**Skewed Normal Distribution Of Return Assets**

 **In Call European Option Pricing**

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**Abstract**

Option is one of security derivates. In financial market, option is a contract that gives a right (not the obligation) for its owner to buy or sell a particular asset for a certain price at a certain time. Option can give a guarantee for a risk that can be faced in a market.This paper studies about the use of Skewed Normal Distribution (SN) in call europeanoption pricing. The SN provides a flexible framework that captures the skewness of log return. We obtain aclosed form solution for the european call option pricing when log return follow the SN. Then, we will compare option prices that is obtained by the SN and the Black-Scholes model with the option prices of market.

**Keywords***:* skewed normaldistribution, log return, options.

**1. INTRODUCTION**

Option is a contract that gives the right (not obligation) to the contract holder (option buyer) to buy (call option) or sell (put option) a particular stock at a certain price within a specified period.

In practice, there are many different types of option. Based on the form of rights that occurs, the option can be divided into two, namely a call option and put option. Whether buying or selling option, they can type in europe and america.

In the pricing of stock option, Black-Scholes model is the first model used in option pricing. This model limited the problem by making an assumption that stock returns follow a Normal Distribution while stock prices follow Lognormal Distribution

In fact, the return data are often not normally distributed. In addition, the return data are often found in non zero skewness, see Hsieh [1], Nelson [3] and Theodossiou [4]. Thus, the Normal Distribution for the return and Lognormal Distribution for stock data which is the assumption in Black-Scholes model is less able to explain the existence of this skewness. This has encouraged researchers to perform calculations of the option price based on distribution approach. They expect that the characteristics of data is used can be modeled well by the approach of the distribution (other than Normal and Lognormal Distribution).One such distribution is Skewed Normal Distribution (SN). Therefore, this paper will discuss the use of SN for stock return on assets in determining the price of european call option.

The remainder of this paper begins with Ito’s Lemma, Brownian Motion and Black-Scholes Model in section 2, while section 3 derives SN approach pricing model for europeancall options and the last section offers the conclusions.

**2. ITO’s LEMMA, BROWNIAN MOTION, ANDBLACK-SCHOLES MODEL**

2.1 Itô’s Lemma

The price of each derivative security is a function of the security stochastic variables underlying the derivative and time. Therefore, it is need an understanding about the functions of stochastic variables. An important result in understanding this area was discovered by the mathematician, K. Ito in 1951, later known as Itô’sLemma.

**Itô’s Lemma.***If F(x,t)be a continuous function, twice differentiable function ofx andt, or*

**

*and*

**

*Defined a stochastic differential equation with driftrate**and variance rate,*

**(2.1)

*where dW is Wiener Process, and are function of  and , then a function F of  and  follow the process:*

 (2.2)

2.2 Brownian Motion of Stock Price Process

 Defined a Geometric Brownian Motion of stock price as follow

(2.3)

where andare constants. Equation (2.3) had been known as a model of stock price behavior.

Using Itô’s Lemma, the solution of equation (2.3) is

(2.4)

**2.3 Black-Scholes Model**

 Black-Scholes model is the first model used in option pricing.

Hull [2] states that  european  call option with the corresponding payoff , rational price set by the Black-Scholes formula is­

= (2.5)

whereis stock price at time 0, is stock price at a future time , is strike price,

(2.6)

(2.7)

and



is the cumulative standard normal distribution.

**3. SNAPPROACH FOR DETERMINING THE PRICE OF EUROPEAN CALL OPTION**

This section describes the calculation of the european call option price using SN approach for return assets. However, the previously disclosed little about the Skewed Generalized Error Distribution (SGED) which is a generalization of SN.

**3.1 Density Function ofSGED andSN**

A continuous random variable *Y*  followsSGEDwith parameter$μ,σ,k,λ$if it

has probability density function which can be expressed as follows:

(3.1)

where

$C=\frac{k}{2θ}Γ\left(\frac{1}{k}\right)^{-1}$ (3.2)

$θ=Γ\left(\frac{1}{k}\right)^{1/2}Γ\left(\frac{3}{k}\right)^{-1/2}S\left(λ\right)^{-1}$ (3.3)

$δ=2λAS\left(λ\right)^{-1}$(3.4)

$S\left(λ\right)=\sqrt{1+3λ^{2}-4A^{2}+λ^{2}}$, and(3.5)

$A=Γ\left(\frac{2}{k}\right)Γ\left(\frac{1}{k}\right)^{-1/2}Γ\left(\frac{3}{k}\right)^{-1/2}$. (3.6)

Notation of   and represent the expected value and standard deviation of random variable  *Y*.

In the equation above, the sign states Signum Function, while **stating Gamma Function. Parameter with constraints to control the height and the tails of density function, while which obey the following constraint is a skewness parameter, as in Theodossiou [4].

In the case of positive skewness$λ>0$, the density function is skewed to the right, whereas for the case of negative skewness, the density function is skewed to the left.Theodossiou and Trigeorgis [5] state that the probabilitydensity function of SN is a special case of  the probabilitydensity function of SGED for parameter values .

Through substitutionis obtained that the probability density function of SN is

(3.7)

where, $θ=Γ\left(\frac{1}{k}\right)^{1/2}Γ\left(\frac{3}{k}\right)^{-1/2}S\left(λ\right)^{-1}$, , $S\left(λ\right)=\sqrt{1+3λ^{2}-4A^{2}+λ^{2}}$, and

Probability of SN density function illustrated by this figure



**Figure 2**Probability of SN density function

Note that, figure 2 only differ in the value of , while for value of  and are the same.

Can be calculated directly that

, , and

**3.2 Estimation of SN Parameter by Moment Method**

This section derives estimating of SN parameters by using moment method.

Let the unknown mean and variance of the SN respectively are and

If skewness is also known, it can be easily calculated thatand

Based on the definition of sample moment, it was known that sample

moments of SN are , , and

Furthermore, the calculation shows that estimates of andrespectively are, and$\hat{SK}$.The details of derivation of ,and can be found in appendix 1, below.

**Appendix 1-Estimation of SN Parameter by Moment Method**

Using Moment Method, mean and variance of SN respectively are





Then, let*skewness* of SN is



On the other hand, we know



so



Consider that moment population of SN are



Based on definition of sample moment of moment method, sample moment of SN are



Furthermore, using moment method we conclude that

(A.2.1)

(A.2.2)

(A.2.3)

Using equation (A.2.3), we get

$\hat{SK}$

Based on the above results, we get and $\hat{SK}$are

(A.2.4)

 (A.2.5)

$\hat{SK}$(A.2.6)

Using equation (A.2.4), (A.2.5) and (A.2.6), we will find the parameter estimator for as follows



So

(A.2.7)

Estimator for  can be found by completing equation (A.2.7) through the use of Taylor Series approach. Founding  is analog with searching of root from equation (A.2.7). This process is done by Scientific Work Place (SWP) with the following stages:

1. For purposes of simplification, the first term on the left side of equation (A.2.7) simplified by utilizing the simplify command with the following results



2. Notice the second term on the left side of equation (A.2.7). Based on previous results, the term that value is determined by equation (A.2.7). Thus, equation (A.2.7) can be written as

$\hat{SK}$

3. Then, the equation obtained in stage 2 above was approached by third-order of Taylor Series through Power Series command.

4. By cutting the error term in the Taylor Series generated in stage 3, then using Polynomial Roots command, we will find some roots of it. At this stage will produced three types of roots that will determine the value of 

 In this case will be chosen real-valued root.

5. Furthermore, since at the fourth stage is still obtained real root whose form is still quite complicated, the roots obtained were approached by a second-order Taylor Series.

 Through some stages above, we obtained estimator for parameteras follows

$\hat{SK}$.

* End of Appendix 1.*

**3.3 SN Approach for Call European Option Pricing**

Call option pricing using SN approach is as follows:



(3.8)

For purposes of simplification calculation of SN, conducted reparameterization equation (3.7)becomes

(3.9)

where, , and.

**Calculation of *p***

This section describes about result from the first stage of theexplicit formula calculation process of the call european option pricing by SN approach.

This stage considered the two cases as follows:

1. for ,

so

2. for,

so

**Calculation of *q***

This section describes about result from the second stage of theexplicit formula calculation process of the call european option pricing by SN approach.

Analogue with the first stage, this stage is also looking at two cases as follows:

1. for ,

where .

1. for ,

where

and 

The details of *p* and *q* calculation of can be found in appendix 2, below.

**Appendix 2-Calculation of *p* and *q***

**Calculation of *p***

1. For ,





So, we can conclude that



1. For

******

 

 

 



So, we get



**Calculation of *q***

1. For ,







 where .

1. For





where

and 

* end of appendix 2*

**Call Option Price with SN Approach**

Based on the result of and calculation, call option price with SN approach is

**

**3.4 Performance of SN Approach of Return Assets and Black-Scholes on European Call Option Pricing**

This section describes the application of european call option pricing with SN approach and Black-Scholes model for return assets.

Based on the discussion in previous section,the calculation process of european call option pricing with SN approachand Black-Scholes model needed some informations that the stock price at the beginning of the contract (), strike price (), risk free interest rates

(), maturity time()and volatility().Especially for the SN model is required to estimate the value of skewness parameter .

The details of the option price obtained by using SN approach and Black-Scholes modelfor log return assets of Dominion Resources, Inc. (D), Tyco International Ltd. (TYC), Chesapeake Energy Corporation (CHK), Home Depot, Inc. (HD), Johnson & Johnson (JNJ), Loews Corporation (L), Qwest Communications International Inc. (Q), Time Warner Inc (TWX), Weatherford International, Ltd (WFT), dan Yahoo! Inc. (YHOO) are compared to option prices in the marketpresented in appendix 3, below.

**Appendix 3**

**Table Comparison of 10 call option prices of stocks with**  **= 0,25% and maturity timeMarch 19, 2010**

| **No** | **Stocks** | **S0** | **K** | **Price of Market Option** | **BS Price****(XB)** | **SN Price****(XSN)** | **Error of BS****(X-XB)2** | **Error of SN** **(X-XSN)2** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **(X)** |
| 1 | D | 39,11 | 35 | 4 | 41,152 | 41,151 | 0,013271 | 0,013248 |
| 2 | TYC | 37,51 | 31 | 4,4 | 65,143 | 62,138 | 44,702,645 | 32,898,704 |
| 3 | HD | 31,8 | 34 | 0,02 | 0,063258 | 0,048302 | 0,0018713 | 0,000801 |
| 4 | L | 37,5 | 30 | 5,3 | 75,042 | 66,834 | 48,584,976 | 19,137,956 |
| 5 | Q | 4,66 | 5 | 0,03 | 0,036271 | 0,034518 | 3,93E-05 | 2,04E-05 |
| 6 | TWX | 30,54 | 33 | 0,05 | 0,082985 | 0,051699 | 0,001088 | 2,89E-06 |
| 7 | WFT | 17,38 | 19 | 0,11 | 0,14228 | 0,11163 | 0,001042 | 2,66E-06 |
| 8 | CHK | 26,31 | 17.5 | 9,37 | 8,81E+00 | 8,90E+00 | 0,3109178 | 0,2179956 |
| 9 | M | 20,45 | 12 | 4,8 | 8,45E+00 | 7,91E+00 | 13,334,913 | 96,814,323 |
| 10 | JNJ | 64,04 | 65 | 0,22 | 0,3206 | 0,27984 | 0,0101204 | 0,0035808 |
|  |  |  |  |  |  | **SSE** | 23 | 151,207 |
|  |  |  |  |  |  | **MSE** | 23,002 | 15,121 |

Notes:

XB= Option price of Black Scholes

XSN = Option price of SN Approach

* end of appendix 3*

From these results, by taking 10 exercise prices () for days, it seems that the price of the option with SN approach gives results that are relatively closer to the option price in the market when compared

to the option price obtained by the approach of the Black-Scholes model. In addition, the SNapproachmodel also gives the value of mean of square error (MSE) is relatively lower than the Black-Scholes model.

Based on the results obtained, SN approach was better able to explain the

existence of skewness in the logreturn data than the Black-Scholes model.

As noted previously, the Black-Scholes model is less able to capture the skewness of logreturn data. This is caused by the distribution used in the model does not have skewness parameter as in the SN which is able to capture the skewness in the data.

**4. CONCLUDING REMARKS**

 Based on result of writing that has been stated previously, we can conclude that Skewed Normal Distribution (SN) is a special case of Skewed Generalized Error Distribution for parameter value *k =2.*

Empirical results show that option pricing with SN approach results option price which is closer to the market price when compared to the option price obtained by Black-Scholes model.This caused by the distribution is used in the model does not have skewness parametersuch as in SN which is able to capture skewness in the data.

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