to the characteristics of students (Fauzi & Arisetyawan, 2020), so the teacher must be able to understand the flow of students' thinking so that the learning plan follows the real conditions experienced by students so that learning gets maximum results.

METHOD

This study employs a qualitative approach to analyze and understand the learning difficulties experienced by students (Palinkas et al., 2015). This study uses a case study approach. It is due to the in-depth understanding of individuals, groups, institutions, or backgrounds (Nugrahani, 2014). This method explores the difficulties of learning to add fractions to elementary school students.

The credibility of this research data using triangulation (Colorafi & Evans, 2016). Triangulation is a method for verifying data from multiple sources in multiple ways and at multiple times (Hardani et al., 2020). The type of triangulation used

is technical triangulation with interviews and written tests.

During the COVID-19 pandemic, schools implemented a limited face-to-face learning system, in which 38 students were divided into two classes in one class. There were 19 students who only three times a week face-to-face learning at school in one session. So that the participants in this study were fourth-grade students at SDN Candinegoro No. 484, totaling 19 students.

Written tests and interviews served as the research tools for this study. The data collection techniques used are: (1) Test, this test aims to find out the learning difficulties experienced by students related to the addition of fractions. This written test is in the form of a visual representation of fractions and students are asked to change to a symbolic representation of fractional values, then add up the values; (2) Interviews, in this study, students were given investigation to assess their thought processes in working on the written test questions, besides that interviews were also given to homeroom teachers who teach mathematics learning to find out how teachers teach fraction addition material.

The indicators used in this study refer to Cooney's theory, namely understanding concepts, applying principles and solving verbal problems (Cooney et al., 1975), see Table 1. The following is an analysis of the data used in this study: (1) Data reduction, the researcher recorded the responses of teachers and students in responding to questions during interviews and to questions pertaining to fraction addition; (2) Display data, clarifying and identifying students' answers in answering the question of adding fractions based on their difficulty; (3) Verification, analyzing in detail the types of student difficulties in answering questions.

Table 1. Difficulty indicator in adding fractions adapted from Cooney theory

Difficulty → Descriptor	Indicator of Difficulty in Adding Fractions	Code
Concept (C): Related to understanding and distinguishing words, symbols and signs.	Can not tell the difference between the numerator and denominator; writes the fraction value in reverse.	C1
	Ignores the fraction symbol ($\frac{a}{b}$), cannot write the shaded part.	C2
	Expresses the value of fractions correctly, but cannot use the fraction addition formula, namely $(\frac{a}{b})+(\frac{c}{b}) = (\frac{d}{b})$.	C3
Principle (P): Related to interpreting the form of the questions that have been presented.	Not careful in interpreting images and adding fractions.	P1
	Can understand the principles related to the addition of fractions but cannot solve the problems contained in the problem.	P2
Verbal (V): Related to understanding various special terms.	Can represent images of fractions in the form of fractional values, but cannot present images of the addition of fractions.	V1
	Can present data in a fractional model.	V2

RESULTS AND DISCUSSION

Based on Cooney's theory, students' learning difficulties are grouped into three types, namely: (1) Students' learning difficulties in understanding concepts; (2) Students' difficulties in applying fundamental principles; (3) Students' learning difficulties in verbal problem-solving. Each type of learning difficulty above is divided into several indicators, namely: (1) Concept, unable to distinguish between numerator and denominator, write down fraction values in reverse (Code C1); ignoring the fraction symbol, namely ($\frac{a}{b}$), cannot write the shaded part (Code C2); state the value of fractions correctly, but cannot use the formula for adding fractions, namely $(\frac{a}{b})+(\frac{c}{b}) = (\frac{d}{b})$ (Code C3); (2) Principle, not careful in interpreting images and adding fractions (Code P1); can understand the principles related to the addition of fractions, but cannot solve the problems contained in the problem (Code P2); and (3) Verbal, students have the ability to represent fractional images as fractional values, but are unable to present images of the addition of fractions (Code V1); students are not able to present data in a fractional model (Code V2). An overview of the learning difficulties experienced by stu-

dents in solving fraction addition problems is presented in Table 2.

Table 2. An overview of the types of student difficulties

Student's name	Difficulties Type		
	Concept (C)	Principle (P)	Verbal (V)
A	C1		
B	C1, C2	P1, P2	V1
C	C1, C2, C3	P1, P2	V2
D	C2, C3	P1, P2	
E	C3	P1	V1
F	C3	P1, P2	
G	C2		V1, V2
H	C1, C2, C3	P2	V1, V2
I		P2	
J			V1
K	C3		V2
L	C1		
M	C2	P1	
N		P2	V2
O	C1		V1
P	C3		V2
Q	C2		V2
R		P1	V1
S	C2		

Table 2 shows that the learning difficulties experienced by students in solving the addition of fractions are different. Based on Table 2. It can be said that there are still many students who experience difficulties with academics based on Cooney's theory. Table 3 provides a sum-

mary of student difficulties in solving addition problems involving fractions.

Table. 3 Recapitulation of types of student difficulties

Difficulties Type		n	%
Concept (C)	C1	6	31,57
	C2	8	42,10
	C3	7	36,84
Principle (P)	P1	7	36,84
	P2	7	36,84
Verbal (V)	V1	7	36,84
	V2	7	36,84

Based on Table 3. It indicates that 42.10% students in code C2 experience learning difficulties, while 31.57% students in code C1 experience learning difficulties. To analyze in detail and in-depth the students' answers based on their respective types of learning difficulties, given the question "State the shaded area in the image below in the form of a fraction, then operate it and draw the result of the fraction operation", see Figure 1.

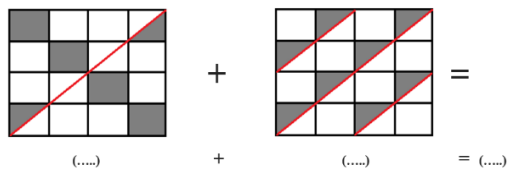


Figure 1. The problem of adding fractions

Students' difficulties in using concepts

Students' difficulties with applying concepts stem from their inability to understand and differentiate words, symbols, and signs. In this study, student B could not distinguish between the quantifier and denominator and wrote the fraction value in reverse. It is motivated by the inability of students to remember names technically. A total of 6 students (31.57%) experienced conceptual difficulties on the C1 indicator, as did student B in Figure 2.

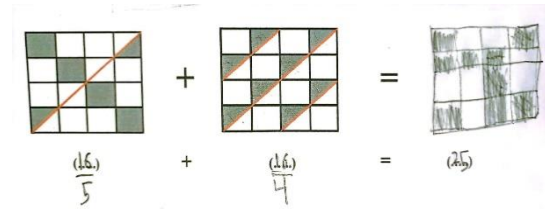


Figure 2. Student's answer B

Based on the interviews with student B, it was determined that students were incapable of differentiating between the numerator and denominator, writing fractional values backward.

- Researcher : Are you familiar with the numerator and denominator?
- Student B : A little.
- Researcher : In the fraction problem, try to mention which part above is the numerator or denominator?
- Student B : The top represents the numerator, while the bottom represents the denominator.
- Researcher : Are you sure?
- Student B : Yes sure.
- Researcher : If from the image, try to show the numerator and denominator.
- Student B : The numerators are all, if the denominators are black only.

The problem presented in Figure 2 is two images of fractions with unequal parts. Student B can understand the problem and answer correctly that by combining 2 triangular parts into 1 whole square part, so that the results in the first black image are 5 and a total of 16 and in the second image the black part is total 4 and the total number of sections is 16. However, student B has difficulty in understanding and distinguishing parts of the numerator and denominator so that they include the value of the numerator and denominator in reverse, namely $(\frac{16}{5})$ on the first image and $(\frac{16}{4})$ in the second image, the correct answer should be $(\frac{5}{16}) + (\frac{4}{16})$.

Students can understand the concept of fractions simply, that there is a numerator and a denominator in a fraction.

A fraction is a number that can be expressed by $(\frac{a}{b})$, where a is the numerator and b is the denominator, both of which are integers, and b is not equal to zero (Kalra et al., 2020).

Students are said to have the ability to understand mathematical concepts if the predetermined indicators are met (Arini et al., 2017). This indicator applies to questions that students must answer to measure their respective abilities. A lack of students' understanding of adding fractions concept will result in difficulties in terms of fractional addition operations (Kusuma & Retnawati, 2019). According to the results of interviews with the fourth-grade homeroom teacher, it was found that not all students understood the denominator and numerator of fractions. Some students had difficulty in adding fractions because each student's ability to understand the material was also different.

The next learning difficulty is that students ignore the fraction symbol, namely $\frac{a}{b}$ and cannot write the shaded part, it can be motivated by the inability of students to state the meaning of the term that represents the fraction concept. A total of 8 students (42.10%) experienced conceptual difficulties on the C2 indicator, as did student C in Figure 3.

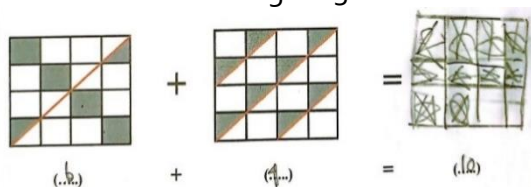


Figure 3. Student's answer C

Based on student C's interviews, it was determined that these students did not understand the material for adding fractions, so students could not pay attention to the fraction symbols and could not write the shaded parts correctly.

Researcher : How can you answer the number 6 in the first image?

Student C : Count the black ones.

Researcher : Some parts are black that are not the same, is that okay?

Student C : I don't know.

Researcher : Then why is it not given a denominator?

Student C : I forgot.

In Figure 3, student C tends to focus on addition operations but cannot correctly state the value of fractions. Students only answer with whole numbers and are not given a denominator (Aksoy & Yazlik, 2017). In the first image, students answer 6, which means students count all the parts shaded in the first image and in the second image, students answer 4, which means students combine two triangular parts into 1 square part, students' thinking patterns are inconsistent in answering the questions that were posed. Additionally, students should include the denominator because the fraction consists of a numerator and a denominator, so the correct answer is $(\frac{5}{16}) + (\frac{4}{16}) = (\frac{9}{16})$.

Furthermore, students are not able to state the value of fractions correctly, but cannot use the fraction addition formula, namely $(\frac{a}{b}) + (\frac{c}{b}) = (\frac{d}{b})$, because this is motivated by the inability of students to remember a sufficient condition of an object to be expressed in terms that represent the fraction concept. A total of 7 students (36.84%) experienced conceptual difficulties on the C3 indicator, as did student D in Figure 4.

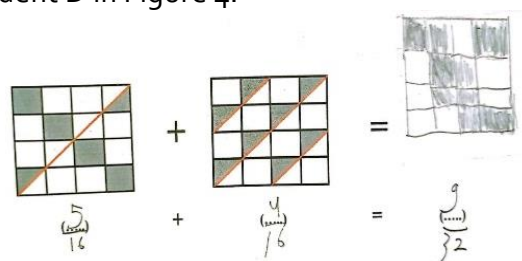


Figure 4 Student's answer D

According to the results of interviews with the student, student D did not understand the formula for adding fractions. So, students could not answer the question of adding fractions correctly.

- Researcher : Are you sure your answer is correct?
 Student D : Yes.
 Researcher : $(\frac{9}{32})$ Where did it come from?
 Student D : 9 is from $5 + 4$, if 32 is from $16 + 16$.
 Researcher : Do you know the formula for adding fractions?
 Student D : I don't know, I just summed it up because there is a (+) sign in the question.

In Figure 4. student D can state the value of fractions correctly, but students cannot use the formula for adding fractions correctly. Student D adds up the denominator with the denominator, which is $16 + 16$. The result is 32 and adds the numerator with the numerator, which is $5 + 4$. The result is 9. It is supported by the previous study, namely many students who have difficulty using the formula (Mulyana et al., 2019). In addition, many students add the numerator and denominator directly (Maelasari & Jupri, 2017). In fact, for the material for adding fractions with the same denominator, only the numerator and denominator are added together, so the correct answer is $(\frac{5}{16}) + (\frac{4}{16}) = (\frac{9}{16})$.

Students' difficulties in using the principle

The difficulty for students in applying the principle stems from their inability to interpret the question's format. In this study, student E was not careful in interpreting images and adding fractions. It is motivated by the inability of students to carry out discovery activities about something because they are not cautious in carrying out calculations or arithmetic operations (Safriani et al., 2019). A total of 7 students (36.84%) had difficulty in principle on the P1 indicator, as did student E in Figure 5.

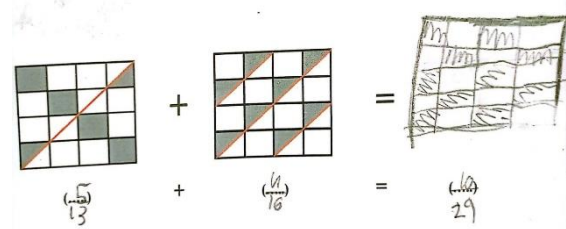


Figure 5. Student's answer E

Based on student E.'s interview results, we can conclude that It was discovered that students had understood addition, but were not careful in counting.

Researcher : Where did you get those 10 answers from?

Student E : Added up, 5 and 4.

Researcher : Is it true that 5 plus 4 is the answer 10.

Student E : Ehh, Yes 9.

Student E can understand addition operations, but students tend to be less thorough in adding them up in Figure 5. Students are wrong in adding up the numerator part $5 + 4 = 10$ while the correct answer is $5 + 4 = 9$. In the denominator section student E has answered with correct, namely $13 + 16 = 29$. It shows that student E is not incapable of calculating fraction addition operations, but students are less careful in carrying out calculations to produce wrong answers. Similar to this finding, grade-fourth elementary school students are often not careful in calculating numbers (Pratiwi et al., 2020).

Learning mathematics will not be separated from the ability to count (Chan & Scalise, 2022). An important aspect that students in the use of principles must master is numeracy. Although learning to count has been introduced to students from an early age, even before school, many students still cannot count, and most of the students have difficulty calculating (Pramesti & Prasetya, 2021). Many students have difficulty operating fractions, especially those with the concept of integers (Fuchs et al., 2016).

According to the results of the interview with the homeroom teacher for the

fourth-grade, it was found that the teaching method of adding fractions was done using the lecture method and did not use any media. So during learning, students tended to be passive and unable to explore their knowledge, the concept being taught. Namely, fractions came from integers which were broken down to produce the fractional part. If you add fractions with the same denominator, only the numerators are added. Still, if the denominators are not the same, you must first equalize the denominators, then add the numerators. Patterned learning like this requires students' ability to memorize principles. This causes students' difficulties in adding fractions when students encounter questions that are different from before or require completion based on conceptual understanding (Ainia & Amir, 2021; Hansen et al., 2017).

Students' difficulty in using the next principle is that students can understand the principles related to the addition of fractions, but is unable to solve the problems in the questions. It can be motivated by the inability of students to determine the relevant factors contained in the fractional image. A total of 7 students (36.84%) had difficulty in principle on the P2 indicator, as did student F in Figure 6.

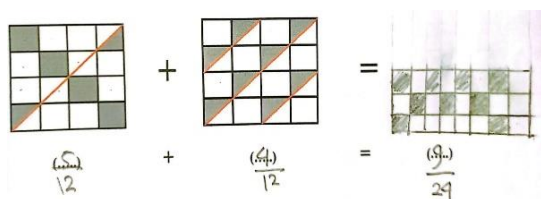


Figure 6. Student's answer F

In Figure 6, student F is able to understand the principle of adding fractions that in addition to fractions with the same denominator, the part that is added is only the numerator. While student F can add up the numerator part correctly, namely $5 + 4 = 9$, but answer incorrectly in the denominator, namely adding $12 + 12 =$

24 so that it becomes $(\frac{9}{24})$, while the correct answer is $(\frac{9}{12})$. So, student F can be said to be unable to apply the principle of adding fractions correctly.

Students' difficulty in performing fractional arithmetic operations can be caused by the imperfect learning process in the classroom (Suarjana et al., 2018). Difficulties arise when students perform arithmetic operations because students have very little understanding of the concepts of arithmetic operations (Hunt et al., 2016).

Students' difficulties in solving verbal problems

Students' difficulties in solving verbal problems are related to understanding various special terms. A total of 7 students (36.84%) experienced verbal difficulties on indicator V1, as did student G, see Figure 7.

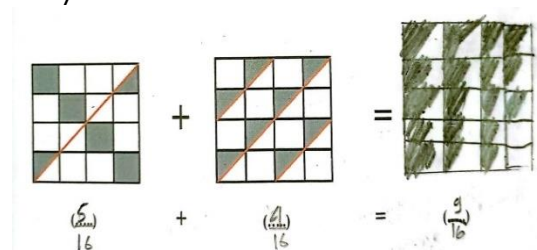


Figure 7. Student's answer G

Based on student G's interview results, it is known that students are less cautious when representing fractional values into fractional images.

Researcher : Where did the answer 9 come from?

Student G : 5 plus 4.

Researcher : That's right. Take a look at the image. Does it already show 9 sections?

Student G : Yes.

Researcher : Why are there 18 shaded ones?

Student G : Yes, I joined together. 2 triangles means 1 quadrilateral.

Researcher : How come there is no dividing line?

Student G : Forgot.

Researcher : In the fraction image, which denominator is it?

- Student G* : The total number of boxes.
Researcher : The denominator is 16, why are there 20 parts drawn in total?
Student G : I'm confused.

In this study, student G was able to represent a fraction image in the form of a fractional value by correctly mentioning the numerator and denominator parts, namely $(\frac{5}{16}) + (\frac{4}{16}) = (\frac{9}{16})$, but unable to present the image of the addition of fractions correctly. Representation is the interpretation of student understanding in the form of ideas generated in the mind of a problem that is communicated in physical form, terms, images, text, objects, or certain symbols to make it easier to find solutions to problems (Ulya & Rahayu, 2020). It can be motivated by the inability of students to understand the context of the presented questions.

Based on the answers above, student G wrote the sum result correctly, namely $(\frac{9}{16})$, while it does not match the fractional value in the image. Student G can correctly shade 9 parts as the numerator and the total number of parts is 20 parts as the denominator. While the correct answer is 16 parts. In learning mathematics, students are said to be able to represent mathematics if they can express mathematical ideas, whether in the form of problems, statements, solutions, definitions, and others (Rahmawati et al., 2015)

An essential math skill to master is the ability to solve problems (Khaesarani, 2021). Verbal ability is the ability to analyze language and translate it into another form that is easier to understand (Irawan & Kencanawaty, 2017).

Students have difficulty solving verbal problems because they cannot present data in a fractional model. A total of 7 students (36.84%) experienced verbal difficulties on the V2 indicator, as did student H in Figure 8.

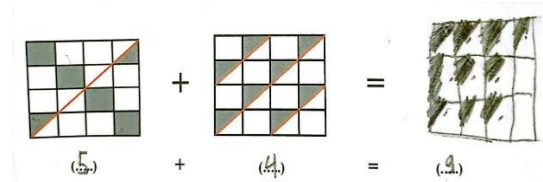


Figure 8. Student's answer H

According to student H's interview results, it is known that students do not understand the question order, so students' answers do not match what was ordered by the questions.

- Researcher* : What is the image in the question?
Student H : Fraction
Researcher : Where is the quantifier and denominator?
Student H : (Silent)
Researcher : Where can you answer 5 and 4 from?
Student H : 5 of the shaded 4, I don't know I saw my friend
Researcher : Does that mean you don't understand this yet?
Student H : Not yet

Students did not understand the question command based on student H's answer in Figure 8. Students did not understand the question command. Namely, students were asked to provide fractional representations of the image's shaded area, but students only answered with the number of shaded areas, namely $5 + 4 = 9$, not in the form of fractions. To learn mathematics, students need numeracy skills and verbal skills (Wahyuddin, 2016).

CONCLUSION

Based on the findings of the data analysis, it can be concluded that students face the following learning difficulties: (1) Students' difficulties in using the concept, namely not being able to distinguish between the numerator and denominator, writing down fractional values in reverse, not paying attention to the fraction symbol, namely $(\frac{a}{b})$, can not write the shaded

part and state the value of the fraction correctly, but cannot use the fraction addition formula, namely $\left(\frac{a}{b}\right) + \left(\frac{c}{b}\right) = \left(\frac{d}{b}\right)$; (2) The difficulty of students in using principles, namely not being careful in interpreting images and adding fractions and being able to understand the principles related to the addition of fractions, but unable to solve the problems contained in the questions; and (3) Students' difficulties in solving verbal problems, namely representing fractional images as fractional values, but are unable to present images of the addition of fractions and students are not able to present data in fractional models.

Based on the findings of this study, the author recommends that teachers must be able to comprehend each student's learning abilities. Learning carried out in schools must be student-centered so that students are more active in learning and the lesson plans used must be by the characteristics of students. Then, further study can also be carried out to analyze the learning difficulties experienced by elementary school students in adding fractions in depth. Furthermore, researchers who conduct similar studies are expected to monitor more closely when collecting data. It aims to reduce the collaboration space between students in the class.

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