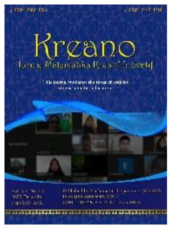
UNNES JOURNALS

Kreano, Jurnal Matematika Kreatif-Inovatif
Volume 14, number 2, 2023, pp. 280-292



Identification of Representation Ability in the Topic of Space Analytical Geometry for Student in Higher Education

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Abstract

The objective of this research is to expose the representation skills of future mathematics educators by delving into the content of Space Analytical Geometry and identifying potential enhancements. The type of research is design research involving 38 Mathematics Department students and FMIPA UNM students. The research instruments used are Lesson Plans and e-tasks. The e-Task is the instrument combining the LMS Syam-OK and Gdrive for collecting the outputs of these activities. The results suggest that 1) students' representation approaches depend on their knowledge level, with broader knowledge leading to diverse presentation information. 2) The complexity of given questions influences the development of students' representation methods. 3) An analytical approach is employed for specific problems requiring visual elements in the solution. The implications call for adjustments by presenting challenging problems that impact the flexibility of methods and the variety of exercise materials supporting the enhancement of representation abilities.

Keywords: Representation Ability; Space Analytical Geometry

Information of Article

Subject classification	*97C30 Cognitive processes, learning theories (aspects of mathematics education)*
Submitted	*20 October 2022*
Review Start	*23 October 2022*
Initial Review Finish	*20 November 2022*
Round 1 Finish	*5 January 2023*
Round 2 Finish	*17 February 2023*
Accepted	*6 June 2023*
Published	*19 November 2023*
Similarity Check	*4%*

Abstrak

Penelitian ini bertujuan untuk mengungkap keterampilan representasi mahasiswa calon guru matematika dengan mendalami materi Geometri Analitis Ruang dan mengidentifikasi peningkatan yang dapat dilakukan. Jenis penelitian ini adalah penelitian desain yang melibatkan 38 mahasiswa Departemen Matematika, FMIPA UNM. Instrumen penelitian yang digunakan adalah Rencana Pelajaran dan e-Task. E-Task merupakan instrumen yang menggabungkan LMS Syam-OK dan Gdrive untuk mengumpulkan hasil kegiatan tersebut. Hasil penelitian menunjukkan bahwa 1) pendekatan representasi mahasiswa bergantung pada tingkat pengetahuan mereka, semakin luas pengetahuan menghasilkan informasi presentasi yang lebih beragam. 2) Kompleksitas pertanyaan memengaruhi perkembangan metode representasi mahasiswa. 3) Pendekatan analitis digunakan untuk masalah tertentu yang memerlukan elemen visual dalam solusinya. Implikasinya menuntut penyesuaian melalui penyajian masalah yang menantang, yang memengaruhi fleksibilitas metode dan ragam materi latihan untuk mendukung peningkatan keterampilan representasi.

INTRODUCTION

Representation ability is an ability that is important not only for students (Kohl & Finkelstein, 2005; Surya et al., 2013) (Nasrullah et al., 2021), especially in learning mathematics, but also this ability is important for students in higher education because they become a central issue in mathematics teaching (Elia & Gagatsis in (Gervasoni, 2006). This ability requires concepts and uses them to communicate the user's understanding of a given problem with mathematical ideas (De Cock, 2012); this is because, unlike other scientific domains, a construct in mathematics is only accessible through its semiotic representation and in addition, one semiotic representation by itself cannot lead to an understanding of the mathematical object it represents (Duval, 2006). In addition to increasing the user's mathematical knowledge, for example, solving arithmetic problems (Elia & Gagatsis in Gervasoni, 2006)), the placement of the proper representation will support mathematical problem solving on the right track where applying these representations requires different strategies (Kohl & Finkelstein, 2005). Not infrequently, students fail to construct the expected problem-solving caused by failure at the representation stage (Surya et al., 2013). Therefore, teachers need to provide opportunities for students to solve mathematical problems as well as mathematical understanding and representation (Minarni et al., 2016); it is even hoped that the student's representation ability should have reached the best level of the specified category.

However, according to (Dewi Sopiany, 2017), students still have low representational abilities to create problem situations based on the data or representations given. Of course, this will impact students' lack of skills in generating ideas, asking questions, and responding to other people's questions or opinions (Widakdo, 2017). Research (Minarni et al., 2016) reveals that students' ability to represent essay questions is still relatively low. In addition, Sari et al. (2018) examined student errors in mathematical representation tests. The results showed that students made mistakes in solving problems involving arithmetic symbols, namely concepts related to characters, and applying other mathematical concepts. Thus, students' mathematical representation skills still require special attention and action to be improved (Saputri & Kamsurya, 2020).

This low ability shows the need for attention because students use representation to support their mathematical understanding by constructing abstract ideas into concrete ideas using logical thinking; representation is something that represents something else (Duval, 2006). Because representation is a sign or configuration of signs, characters, or objects that mark and configure to represent, describe, or represent something other than itself. So that it will support students to learn and communicate, connecting mathematical concepts to solve problems in each project. Representation as one of the standard processes shows that the process of learning mathematics in schools must develop students' representational abilities. Mathematical representation is the ability to represent a mathematical problem through symbols, images, manipulative objects, and other mathematical ideas (Farokhah et al., 2019).

This problem does not need to occur for prospective teacher students because it will facilitate students to learn mathematics later. For this reason, when becoming a teacher, the representation ability has developed very well so that it does not experience difficulties directing students to mobilize these abilities. This avoids what happened to South Sumatra and Bangka Belitung teachers, where 48.4% could correctly represent symbolic representations into graphic representations (Hapizah et al., 2019). Undoubtedly, teachers should teach students in such a way that students can solve mathematical problems as well as mathematical understanding and representation (Minarni et al., 2016). One of the challenges they will face is that students can use symbolic representations to find solutions to problems. They can perform mathematical operations based on known data from the problem. However, this ability is not well developed for all students; most students still struggle to solve problems using symbolic representations correctly. This makes it difficult for students to solve problems involving mathematical expressions or symbolic representations (Farokhah et al., 2019) (Ruslan et al., 2017).

For this reason, the transposition of student to think as teacher candidates need to be trained as suggested by (Utami et al., 2019) (Nasrullah et al., 2017). It is possible that most students experienced the same as those who still have low mathematical representation skills and have difficulty understanding problems and correctly writing equations. Students are not used to solving problems through visual, verbal, and symbolic representations. Therefore, prospective teachers must be trained in students' mathematical representation skills by applying the multiple representation learning model, which uses transpositive work in order to reach they can reach the point that knowledge transposed is itself bettered (Chevallard & Bosch, 2020). Farokhah et al. (2019) in their research, the results showed that the obstacles experienced by students in representing a mathematical problem were triggered by the limited ability to visualize a problem in the form of other mathematical models, the limitations of students in connecting the knowledge they already had with the form of representation. A mathematical problem and limitations in applying mathematical concepts so that they cannot be represented correctly. Another factor that also affects students' representational abilities is student flexibility. Thus, a prospective teacher who will facilitate students in learning activities needs to improve the way in visualizing, connecting, and applying mathematical concepts flexibly so that the process of interpreting and constructing knowledge based on the given problem can develop properly.
Therefore, this study works to explore the extent to which student-teacher candidates progress in developing their representational abilities to solve spatial analytic geometry problems.

METHOD

To carry out this research, the initial activity was to design learning activities that prospective teacher students would participate in. Their activities for this course contain the following materials: 1) Three-Dimensional Space, 2) Point Distances in Three-Dimensional Space, 3) Coordinates of Points in Cartesian, Tube, and Spherical Spaces, and 4) Vectors in Three-Dimensional Space. To observe the progress of students' learning in this study, observations were made not only when they responded to problems given in class but also through tasks that were collected virtually. Students collect assignments through LMS Syam-OK and connect them to GDrive or e-Task (Nasrullah & Baharman, 2018). Through this e-Task, we can easily make virtual observations and facilitate students to collect assignments anytime and anywhere. The research subjects involved were 38 people from students of the mathematics education study program majoring in mathematics, FMIPA UNM. Therefore, this research is supported by a research instrument in the form of a compilation of student assignments stored on the G-Drive.

In the learning activity design process, the learning environment contains three entities: learning objects, learning services, and sub-environments (Koper & Olivier, 2004). In this case, the learning objects are entities used in learning, such as books, websites, and other learning resources. Some learning resources other than the books used, for example, brilliant, topper, cuemath, and others. Then, the learning services used are in the form of services needed for learning, such as learning resource services, communication services, and monitoring services. This learning service tool involves Google Drive and LMS Syam-OK. While in the activity design, there are three components to be considered: the name of the activity, the goal, and the output. Overall, this process is shown in the following diagram.

Figure 1 shows how to provide lecture activities. In lecture activities, several stages are carried out, starting from 1) Preparation, a stage where the activity begins with preparing all lecture needs, including Semester Learning Plans (RPS), Lecture Materials, and Publication Media (for example, e-Task). 2) Input Lecture Materials, the next stage where lecture materials are presented through publication media and prepared to be distributed to students. 3) Learning Interaction: There

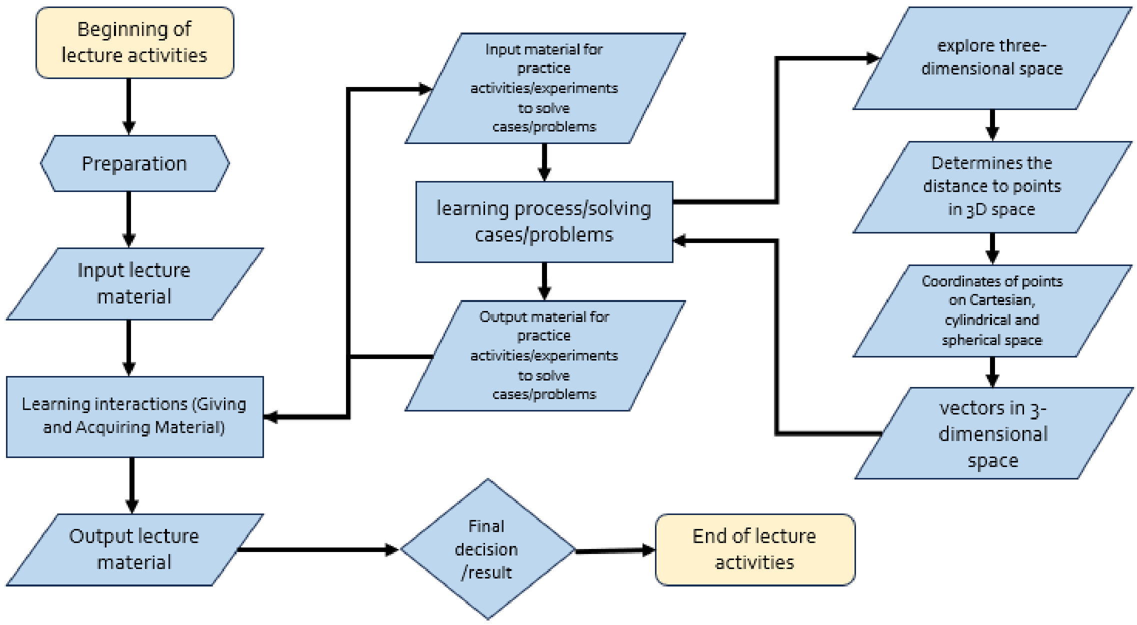


Figure 1. Flowchart of Giving Lecture Activities

are two main processes at this stage: provision and acquisition of material. In this case, the provision of material meant that students were given material prepared by the teacher in various forms, written, presented through an electronic LMS, and others. When material input is carried out, the expected activities are oriented toward practice activities, experiments, and case/problem-solving. Then acquisition can mean that students actively get various materials determined and prepared for lecture activities through the LMS or in the e-Task system. This acquisition activity is directed toward learning or solving cases/ problems. Therefore, the targeted outputs are activities and outcomes related to exercises, experiments, case-solving, or problem-solving. The sub-environment in lecture activities is designed in such a way that it supports the expected learning process. To achieve the expected target, as shown in the picture, four lecture activity topics are placed in the learning process. These four topics are directed at providing learning opportunities for students so that the expected representational abilities and information support the target of this research.

To support the trustworthiness of this research, four criteria of the approach used are credibility, transferability, dependability, and confirmability (Stahl & King, 2020). For the credibility criteria, the consistency of the research findings is shown by comparing the artifacts of analytic geometry space learning with the observations in the classroom. This process takes place for transferability by observing the tendencies for the four given activity contexts. Students' response to the situation is the essence of the transfer process in the activity. In fulfilling the dependability criteria, peer debriefing is one stage to ensure that the entire process is carried out with the correct approach. However, confirmability is not fully passed to ensure that the scope of objective reality is as far as possible. In general, to ensure that the entire process has placed the trustworthiness criteria as a complementary part of the reliability of this research result.

RESULTS AND DISCUSSION

Results

To obtain the expected research results, first, design learning activities that course participants will follow before the lecture begins. The following activities are designed to support research:

Table 1 Learning Activities

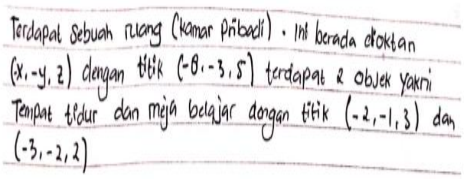
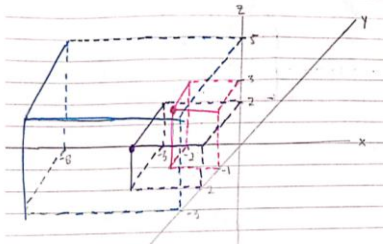
Activity Name	Goal	Output
Explore Three-Dimensional Space	Students can explore the characteristics of three-dimensional space	The ability to represent three-dimensional space and its characteristics.
Determining the Distance of Points in Three-Dimensional Space	Students can determine and predict distances in three-dimensional space	The ability of representation in determining and predicting distances in three-dimensional space
Coordinates of Points in Cartesian, Tube, and Spherical Space	Students can use Cartesian, cylinder, and spherical coordinates to place points.	Representational ability to use Cartesian, cylindrical, and spherical coordinates to place points.
Vectors in Three-Dimensional Space	Students can be able to define and operate vectors in three-dimensional space.	Representational ability to define and operate vectors in three-dimensional space

These activities include lecture activities, which were carried out for eight meetings; the research results are shown in the following figure.

Activity 1: Explore Three-Dimensional Space

In this activity, students are asked to identify objects around them. After that, they use the object as a representation of three-dimensional space. Like what they illustrate, this will be interesting as the development of ideas or bringing objects into the surrounding environment.

Table 2. Construction of Student Answers on the Topic of Exploring Three-Dimensional Space

No	Student Answer Construction	Description
1	You are asked to look for everyday objects that are then used to illustrate three-dimensional space. Show the three-dimensional space on the object!	Problems given to students
2	 Terdapat sebuah ruang (kamar pribadi). Ini berada dioktan (-x, -y, z) dengan titik (-8, -3, 5) terdapat 2 objek yakni Tempat tidur dan meja belajar dengan titik (-2, -1, 3) dan (-3, -2, 2)	
	There is a room (private room). It is in octane (-x, -y, z) with dots (-8, -3, 5). There are two objects, namely the bed and study table, with dots (-2, -1, 3) and (-3, - 2, 2)	Solving a given problem is by selecting the context of a private room. Three objects are illustrated in the room: a bed and a study table. These two objects are represented by two dots (-2, -1, 3) and (-3, -2, 2).
3	 The dot image in the three-dimensional Cartesian space above is shown to place objects in the private room.	

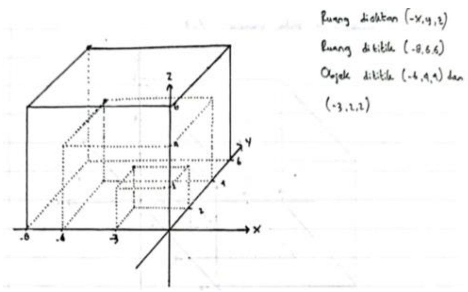
In Table 2, the problem given is looking for objects that exist in everyday life. The object relates to the use of three-dimensional space. A private room is used as an illustrated situation for a three-dimensional space in a student's work. This room represents the concept of a three-dimensional space built by the student. However, the object introduced in this case is only in the form of a point. At least the placement of these objects initiates the use of the concept of space in the study of analytic geometry. Of course, the ability that is built with this activity is exploratory. Not infrequently, this ability is used mainly in situations of recognizing space. With good space recognition, the benefit that can be obtained is the placement of the proper object at the right point so that, in turn, it will impact optimizing the space based on its designation.

From this description, what is shown by the students from their work follows the following stages: 1) Identification of objects related to daily life; in this case, the chosen one is a private room. 2) Connect the situation of the private room with the illustration of the three-dimensional space. 3) Using the concept of three-dimensional space and objects in the private room, the object is represented by a point. 4) Use pictures to visualize what was done in steps 1, 2, and 3.

Activity 2: Determining the Distance of Points in Three-Dimensional Space

Distance is an essential part of the use of space. Determining the distance between two or more objects requires recognizing the object's distance. The distance norm can mathematically calculate the distance between two objects (|d|, d = distance). For three-dimensional space, the object's position is different compared to two-dimensional space. Even so, the position of objects in space is determined by the three dimensions that place the object in its place. For students in Space Analytical Geometry learning, distance learning is not only training to determine the distance norm, but they need to take advantage of space to place objects and calculate the distance between them—object loci in space.

Table 3. Construction of Student Answers Topic Determining Distances to Points in Three-Dimensional Space

No	Student Answer Construction	Description
1	Hitunglah·jarak·kedua·titik·pada·gambar·tugas·sebelumnya¶ Calculate the distance between the two points in the previous task drawing!	Problems given to students
2	Diketahui·:·A=(-6,4,4)·dan·B=(-3,2,2)¶ The prepared completion plan begins by identifying the information from the given problem.	
3	d(AB)·=·$\sqrt{\lvert(-3)-(-6)\rvert^2+\lvert 2-4\rvert^2+\lvert 2-4\rvert^2}$ =$\sqrt{3^2+2^2+2^2}$¶ =$\sqrt{9+4+4}$¶ =$\sqrt{17}$¶ The problem-solving process is applied using the known distance formula. Applying the formula used shows no problems running well and smoothly from the work shown.	
4	 Draw points in three-dimensional Cartesian space.	

In Table 3, the challenge given to students is determining the distance by placing objects in the previous assignment. Based on these questions, as seen from what the students did, before determining the distance between objects in space, the placement of objects in space, for example, Cartesian coordinates, was done as a form of object orientation. In this orientation activity, it is not easy to orient this object, where three dimensions of space must be considered to ensure that the object's placement is in the right place. As described in Table 3 above, the usual method follows the following procedure: 1) Plot the points on the x, y, and z axes; 2) Determine the position where the x and y points meet in the XY plane; 3) Lift the point (x, y) corresponds to the position of the point on the z-axis. This is also done at other points; for example, points A(-6, 4, 4) and B(-3, 2, 2) are in the task.

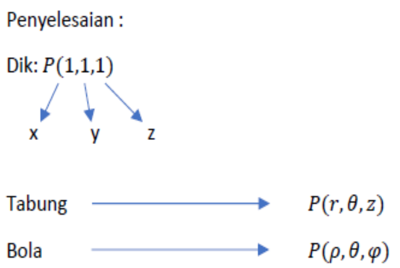
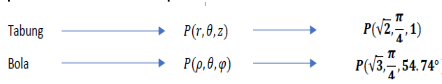
After the position of the two points is placed in space, the distance determination (d(AB)) is carried out using the distance formula in space, and as shown in the figure, the distance AB is the $\sqrt{17}$.

Some of the representations used in this procedure stem from the placement of points on the axes of the coordinate system. The representation in the same way is also applied to other points. Because determining the distance requires more than one point, it takes a minimum of 2 points. To make it look more precise, the representation of these points is visible using pictures. So, object representation is done in addition to placing points or objects; it is also equipped with forming images using dotted lines to ensure the accuracy of objects in space.

Activity 3: Coordinates of Points on Cartesian, Cylindrical Spaces, and Spheres

For the activity in this section, students are given problems in the form of questions that expect them to convert points in the coordinate system. The conversion is directed from three-dimensional Cartesian coordinates to cylindrical and spherical space coordinates.

Table 4. Student Answer Construction Topic Coordinates of Points on Cartesian, Cylindrical, and Spherical

No	Student Answer Construction	Description
1	Diketahui titik P(1,1,1) pada sistem koordinat ubahlah posisi titik tersebut untuk sistem koordinat tabung & bola! Given the point P(1, 1, 1) in the coordinate system, change the position of the point for the cylindrical and spherical coordinate system!	Problems given to students
2	 Completion plans are arranged in an excellent systematic manner. It begins by presenting the known information, and the representation of the question in the problem is made in the form of an arrow diagram that connects the type of space and the arrangement of its coordinate points.	
3	Mencari komponen yang diperlukan untuk menentukan koordinat dari tabung dan bola • $r = \sqrt{x^2+y^2}$ $= \sqrt{1^2+1^2}$ $= \sqrt{2}$ $Tan\,\theta = \frac{x}{y}$ $Tan\,\theta = 1$ $\theta = 45° = \frac{\pi}{4}$ • $\rho = \sqrt{x^2+y^2+z^2}$ $= \sqrt{1^2+1^2+1^2}$ $= \sqrt{3}$ • $r = \rho \sin\varphi$ $\sin\varphi = \frac{r}{\rho}$ $\sin\varphi = \frac{\sqrt{2}}{\sqrt{3}}$ $\varphi = \arcsin\frac{\sqrt{2}}{\sqrt{3}}$ $\varphi = 54{,}74°$ • $z = \rho\cos\varphi$ $\cos\varphi = \frac{r}{\rho}$ $\cos\varphi = \frac{1}{\sqrt{3}}$ $\varphi = \arccos\frac{1}{\sqrt{3}}$ $\varphi = 54{,}74°$ The problem-solving process shown in the figure to the right is to parse any components and determine their values. The description of the values obtained is then used to answer the core questions of this problem.	
4	 The conclusions obtained from the problem-solving process are based on the questions attached to the problem and adjusted using the results of the problem-solving process shown previously.	

Understanding how to convert point coordinates from Cartesian to cylindrical and spherical coordinates requires knowledge related to it. The knowledge in question is what the coordinates for the cylindrical space and spherical space look like. For the cylindrical space coordinates it is filled by (r, θ, z), and the spherical coordinates are filled by (ρ, θ, ϕ). R = ρ; therefore, θ in cylindrical space is also used for spherical space, but spherical space is formed from the angle from the xy-axis region and the z-axis. Then, for the cylinder space, the height of the cylinder is none other than the z-axis, so the third point in the coordinates of the tube space is the z-axis. Meanwhile, for a spherical space where the symbol used is ϕ, the angle formed between the z-axis and ρ.

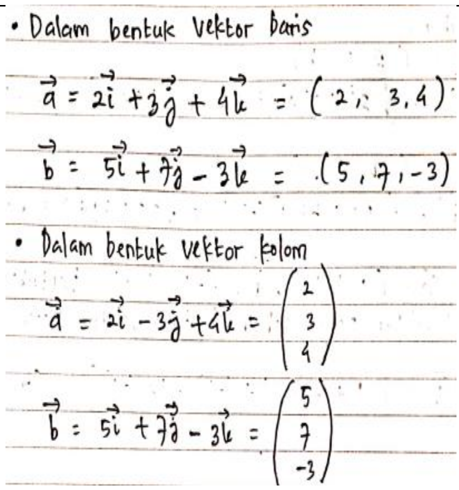
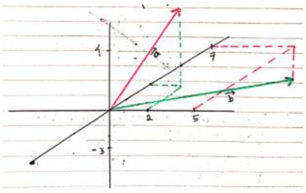
Based on this review, the working steps taken by students were as follows: 1) Identifying the coordinates of each space converting Cartesian to cylinders and spheres. 2) Determine some required values, for example, for tubes r and θ, while for balls, namely ρ dan ϕ. After all that is needed is complete, these values are used to complete the coordinate components of each space.

Activity 4: Vectors in Three-Dimensional Space

For the activity in this section, students are given problems in the form of questions that expect them to create vectors placed in a three-dimensional space coordinate system. The results are shown in the table below.

Table 5. Construction of Student Answers on the Topic of Vectors in Three-Dimensional Space

No	Student Answer Construction	Description
1	. Silahkan buat vektor pada ruang tiga dimensi ! Please vector in three-dimensional space!	Problems given to students

No	Student Answer Construction	Description
2		The problem-solving process shown in this issue is very well developed. The presentation of the given vectors begins by giving a general shape before being converted into row vectors and column vectors. In other words, student work has progressed with some review, such as column and row vectors.
3		Draw points in three-dimensional Cartesian space. This picture shows two octane chambers, namely octane 1 (xyz) and 5 (xy-z), are used. The use of 2 different colors, red and green, indicates these vectors' presence. This is a good attempt at demonstrating the quality of performance.

Although the questions given explore students' knowledge and understanding by asking, "Please create a vector in a three-dimensional space!" this question requires creative responses from students where they are expected to create a three-dimensional space whose contents are vectors. Knowledge is needed in addition to three-dimensional space and vectors with values and directions to answer this question. In addition to writing examples of vector numbers, students are also expected to be able to visualize the vector's shape depicted in the three-dimensional space.

Table 5 shows students' declarative knowledge through how the answers are given, namely row and column vectors. In other words, the description of the answer shows what the student knows. Then, the response to the questions is continued by visualizing the vector in the three-dimensional space. For this reason, number 3 in Table 5 shows a three-dimensional space image with vectors given in red and green colors.

Discussion

From the research findings stated above, the representation shown by students in their work is described in the following description:

The methods and materials of student representation depend on their knowledge and the broader knowledge of the various presentation information provided. What is meant in identifying objects where the selected object is related to daily life? In this case, the chosen one is a private room. Then, connecting the situation of the private room with the illustration of a three-dimensional space involves representation, such as the representation of objects in the private room in the form of dots. With this object, representation supports the visualization of images in three-dimensional space. Developing representation skills requires understanding mathematical concepts and attaching their relation to other mathematical concepts (Allen et al., 2020). Through the relationships between these concepts, representations can be used to recognize the relationships between concepts and apply them to realistic problems through modeling (Meilon et al., 2019). In its implementation, it emphasizes that the knowledge and learning experiences possessed and developed by students can lead them to demonstrate their representational abilities to be richer and support ways of solving problems or experiences to develop knowledge, use media, and practice problem-solving (Astuti, 2017). (Kozma, 2003) suggests that this representational ability provides learners with an authentic learning environment in which they use these abilities to explain phenomena. The ability to represent an object is also a form of confidence in interpreting and communicating mathematical concepts through abstract symbols that can be understood logically (Nurlisna et al., 2020). Indirectly, the learning process built with representational abilities forms a learning environment that enriches the learner's experience.

Then, object representation is usually supported by treatment in the form of placing points or objects that are equipped with forming images using dotted lines to ensure the accuracy of objects in space. Whatever is done is part of the form of representation where they think about the problem and valuable tools to solve the problem (Sabirin, 2014). In line with the explanation (De Cock, 2012) that the ability of representation or the ability of multiple representations can be a valuable tool to facilitate problem-solving. Although it is not stated that the goal is an abstraction, the formation of valuable learning experiences may be in line with the argument (Kaput, 1989) that the learning outcomes can be used for further learning activities where it is possible to see complex ideas in new ways and apply them more effectively. Mathematical understanding and representation are integral to problem-solving (Minarni et al., 2016). That way, improving mathematical representation skills will lead students to improve their problem-solving abilities.

Representation activities become complex when converting point coordinates from Cartesian to tube and spherical coordinates. In addition to requiring knowledge related to this, students will also explore the components needed to obtain converted points. (Kaput, 1989) found that using multiple representations helped students better understand mathematical concepts and provided various concretizations of concepts. This description relies on differences in the performance shown by students (Kohl & Finkelstein, 2005), suggesting that differences in performance depend on several things, including student expectations, prior knowledge, metacognitive skills, and specific contextual features of the problem and representation.

The development of previous knowledge can be done by asking questions that require creative responses from students or users; if this is done, student performance will develop according to the challenges given. Working on the problem of representation ability will direct the user to propose strategic considerations based on the characteristics of the problem at hand (Munn et al., n.d.). Although it is different from (Hegarty & Kozhevnikov, 1999), where the use of pictorial representations is negatively related to success in solving mathematical problems, in student performance, this is getting clearer and better developed using pictures and connecting the reasonable opinion that has been put forward where representation is a kind of configuration process to present something in different situations which involves identification, selection, and conveying of ideas (Goldin, 2015; Fennema et al., 1999); (Seeger et al., 1998). Other implementations were Nurlisna et al. (2020) by utilizing software and developing its function as a medium to present other learning tools in the form of a worksheet. The goal is that mathematical representation skills developed in learning activities are not only the instructions needed to be precise but also to facilitate the development of these skills to support the achievement of the expected learning objectives. The research implies that this kind of learning opportunity must be prepared by teachers to students to prospective teacher students so that they not only hone their mathematical understanding and representation (Minarni et al., 2016), especially students as prospective teachers who are enriched with knowledge leads to creative skills. Also, enhancing skills that utilize such knowledge enriches the learning experience (Cambaya & Tan, 2022). However, the research has several constraints and limitations regarding developing students' knowledge, where representation activities would further flourish if supported by visualization activities. Hence, this study would be interesting to explore visualization activities. Naturally, other skills would develop as part of the representation ability.

Implication

The importance of nurturing representation skills in mathematics education cannot be overstated. These skills involve comprehending mathematical concepts and establishing connections with other mathematical ideas. By refining representation skills, students can recognize the relationships between concepts and apply them to solve real-world problems through modeling. Implementing learning underscores that students' knowledge and learning experiences can lead them to demonstrate more robust representational abilities, supporting effective problem-solving and knowledge development using media and practical application in problem-solving. Representation skills also foster an authentic learning environment where students can elucidate phenomena and build confidence in conveying mathematical concepts through logically comprehensible symbols.

The complexity of representation activities in the mathematical context, especially in point coordinate conversion and the use of multiple representations, aids in developing prior knowledge and leveraging visuals to enhance the understanding of mathematical concepts. Despite constraints and limitations in students' knowledge development, there is potential for further enhancement through visualization activities. This writing emphasizes the urgency and positive impact of developing representation skills in improving students' understanding and mathematical proficiency.

Limitation

However, the limitations of this study lie in the lack of detailed exploration regarding which aspects of students' representation skills have reached an optimal level. Therefore, the foundation for improvement and skill enhancement can be identified by developing problems or learning activities. Additionally, the insights gained from this research are expected to reveal the extent of students' representation skills, shedding light on areas that may require improvement, particularly concerning the complexity or visualization aspects of representation skills.

CONCLUSION

Based on the discussion stated above, the research results obtained lead to conclusions that can be drawn as follows: 1) Involving problems in everyday life is one way to explore students' representational abilities, 2) Complex and non-routine problems will trigger students to develop representational skills where they will explore various related information to solve the problem. 3) The challenges given to students should trigger creative responses from students so that they not only form knowledge but also have a good learning experience. 4) Visualization techniques in pictures, diagrams, or other similar are the right opportunity for students to develop representational abilities.

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