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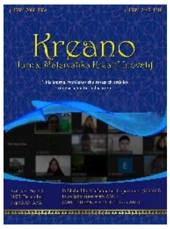

Learning Obstacles of Prospective Mathematics Teachers: A Case Study on the Topic of Implicit Derivatives

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Abstract

This research is a Didactical Design Research (DDR), aiming to identify various learning barriers for prospective mathematics teachers. Some students still experience learning difficulties in derived concepts which are pre-requisites for other concepts or other subjects, didactic and pedagogical anticipation can be prepared to over-come them. Based on the learning design, various learning barriers were identified, especially in the implicit derivative concept. The research participants consisted of 3 lecturers and 46 second-semester prospective teacher students at one of the tertiary institutions in Indonesia. The results of interviews and questionnaires were analyzed through identification, clarification, reduction and verification techniques and then presented narratively. The results showed that some prospective teachers experienced learning barriers 1) ontogenic instrumental, conceptual, and psychological types, 2) didactic, students could not identify contextual relationships in the structure of answers, indicating that the material was not by the continuity of students' thinking, and 3) epistemological, the lack of understanding of explicit and implicit similarities shows the limitations of the context that students have. Based on the research findings, a learning design will be developed based on the theory of a didactic situation with the stages of action situations, formulation, validation, and institutionalization, which are thought to be able to overcome the findings of learning obstacles.

Keywords: *Didactical Design Research; Implicit Derivatives; Learning Obstacle; Prospective Mathematics Teachers*

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Abstrak

Penelitian ini merupakan Didactical Design Research (DDR), bertujuan untuk mengidentifikasi berbagai hambatan belajar calon guru matematika. Sebagian mahasiswa masih mengalami hambatan belajar pada konsep turunan yang merupakan prasyarat konsep lain atau matakuliah lain, dapat disiapkan antisipasi didaktis maupun pedagogis untuk mengatasinya. Berdasarkan rancangan pembelajaran, diidentifikasi berbagai hambatan belajar khususnya pada konsep turunan implisit. Partisipan penelitian terdiri dari 3 dosen dan 46 mahasiswa calon guru semester dua di salah satu perguruan tinggi di Indonesia. Hasil wawancara dan angket dianalisis melalui teknik identifikasi, klarifikasi, reduksi, dan verifikasi, selanjutnya disajikan secara naratif. Hasil penelitian menunjukkan bahwa beberapa calon guru mengalami hambatan belajar 1) ontogenik tipe instrumental, konseptual, dan psikologis, 2) didaktis, mahasiswa tidak dapat mengidentifikasi hubungan kontekstual dalam struktur jawaban, menunjukkan bahwa materi tidak sesuai dengan kesinambungan proses berpikir mahasiswa, dan 3) epistemologis, kurangnya pemahaman persamaan eksplisit dan implisit menunjukkan keterbatasan konteks yang dimiliki mahasiswa. Berdasarkan temuan penelitian, akan dikembangkan desain pembelajaran berdasarkan theory of didactical situation dengan tahapan situasi aksi, formulasi, validasi, dan institusionalisasi yang diduga dapat mengatasi temuan hambatan belajar.

INTRODUCTION

Calculus is the study of change that uses derivation as the main tool. Many things related to solving mathematics, physics, and other branches of science cannot be solved by geometry and algebra, hence, calculus is needed (Rohde et al., 2012). Various fields apply derivatives, including economics, biology, physics, geography, and sociology. The derivation is among the mathematical concepts taught at the university to learn different concepts, other subjects, or real-world applications (Tarmizi, 2010; Tall, 2012; Pepper et al., 2012). Specifically, it is essential in differential calculus courses and a prerequisite for several courses in the Mathematics Education Study Program. Most undergraduate students find derivatives difficult due to a lack of conceptual understanding (Willcox & Bounova, 2004; Tarmizi, 2010; Tall, 2012; Pepper et al., 2012). Hashemi *et al*. (2014) concluded that students had a weak conceptual understanding, scoring 8.7 out of 20. Sahin *et al*. (2015) showed that second-year postgraduate students had fewer derivatives understanding. The explanations did not reveal the role of big ideas or modeling connections in derivatives. Unver *et al*. (2018) suggested that prospective mathematics teachers require modeling training with feedback at each stage.

Students experience difficulties with the derivative concept, including determining the derivative of rational functions and chain rules (Tokgöz, 2012) and maximum and minimum values (Fatimah & Yerizon, 2019). In general, they make fundamental errors in derivatives and those that cannot be understood conceptually (Orton, 1983). They have difficulties applying the calculus concepts in modeling and applications, especially in the real world (Roorda et al., 2007). Various factors cause difficulties in derivative concepts, such as the learning system (Habre & Abboud, 2006) and students' ability (Gray & Tall, 1991). The lecture's design that cannot help students connect implicit differentiation components causes difficulties in understanding derivatives (Borji, V., & Martinez-Planell, 2020).

Prospective mathematics teachers have difficulty constructing evidence due to inability to apply definitions and concepts (Noto et al., 2019), lack of prerequisite knowledge (Guler, 2016), negative attitude (Doruk & Kaplan, 2015), and lack of experience and knowledge (Şengül & Katranci, 2015). Gurefe (2018) stated that prospective mathematics teachers could not explain concepts using symbols. The order of the concept of derivatives in textbooks generally starts with the definition of derivatives, unilateral derivatives, derivatives of polynomial functions, derivatives of products and quotients of two functions, higher order derivatives, chain theorem, implicit derivatives, and applications of derivatives. Students' ability to determine implicit derivatives depends on their understanding of the types of underlying equations and various rules for finding explicit derivatives. In general, students can determine the derivative of an implicit function by first converting it into an explicit form. Problems arise when the implicit equation given cannot be presented in an explicit form. The presentation of the material in the learning design is not following the students' experience in learning the concept. The concepts of functions, quotients, and product derivatives are given long before the implicit derivatives. When the implicit derivatives and these concepts become the primary tools, some students experience difficulties because the questions provided do not follow their thinking experience.

This implies that high school students and mathematics or engineering majors have learning difficulties in derivatives. Aspiring math teachers experience similar difficulties as their peers from other programs. The learning experience of prospective teachers positively impacts their beliefs (Lo, 2020), helping students overcome various difficulties. This is in line in with Mufidah *et al.* (2019), which stated that there is a difference between teacher image and scientific derivatives conceptions.

Brosseau’s (2005) Theory of Didactical Situation (TDS) states that external factors cause learning obstacles, specifically didactical design (Suryadi, 2019). Since there is no best learning process, the teacher’s didactic design does not follow the level of thinking, profile, and students’ learning style. Additionally, the material does not follow the continuity of students' thinking, or the didactic design has limited context. Jaafar and Lin (2017) stated that there is no a suitable approach/intervention for students. Their profiles should be determined at the beginning of the semester to understand their weaknesses.

Brousseau (2002) divided learning obstacles into ontogenic, didactical, and epistemological. Ontogenic obstacles are related to students' mental readiness and cognitive maturity to receive knowledge. Furthermore, Suryadi (2019) stated that ontogenic obstacles reflect the difficulty of didactic situations, inhibiting students' learning participation. Didactical obstacles are caused by sequence factors and curriculum stages, including classroom presentation. They are minimized by arranging the material structurally (connections between concepts) and functionally (continuation of the thought process). The third type of obstacles is caused by limited understanding and mastery of concepts, problems, or others associated with a narrow context based on experience.

Suryadi (2019) categorized ontogenic learning obstacles into three, psychological, instrumental, and conceptual. Duroux (in Suryadi, 2010) stated that epistemological obstacles reflect a person's knowledge limited to a specific context. Students experience learning obstacles caused by mental readiness and cognitive maturity in receiving knowledge, the order of textbook material, or lecturer's presentation. Furthermore, these obstacles are caused by limited understanding and mastery of concepts, problems, or others with specific contexts.

Harel (2009) stated that mathematics has two complementary subsets: first, a collection or structure, including axioms, definitions, theorems, proofs, problems, and solutions. Second, ways of thinking through mental action characteristics of the first subset product. Following Harel's opinion, NCTM (2014) stated that mathematics is not a collection of separate topics and abilities but a unified whole. Furthermore, it should not be viewed as a product but an activity. Students construct their mathematical knowledge through various activities, including patterns, generalizing, and abstractions to form a concept (Suryadi, 2010). Based on this, a prospective teacher should be equipped to provide students opportunities to construct knowledge from a series of expertise. Kirschner *et al.* (2006) stated that students must build mental representations or schemas regardless of complete or partial information, where complete information gives accurate and easy terms.

Previous studies did not identify the types of obstacles, hence the need to categorize problems or obstacles to anticipate and overcome multiple student obstacles, such as didactic design. Previous research focused on students, disregarding prospective mathematics teachers. Various difficulties related to derivatives revealed in previous research were found in high school students, engineering students, and mathematics students. This research will focus on student math teacher candidates, considering that they will spearhead the success of a learning process if they are already teachers. The experience of prospective mathematics teachers in overcoming various learning obstacles will be helpful for them in helping overcome student learning obstacles at the school level. Knowing the various learning obstacles experienced by prospective mathematics teacher students is an effort by a lecturer to understand from a student's point of view how they carry out their thinking processes so that they can be taken into consideration in designing a learning process, including creating learning designs along with their didactic and pedagogical anticipations.

Therefore, this specific research questions include: (1) What are the types of ontogenic learning obstacles experienced by prospective mathematics teachers on implicit derivatives? (2) What are the types of didactical learning obstacles experienced by prospective mathematics teachers on implicit derivatives? (3) What are the types of epistemological learning obstacles experienced by prospective mathematics teachers on implicit derivatives?

METHOD

This qualitative research used the Didactical Design Research (DDR) design. Indonesia developed DDR in 2010 (Suryadi, 2019), as a form of educational innovation (Sidik et al., 2021), exploring the teachers' learning designs characteristics and impact on students' thinking development (Fuadiah et al., 2019; Suryadi, 2019). DDR's philosophy provides active learning situations to construct learners' ways of thinking and understanding mathematics knowledge (Marfuah et al., 2022). The interpretive paradigm was applied to examine the impact of didactic design on student thinking (Creswell, 2014; Denzin, Norman and Lincoln, 2018; Suryadi, 2019). According to Suryadi (2011), DDR includes didactic analysis before learning through a Hypothetical Didactic Design, including the Anticipation of Pedagogical Didactics (ADP), considering the learning obstacle. This research identified various learning obstacles for prospective mathematics teachers on implicit derivatives from the aspect of learning design developed by lecturers. n simple terms, the research stages are presented in Figure 1 below.

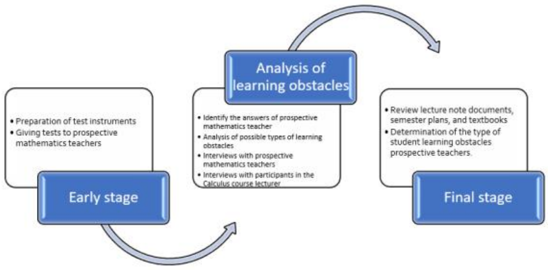


Figure 1. Research Step

Brosseau (2002) identified three learning obstacles, ontogenic, epistemological, and didactic. DDR revealed various students' thinking and obstacles, especially prospective mathematics teachers. The learning obstacles for prospective mathematics teachers were identified based on the lecturers' learning design and effect on students' implicit derivatives understanding.

Learning obstacles caused by cognitive maturity and mental readiness of students to receive knowledge are called ontogenic learning obstacles; if a didactical system causes it, for example, sequence factors, stages in the curriculum, including presentation in the learning process, then learning barriers are called didactical learning obstacles; and if it caused by limitations in students' mastery and understanding of something that is limitedly associated with a particular context adapted to the experiences they experience, then learning barriers are called epistemological learning obstacles. Ontogenic learning obstacles are divided into three types, namely (1) psychological types, learning barriers caused by cognitive maturity and psychological factors of students in acquiring knowledge; (2) instrumental types, learning barriers caused by not mastering critical technical matters of a problem being solved; (3) conceptual type, learning barriers caused by the conceptual level contained in the learning design.

The participants consisted of three female lecturers aged 40 – 57 who were interviewed and provided a semester course plan for the Differential Calculus course, needed for analysis. This study focused on the concept of implicit derivatives. The lecturers had relevant educational backgrounds, teaching Differential Calculus for 5-25 years. Furthermore, the participants included 46 second-semester students of the Mathematics Education Study Program at a university in West Java, Indonesia aged between 18-21, where 39 were female and 7 males. The students involved had taken 40 of the required total credits (145 credits), 3 of which are Differential Calculus courses.

The research was conducted during the Covid-19 pandemic; hence the online lectures were based on distance learning. Therefore, educators utilize technological advances to create a practical and satisfying learning experience (Burdina et al., 2019). The data were collected from test results, questionnaires, interviews, and document reviews.

In qualitative research, there are four criteria for data validity: credibility, transferability, dependability, and confirmability (Denzin, Norman, and Lincoln, 2018; Moleong, 2017; Sugiyono, 2016). Credibility is synonymous with external validity in quantitative research, carried out through the following steps: (a) directly involved in the research site during the data collection process; (b) thorough and detailed at the time of data collection and analysis adapted to the research objectives; (c) using technique and source triangulation; (d) peer review; (c) adequacy of references, keeping authentic evidence of the results of research data collection. With these stages, the researcher can obtain complete data and be accounted for so that the research findings have the correct accuracy from the perspective of researchers, participants, and readers. Transferability, synonymous with external validity in quantitative research, can be seen from the research setting, determination of participants, and data processing. This stage ensures that this research provides sufficient information to the reader about the cases studied to determine the degree of similarity between the cases studied and cases whose findings can be transferred to other issues. Dependability is synonymous with reliability in quantitative research, carried out by examining the entire research process, starting from problem identification, preparation of research instruments, checking data accuracy, and data analysis quality. Confirmability in quantitative research is known as objectivity, carried out by examining the objectivity and transparency of the findings and discussion of the research.

The students determine implicit derivatives depending on their understanding of types of equations, including 1) explicit, with variables x and y on different sides, 2) implicit, with the variables on the same side. In the second case, there are those presented in an explicit form and those that cannot be delivered in an explicit condition. Students have studied how to find and determine derivatives of implicit functions by converting them into explicit forms. The problems arise when the implicit equation cannot be presented in an explicit form, as discussed in this research. For this reason, a test was designed to assess various prospective mathematics teachers' obstacles detected through the stages presented in the questions. The student's understanding depends on prerequisite materials, including explicit equation derivatives, rules, basic ideas, and application. The test questions were structured to identify the continuity of students' thinking patterns, starting with their ability to relate and utilize prerequisite material and determine the derivatives of various implicit equations and applications.

The test had 3 questions, including 1) implicit equation presented in explicit form. Part a guided the students to convert it into an explicit form, while part b asked them to determine the derivative based on a. Part c asked them to determine the equation's derivative without first converting it into explicit form. They were asked to conclude the b and c results in the last section. This question determined the students' understanding of prerequisite knowledge and implicit derivatives. 2) an equation in an implicit form not explicitly stated, asking the students to determine the equation's derivative using the implicit derivative. This question determined their understanding of derivatives, solved with implicit derivatives. 3) the implicit derivative application problem determines the Tangent and Normal Line Equation. This question determined students' abilities in applying implicit derivatives to problems.

The test was given to student participants to determine various obstacles and their understanding of implicit derivatives. Furthermore, unstructured interviews were conducted on several students with specific characteristics for further exploration. Lecturers were interviewed to determine their learning designs, obstacles in presenting the material, student learning obstacles, solutions, and input for effective lectures.

The student participants filled out questionnaires to substitute class observations. The questions were arranged to describe the studied situation, providing equivalent information as direct observations. However, further information was obtained by interviewing several students for clarification. The questionnaires and interviews followed the tests on implicit derivatives. Therefore, the researcher was not involved and did not treat the participants' class activities. Subsequently, field observations involved document reviews of student test results, lecture notes, semester lecture plans, and textbooks. The data analysis techniques included identification, clarification, reduction, and verification, presented narratively.

RESULTS AND DISCUSSION

Results

The identification and analysis of various learning obstacles for prospective mathematics teachers were categorized following the research questions. The first research question showed three ontogenic obstacles, namely instrumental, conceptual, and psychological types. Meanwhile, the second research question only revealed the didactic type of learning obstacles. Finally, the third research question revealed the epistemological type of learning obstacles. Each obstacle included screenshots of students' work and interview excerpts, as described in the following sections.

What Are the Types of Ontogenic Learning Obstacles Experienced by Prospective Mathematics Teachers on Implicit Derivatives?

Prospective mathematics teachers experience technical errors that hinder problems solving. This problem was detected when students solved the implicit derivative application question to determine the Tangent and Normal Line Equation of $\frac{x^2}{9}+\frac{y^2}{36}=1$ at the point $(-1, 4\sqrt{2}\,)$. Some of the answers shown in Figure 2 (See Appendix).

These answers indicate difficulty in determining the form of $\frac{dy}{dx}$ and the tangent line equation. Students could not solve the problems due to a lack of key technical matters mastery, including the real numbers of properties, Cartesian coordinates, and gradient. The questionnaire results reinforced the learning obstacles experienced.

Researcher: What difficulties have you experienced in determining the derivative of an implicit function?
Student-1: Missed item when determining derivatives of an implicit function, using the product derivative of two functions.
Student-3: Difficulty in applying derivatives.
Student-15: Likes to be wrong when determining $\frac{dy}{dx}$.
Student-28: In the implicit derivative, I thought y as x function.

The responses of Student-1, Student-3, and Student-15 indicated a lack of key technical understanding of the problems, namely the product derivative adaptation in implicit derivatives with two variables simultaneously. The student-28 response indicated a lack of technical understanding of the general principle of implicit derivatives; hence both problems were not solved properly.

The test and questionnaire analysis showed that technical errors often cause errors in solving questions. Therefore, the prospective teachers' mistakes in determining the tangent and normal lines included ontogenic obstacles of the instrumental type.

The ontogenic obstacle with conceptual type, relates to the question's conceptual level, not the student's thinking experience. Weak understanding of rational function derivatives causes difficulties, as shown in Figure 3 (See Appendix).

Several answers identified a weak understanding of the rational function derivative $y=\frac{u}{v}$. Some students understood the derivative function as $y'=\frac{u\prime}{v\prime}$. The errors occurred because the lecture on implicit derivatives provided questions in polynomial functions, causing difficulties when solving a rational function. The problem was traced in the students' lecture notes. Apart from the derivative of the rational function, several errors were identified due to a weak understanding of the product derivative for two functions. The problem was revealed when determining the derivative of $x^2y + xy^2 = 3x$, as shown in Figure 4 (See Appendix).

Some answers showed a weak understanding of the product derivative of two functions, namely $y = u.v$. Some students lack understanding of the function's derivative as $y' = u'v + v'u$. There were different errors, specifying $u'.v$ and $v'.u$ due to a weak understanding of implicit derivatives. Students lack understanding of implicit derivatives as a concept that requires another, namely chain proposition, and considering variable y as a function of x.

The interviews showed that some students did not know the derivative of a rational function and product of two functions, while some could not apply them to implicit derivatives. Therefore, the lack of this understanding affects other topics with similar concepts. Students should understand the concept before discussing implicit derivatives. The Differential Calculus lecturer interviews confirmed students' difficulties, as shown below:

Researcher: According to your experience, what are the students' difficulties when learning implicit derivatives?
Lecturer-1: The variables x and y often confuse students in determining the correct derivation rule.
Lecturer-2: Some students forget to derive implicit functions using the derivative search and chain rule.
Lecturer-3: Students have difficulty deriving the multiplication and division of functions.

The questionnaires showed these difficulties due to a weak understanding of implicit derivatives, as shown below:

Researcher: Difficulties encountered when determining the implicit function derivative.
Student-10: The y-derived symbol to x is not written because y' is alternated with y, despite having different meanings.
Student-25: I am confused about what to take first when using the chain theorem.
Student-28: Forgetting that I was considering y as a function of x.
Student-39: Determining implicit derivatives using the chain theorem.

Weak understanding of algebraic concepts and characteristics of function derivative rules cause obstacles, as revealed answers in Figure 4 (See Appendix).

Various difficulties were caused by the conceptual level in the lecturers' learning design, disregarding the student's learning experience. Students have obtained the derivative of rational functions, the derivative of the product of two functions, and the chain theorems before studying implicit derivatives. The designs lacked an explicit effort to link the studied concepts with the student's previous knowledge. The tests, interviews, and questionnaires analyzed the difficulties as conceptual ontogenic obstacles. Furthermore, the obstacles were due to the question's unmatched conceptual level with the student's thinking experience.

This research did not find the explicit ontogenic constraints with psychological type. The lack of mastery of prerequisite materials on the basic concepts of derivatives and various rules has hindered the study of implicit derivative. These problems are attributed to the lack of student interest in implicit derivative concepts, classified as a psychological ontogenic learning obstacle.

What Are the Types of Didactical Learning Obstacles Experienced by Prospective Mathematics Teachers on Implicit Derivatives?

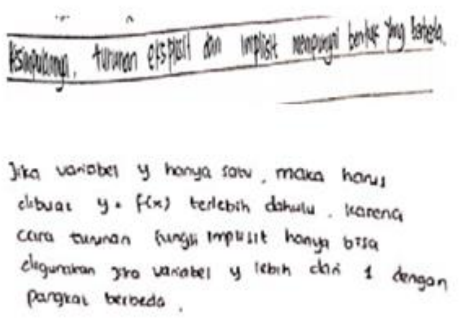
Obstacles can occur when the learning materials lack the continuity of students' thinking. Some students do not understand the questions, providing wrong answers (see below).

Given the function $xy + 2x + 3x^2 = 4$
a. changes the equation in the form $y = f(x)$;
b. based on part a determine the value of $\frac{dy}{dx}$;
c. without part a, can $\frac{dy}{dx}$ be determined? if yes, determine the value.
d. conclusions from sections b and c results.

Some students solved part a using implicit derivative (Figure 5 part (i)). However, when answering part c, they wrote it in the form $y = f(x)$ before determining the derivative (shown in Figure 6. Part (ii). - See Appendix). Some students determined the explicit form in part a but used implicit derivative to determine part b (Figure 6. part (iii) - See Appendix).

The obstacles were caused by order of the material, disregarding the continuity of students' thinking. This affected their knowledge reflected in their work and lecture notes. There was inadequate explanation of the relationship between implicit and explicit equations and utilizing explicit equation derivatives to determine implicit equations derivatives converted into explicit equations.

The weak understanding of implicit equations and derivatives affected part d. Some students cannot conclude whether the implicit equation derivative is the same, first converted into $y = f(x)$, or directly uses implicit derivative. Some answers were as follows.



Author's translation:

- Conclusion: Implicit and explicit derivatives have different forms
- In case there is only one variable, y, the equation needs to be changed to the form $y = f(x)$, because the implicit derivative can only be used if the variable y is more than one with different ranks.

Figure 6. Examples of errors in drawing conclusions

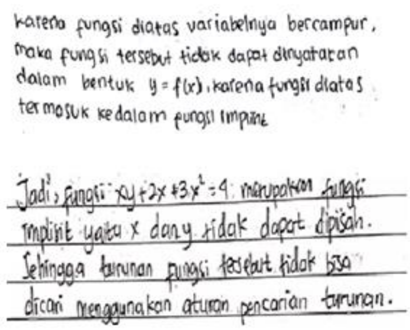
Obstacles for prospective teachers cannot conclude whether the results were similar. The continuity of students' thinking is not by the problem's context. Students did not complete the situation, determine the contextual relevance in the answer, or have limited context. They solved problems following the work examples, as reinforced by the lecturer's interviews below.

Researcher:	When teaching and learning "Implicit Derivatives," what is the order of the applied materials?
Lecturer-1:	Define implicit function and find its derivative.
Lecturer-2:	Derivative, derivation search rule, trigonometric functions, implicit derivative, and chain rule.

The interview results showed that the order of the material lacked an explicit description of the relationship between implicit derivatives and functions. The lecture notes lacked stimulating examples for students to conclude the derivative functions determined in two ways, either convert into an explicit equation (that can be changed) or directly use implicit derivatives. This condition is shown in the order of material on the applied syllabus. Furthermore, the problem is well presented when traced in the sourcebook. The results showed that students experience didactical obstacles due to the sequence of material, exercises, and types of questions.

What Are the Types of Epistemological Learning Obstacles Experienced by Prospective Mathematics Teachers on Implicit Derivatives?

Some students experienced obstacles due to a lack of understanding of the two types of equations meaning when solving an implicit equation $xy + 2x + 3x^2 = 4$, converting it into an explicit equation $y = f(x)$ to determine the derivative function. Some of the answers included:


karena fungsi diatas variabelnya bercampur, maka fungsi tersebut tidak dapat dinyatakan dalam bentuk y = f(x), karena fungsi diatas termasuk kedalam fungsi impilit

Jadi, fungsi xy+2x+3x²=4 merupakan fungsi implisit yaitu x dan y tidak dapat dipisah. Sehingga turunan fungsi tersebut tidak bisa dicari menggunakan aturan pencarian turunan.

Author's translation:

- The function above has mixed variables, hence cannot be expressed as $y = f(x)$, and is included in the implicit function.
- The equation $xy + 2x + 3x^2 = 4$ is an implicit function because the x and y variables are inseparable, hence the derivative search rule cannot determine its derivative.

Figure 7. Examples of the epistemological type of learning obstacle

Weak understanding of the explicit and implicit equations differences causes obstacles when solving implicit function derivatives, as revealed in the student's interviews below:

Researcher: What is the difference between explicit and implicit equations?
Student-11: Explicit and implicit equations have separable and inseparable coefficients, respectively.
Student-4: The explicit equation is $y = f(x)$ with arbitrary variables.
Student-27: An explicit equation has one variable, while an implicit has two.
Student-40: Explicit equations are solved faster than implicit.
Student-44: Implicit functions cannot be represented by $y = f(x)$.

The questionnaires indicated a weak understanding of explicit and implicit equations. Some students had difficulties distinguishing equations' variables, coefficients, and constants. The student's lecture notes showed the explicit equation directly presented as $y = f(x)$; no examples showed whether the explicit equation could be obtained from the implicit. Furthermore, there was a minimal explanation of implicit equations, giving direct examples of implicit derivatives without associating explicit derivatives of functions. A student stated that they had difficulty "solving a different question from the example given." Therefore, they cannot solve problems due to limited context, causing epistemological obstacles. Some students could change the implicit equation into the form $y = f(x)$ but made an error in determining the derivative, as shown in Figure 7 (See Appendix).

The mistakes are attributed to the weak mastery of basic quotient derivatives concepts, as revealed from the student's interviews: they can easily determine the quotient derivative when the numerator and denominator contain variable x but have problems with the variable only appearing in the numerator or denominator. Some students stated that they could not use the quotient derivative when determining the derivative of $y = \frac{1}{x}$, given the derivative of 1 as 0. Furthermore, they believed that the derivative of $y = \frac{x}{3}$ could be determined using the quotient derivative rule. They did not know another way of determining the second derivative of a given function. Some realized that the two functions derivatives could be determined using the power derivative $y = x^n$ guided by the rational and linear function's derivative questions. Limited context caused a weak understanding of rational function derivatives, where the given examples contained variable x in the numerator and denominator; hence a different form caused obstacles. Therefore, the students experience epistemological obstacles in determining implicit derivatives.

Discussion

The findings revealed that prospective mathematics teachers experience various learning obstacles on implicit derivatives. This is caused by external factors such as mental readiness to receive knowledge, lecturers' didactical designs, and limited context. Various obstacles were due to the weak understanding of rational function derivative and product of two functions. The results are in line with Tokgoz (2012), which stated that students have difficulty in rational function derivatives and chain theorems. Tarmizi (2010), Tall (2011), Pepper (2012), dan Hashemi (2014) found a weak understanding of derivative concepts considered difficult by students. Orton (1983) concluded that students from the mathematics department made fundamental errors in the concept of the derivative. Furthermore, the weak mastery of prerequisite material hindered implicit derivative problem-solving. This supports other studies that students have difficulty understanding limits, requiring prerequisite mastery before the derivative concept (Kim et al., 2015; Wahyuni, 2017; Fatimah & Yerizon, 2019). Nurwahyu *et al.* (2020) concluded that students experience misconceptions using the basic formula for derivative functions.

The tests and interviews revealed that some students experienced obstacles solving implicit derivative problems due to a limited understanding of chain theorem and keywords of implicit derivatives. This is in line with Borji and Martinez-Planell (2020), which stated that chain rule and implicit function are crucial in achieving coherence of implicit derivative schemes, namely explicit functions, derivatives, and search rules. An implicit function process conception and chain rule schema should be constructed based on the function composition coherence.

The lecturers' didactical design causes a didactical learning obstacle, including the sequence of material, position of prerequisite material, and the question. Lecturers should develop a didactic design, allowing students to construct knowledge without significant obstacles. This supports Amzat *et al.* (2021) that teachers should educate and develop the curriculum. In addition to the order of the material, the didactic design needs to contain didactic and pedagogical anticipations that make it easier for students to construct knowledge. The results of this study are in line with the research of Darmawan et al. (2021). They concluded that students often feel doubtful about the answers given after the teacher provides intervention in the form of dialogical questions.

This research identified and analyzed various learning obstacles in the implicit derivative concept based on Brousseau (2002) and determined various causes. Specifically, this result can help prepare didactic and pedagogical anticipations for future learning.

Implication of Research

The various obstacles identified show the need for good planning, including alternative didactic designs to assist students overcome various learning obstacles. The recommended didactic design contains four steps allowing students to construct knowledge; First, presenting the problem through discovery, solving strategies, or rules on implicit derivatives that stimulate students' thinking. Second, understanding the problems in the first stage, with didactic and pedagogical anticipation to assist students to achieve knowledge. Third, students are faced with situations that allow improvement or strengthening of the concepts learned. Fourth, presenting various problems to measure students' ability to apply concepts in different contexts. This supports the Theory of Didactical Situations with four stages, action situation, formulation, validation, and institutionalization.

Limitation

This research focused on second-semester students of the Mathematics Education Study Program, using a small part of the courses in the curriculum structure. Furthermore, online lectures hinder individual services. The research focused on the first stage of DDR by identifying various learning obstacles on implicit derivatives from the lecturers' learning design.

CONCLUSION

The prospective mathematics teacher students experienced difficulties caused by external factors categorized as obstacles. Instrumental type of ontogenic Learning Obstacles causes difficulties in derivatives functions and tangents equations. The ontogenic learning obstacle conceptual type causes difficulties in determining the derivative of an implicit function because of a weak understanding of the derivative of a rational function and the derivative of the product of two functions. Psychological obstacles reflect minimal interest in implicit derivatives due to lack of prerequisite material mastery. In contrast, didactical learning obstacles are caused by lack of material continuity based on students' thinking processes. This is identified when students cannot conclude the derivative of an implicit function, solve the problem, and find the contextual relationship. Finally, epistemological learning obstacles were due to limited context, showing a weak understanding of explicit and implicit equations and difficulties using rational function derivatives.

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Appendix

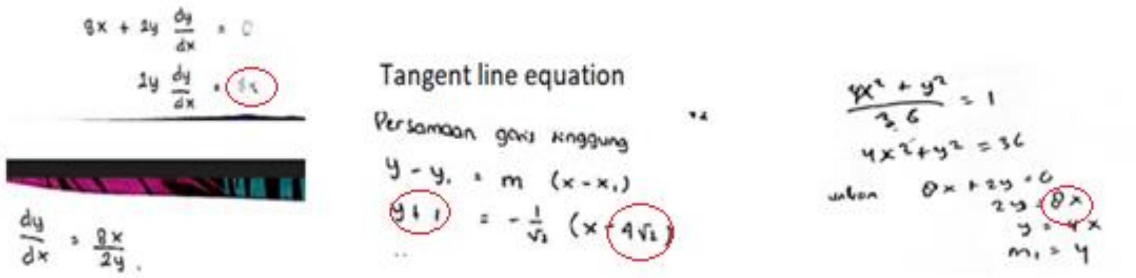

Figure 2. Examples of instrumental type ontogenic learning obstacle

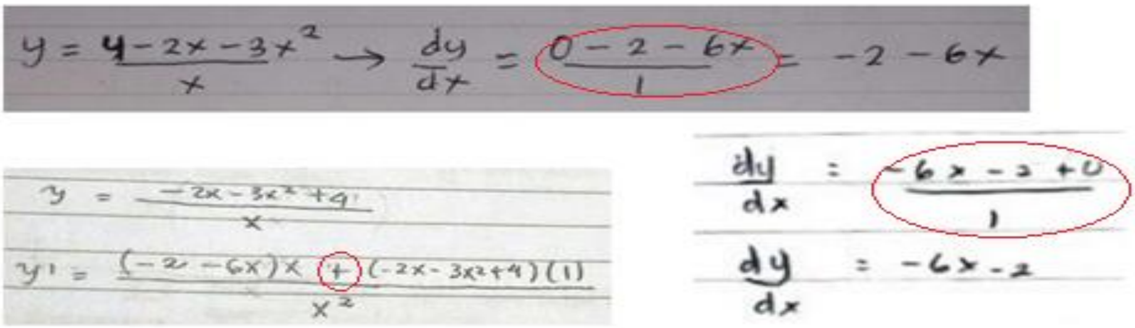

Figure 3. Examples of conceptual-type ontogenic learning obstacle on the derivative of rational functions

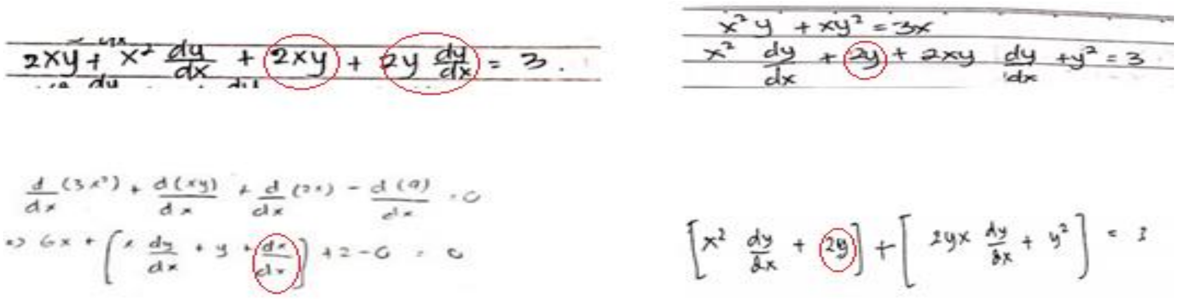

Figure 4. Example of conceptual type ontogenic learning obstacle on the product derivative of two functions

Figure 5. Examples of weak understanding of algebraic concepts

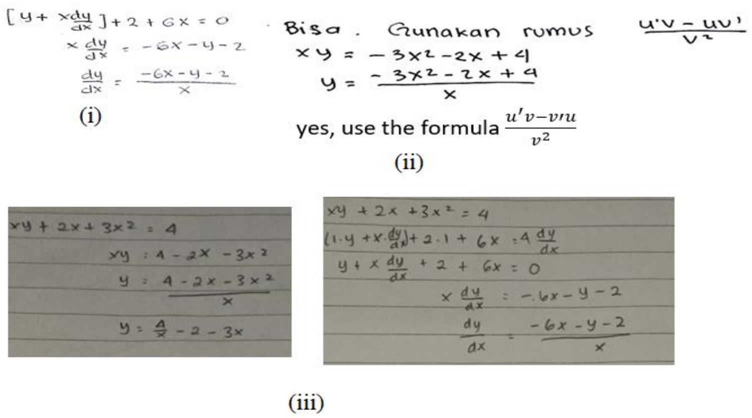

Figure 6. Examples of weak understanding of implicit equations and derivatives.

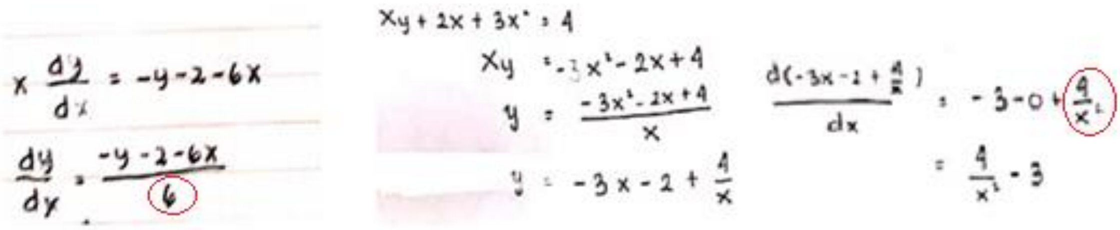

Figure 7. Examples of the epistemological type of learning obstacle