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WEBSITE RANKING ESTIMATION THROUGH FUZZY LOGICAL HIERARCHY PROCESS

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Abstract

The purpose of this paper is to provide case study on website evaluation and ranking by fuzzy analytic hierarchy process (FAHP). Many people have misconception about website evaluation and ranking mistaking good design and user interface alone as high ranking. To measure the website quality, various factors are taken into consideration. The quality metrics that are considered for evaluating website are accessibility, performance, usability, search engine optimization (SEO), website visibility, security, technology and design used for developing website. Analytic hierarchy process (AHP) breaks down complexity into simple hierarchical decision making methods. Fuzzy AHP is a multi-purpose judgement technique which is often used to guide website creation and can further be used to find out website ranking. This approach tells the ranking of website based on multiple conflicting criteria of the website. But all these parameters and metrics considered are qualitative and not measurable which is slight challenging to deal with traditional theory. The drawbacks of AHP is eliminated by use of FAHP. Here, fuzzy decision matrix is constructed by using multi-criteria decision making (MCDM) approach and final weight is calculated of each website based on their qualities.

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1. INTRODUCTION

The Analytical Hierarchy Processes AHP, introduced by Saaty (2001) , is studied extensively and used in the applications where problems related to multiple criteria decision making arises. Aikhuele, D.O *et al,* (2014) presented a framework for evaluating and ranking the contribution of critical success factor based on successful implementation of lean production technique with the help of Fuzzy Analytic Hierarchy Process (FAHP) approach (Aikhuele *et al*, 2014; Ribeiro, 1996). Feng Kong and Hongyan Liu (2005) aimed to find out the factors affecting success in E-commerce by using Fuzzy Analytic Hierarchy process. Also they evaluated method for E-commerce which can help the researchers to determine opportunities and drawback (Kong *et al*, 2005; Bard, 1990). F. Tunc Bozbura *et al,* (2007) prioritized human capital (HC) measurement indicators using fuzzy AHP. They defined a methodology to increase the quality of prioritizing HC measurement indicators under fuzziness. The attributes considered were talent. Strategical integration, cultural relevance, knowledge management and leadership. William Ho (2008) proposed five tools that commonly can combine with AHP include quality function deployment (QFD), mathematical programming, meta-heuristics, data envelopment analysis (DEA) and Strength, Weaknesses, Opportunity, Threats (SWOT) analysis. The AHP earned lot of applications in the field of engineering, personal and social categories to take meaningful decisions.

In general term the quality of website can be expressed as best, better, good, bad and worst. These five different approaches are being evaluated for this paper to check the efficiency of the Fuzzy AHP approach. In this paper we used FAHP method to evaluate website based on accessibility, performance and design & content. The pairwise is done between each alternative to alternative for each parameter and weights are obtained which decides the ranking of website.

The research work can be extended in future by considering more number of website for evaluation using various parameters.

METHODS

This section starts with basic definitions of fuzzy set theory and fuzzy numbers and further gives methods and a demonstration of its application.

Definition 2.1.1. A fuzzy set $\tilde{A}$ is a subset of a universe of discourse *X*, which is characterized by a membership function $\mu_{\tilde{A}}(x)$ representing a mapping $\mu_{\tilde{A}} : X \to [0,1]$. The function value of $\mu_{\tilde{A}}(x)$ is called the membership value, which represents the degree of truth that *x* is an element of fuzzy set $\tilde{A}$.

Definition 2.1.2. A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A} : R \to [0,1]$ has the following characteristics,

2.1.2.1. $\tilde{A}$ is convex.

$$\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)),$$

$$\forall x \in \left[x_1, x_2\right], \lambda \in \left[0,1\right].$$

2.1.2.2. $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(x) = 1$.

2.1.2.3. $\tilde{A}$ is a piecewise continuous.

Definition 2.1.3. A triangular fuzzy number $\tilde{N}$ can be defined as a triplet (l, m, r) and the membership function $\mu_{\tilde{N}}(x)$ is defined as:

$$\mu_{\tilde{N}}(x) = \begin{cases} 0 & x < l \\ \left(\dfrac{x-l}{m-l}\right) & l \leq x \leq m \\ \left(\dfrac{r-x}{x-m}\right) & m \leq x \leq r \\ 0 & x > r \end{cases}$$

Where l, m, r are real numbers and $l \leq m \leq r$.

Definition 2.1.4. A linguistic variable is a variable whose values are linguistic terms (Zadeh, 1975; Bouyssou, 2000).

2.2. Multi Criteria Decision Making (MCDM) Method

Multi criteria decision making (MCDM) is one of the method which can be used for evaluating the website quality and finally decide the ranking of website based on various parameters considered (Pakkar, 2014; Kong, 2005). Main features of MCDM are multiple attributes often for a hierarchy, conflict among criteria, hybrid nature, uncertainty, large scale assessment may not be conclusive, etc. Since the above mentioned qualities for website evaluation are qualitative, we need fuzzy theory for explaining in form of fuzzy linguistic variables.

In general term the quality of website can be expressed as best, better, good, bad and worst. Five different is being evaluated for this paper to check the efficiency of the Fuzzy AHP approach. The research work can be extended in future by considering more number of website for evaluation using various parameters.

2.3. Fuzzy Analytic Hierarchy Process (FAHP) Method

This method is popular analytical technique known for solving decision making problem. It consists of three main operations which are priority analysis, hierarchy construction and consistency verification. Initially, decision makers need to break down complex decision problems into its component parts of which all possible attributes are arranged into multiple hierarchy levels. Then decision makers are compared with each cluster in the similar level in a pair wise fashion based on their own knowledge and skill. The output of AHP is ordered ranking specifying the complete preference for each of the decision alternatives ultimately help the decision maker for selecting the best approach.

The Fuzzy AHP method is developed from the AHP method and now it's proved as an advanced analytical method. This method is systematic method to the alternative selection and justification problem with the ideas of fuzzy set theory and hierarchical structure analysis. Decision makers generally find that it is more assured to give interval decisions than fixed value decisions. The fuzzy comparison fractions are taken into consideration to be able to bear vagueness. Decision maker uses the uncertainty while accomplishing the comparisons of the alternatives. For consideration ration fuzzy numbers are used instead of crisp numbers. The technique proposed by Chen and Hwang (1992) involves following steps:

2.3.1. Changing linguistic terms to fuzzy numbers: At this stage linguistic terms are converted into respective fuzzy numbers. It comprises of eight conversion scales. The conversion scales were suggested by synthesizing and modifying the works (Boender, 1989; Hwang, 1992).

2.3.2. Converting Fuzzy Numbers to Crisp Scores:- This step uses a fuzzy scoring approach that is a modification of the fuzzy ranking approaches proposed by Chen (1992).The crisp score of fuzzy min and fuzzy max of fuzzy number "M" is obtained as follows:

$$\mu_{max}(xx) = \begin{cases} x, 0 \leq x \leq 1 \\ 0, otherwise \end{cases}$$

$$\mu_{min}(x) = \begin{cases} 1 - x, 0 \leq x \leq 1 \\ 0, otherwise \end{cases}$$

y numbers are well-defined in a manner such that absolute position of fuzzy numbers can be spontaneously incorporated in the comparison cases.

The right score of individual fuzzy number Mi is termed as :-

$$\mu_R M1 = Sup\ [\mu_{max}(\chi) \wedge \mu M1(\chi)]$$

And the left score is:

$$\mu_L M1 = Sup[\mu_{min}(\chi x) \wedge \mu M1(x)]$$

The total score of a fuzzy number Mi is termed as: -

$\mu TM1 = [(\mu RM1 + 1 - \mu LM1)\ /2]$

2.3.3. Demonstration of the process: -

5-point scale is taken into consideration to present the conversion of fuzzy number into crisp scores. To demonstrate this method, a 5-point scale linguistic terms to determine the website quality are considered like worst, bad, good, better and best as shown in Figure 1.

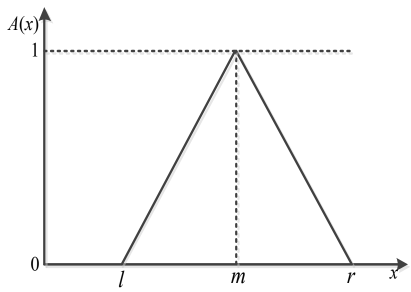

Figure 1: Triangular Fuzzy Number

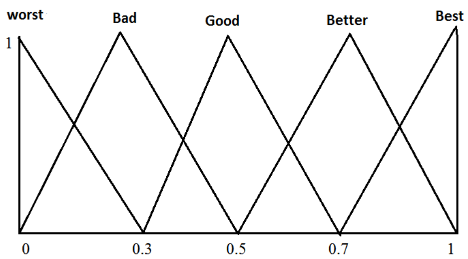

Figure 2: Linguistic classification

Table 1: Point scale of linguistic terms to fuzzy number conversion.

Linguistic Terms	Fuzzy Number
Worst	M1
Bad	M2
Good	M3
Better	M4
Best	M5

From Figure 1, membership functions of M1, M2, M3, M4 and M5 are written as:

$$\mu_{M1(x)} = \begin{cases} 1, & x = 0 \\ \frac{0.3-x}{0.3}, & 0 \le x \le 0.3 \end{cases}$$

$$\mu_{M2(x)} = \begin{cases} \frac{x-0}{0.25}, & 0 \le x \le 0.25 \\ \frac{0.5-x}{0.25}, & 0.25 \le x \le 0.5 \end{cases}$$

$$\mu_{M3(x)} = \begin{cases} \frac{x-0.3}{0.2}, & 0.3 \le x \le 0.5 \\ \frac{0.7-x}{0.2}, & 0.5 \le x \le 0.7 \end{cases}$$

$$\mu_{M4(x)} = \begin{cases} \frac{x-0.5}{0.25}, & 0.5 \le x \le 0.75 \\ \frac{1.0-x}{0.25}, & 0.75 \le x \le 1.0 \end{cases}$$

$$\mu_{M5(x)} = \begin{cases} \frac{x-0.7}{0.3}, & 0.7 \le x \le 1.0 \\ 1, & x = 1.0 \end{cases}$$

The right, left and total scores are computed as follows for M1

$\mu R\ (M1) = Sup\ [\mu max\ (x)^\wedge\ \mu M1(x)] = 0.21$; $\mu L\ (M1) = Sup\ [\ \mu min\ (x)^\wedge\ \mu M1(x)] = 0.93$

$\mu T\ (M1) = [\mu R\ (M1) + 1 - \mu L\ (M_1)] = 0.14$

Similarly, the left, right and total scores are computed for M2, M3, M4 and M5 and are populated in Table 2 (a). Instead of allocating arbitrary values for several attributes, this fuzzy method reflects the exact linguistic reports in terms of crisp scores. Hence, it gives better estimation that are widely used.

Table 2(a): Membership function of M1, M2, M3, M4 and M5

	$\mu_R(M_I)$	$\mu_L(M_I)$	$\mu_T(M_I)$
1	0.21	0.93	0.14
2	0.25	0.75	0.25
3	0.63	0.77	0.43
4	0.75	0.38	0.68
5	0.93	0.21	0.86

Table 2(b): Linguistic terms with their corresponding crisp scores.

Linguistic term	**Fuzzy Number**	**Crisp Score**
Worst	M1	0.14
Bad	M2	0.25
Good	M3	0.43
Better	M4	0.68
Best	M5	0.86

RESULTS

The hierarchy to evaluate website ranking based on various parameters is shown in Figure 3, at the top of hierarchy the objective is placed to decide ranking with three different parameters at the next level, these are: accessibility, performance and design and content. At last the various attributes: website 1, website 2 and website 3 are placed. After hierarchy formation for the problem fuzzy AHP method can be applied to evaluate website ranking which consists of various steps, these are:

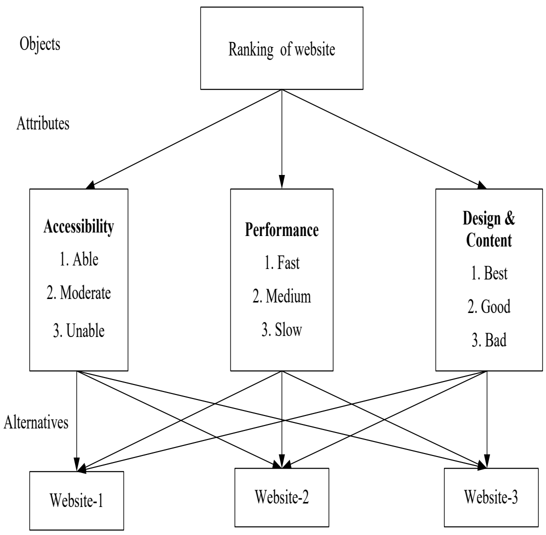

Figure 3: Hierarchy of evaluation of website

Step 1: Various parameters and linguistic terms along with alternatives are populated in Table 3. For calculation purpose in this paper, we have included only three parameters. Problems can be extended by adding more number of parameters and alternatives in the same manner and can be used in real sense for complex decision making process.

Table 3: Various parameters and alternatives for fuzzy AHP

	Accessibility	**Performance**	**Design & Content**
W_1	Able	Fast	Best
W_2	Moderate	Medium	Good
W_3	Unable	Slow	Bad

Instead of considering 5-point scale as explained above, we have considered 3-point scale for fuzzy linguistic term into crisp conversion also corresponding crisp value is calculated in a similar way which is illustrated in Table 4 . Various linguistic terms of three different parameters are also displayed in Table 3. According to Chen and Hwang (1992) method, membership function of M1,M2 and M3 can be written as:-

$$\mu_{M1(x)} = \begin{cases} 1, & x = 0 \\ \frac{0.3-x}{0.3} & 0 \le x \le 0.3 \end{cases}$$

$$\mu_{M2}(x) = \begin{cases} \frac{x-0.3}{0.2}, & 0.7 \le x \le 1 \\ \frac{0.7-x}{0.2}, & 0.5 \le x \le 0.7 \end{cases}$$

$$\mu_{M3}(x) = \begin{cases} \frac{x-0.7}{0.3}, & 0.7 \le x \le 1 \\ 1 & , x = 1 \end{cases}$$

Table 4: Crisp Conversion

Websites	**Accessibility**	**Performance**	**Design &Content**
W1	0.33	0.78	1
W2	0.85	1	0.56
W3	1	0.35	0.42

Step 2: **Checking consistency:** At this step consistency ratio (CR) is checked to know whether the considered weights and results by an expert or user is correct or not, usually the value is less than 0.1 which proves that the weights are consistent. For comparing the parameters, a relative matrix to assign weights is shown below. In this matrix, diagonal elements are always one because the parameters compared with same will

be always be equal to 1. Also $a_{ij}=a_{ji}$, where a is the matrix element of criteria matrix C.

Step 2.1: A = access perform D&C

$$\begin{bmatrix} 1 & 6 & 4 \\ \frac{1}{6} & 1 & \frac{1}{3} \\ \frac{1}{4} & 3 & 1 \end{bmatrix}$$

Sum: 1.417 10 5.33

Step 2.2: Calculation of normalized Matrix:

$$\begin{bmatrix} \frac{1}{1.417} & \frac{6}{10} & \frac{4}{5.33} \\ \frac{1}{6}*\frac{1}{1.417} & \frac{1}{10} & \frac{1}{3}*\frac{1}{5.33} \\ \frac{1}{4}*\frac{1}{1.417} & \frac{3}{10} & \frac{1}{5.33} \end{bmatrix}$$

$$= \begin{bmatrix} 0.706 & 0.6 & 0.705 \\ 0.118 & 0.1 & 0.0625 \\ 0.176 & 0.3 & 0.188 \end{bmatrix}$$

Step 2.3: Calculate parameter weight by taking average of each rows as shown below.

W=

$$\begin{bmatrix} \frac{(0.706+0.6+0.750)}{3} = 0.6853 \\ \frac{(0.118+0.1+0.0625)}{3} = 0.0935 \\ \frac{(0.176+0.3+0.188)}{3} = 0.2213 \end{bmatrix}$$

Step 2.4: Determinant of A = A - λi =

$$\begin{vmatrix} 1-\lambda & 6 & 4 \\ \frac{1}{6} & 1-\lambda & \frac{1}{3} \\ \frac{1}{4} & 3 & 1-\lambda \end{vmatrix}$$

= *(1- λ)* [(1- λ) (1- λ) - 1] – 6 [(1/6) (1- *λ*) - 1/4*1/3] + 4 [(1/6)*3 – (1/4)*(1- *λ)*]

= (1- *λ*) [1 - 2*λ* + λ^2 – *1*] – 6 [(1/6) – (1/6) *λ* – (1/12)] + 4 [(1/2) – (1/4) + (1/4) *λ*]

$$\Rightarrow -2\lambda^3 + 6\lambda^2 + 1 = 0$$

After solving the above equation, we get λ values as follows:

λ = 3.0542

λ = -0.271+0.4047 i

λ = -0.0271-0.4047 i

From the above Eigen values, we need to get "λmax". Therefore, λmax = 3.0542

Then the consistency index is calculated using formula:

$$CI = \frac{\lambda - n}{n-1}$$

Where n is number of criteria.

$$\text{CI} = \frac{3{\cdot}0542-3}{3-1} = 0.0271$$

Now consistency ratio CR is calculated as: $CR = \frac{CI}{RI}$

Where RI is Random Index already given for specified number of criteria for three criteria value is 0.52.

$$CR = \frac{0.0271}{0.52} = 0.052 < 0.1$$

Since the value of CR is less than 0.1 hence the weights are consistent.

Step 2.5: Now we calculate for Alternatives among Website 1, website 2 and website 3.

Accessibility: Pair wise comparison:

Website1 *Website2* *Website3*

$$\begin{bmatrix} 1 & 0.653 & 0.898 \\ \frac{1}{0.653} & 1 & 0.321 \\ \frac{1}{0.898} & \frac{1}{0.321} & 1 \end{bmatrix}$$

Sum 3.644 4.768 2.219

As Shown above similarly normalized matrix for accessibility parameter matrix:

$$\begin{bmatrix} 0.2744 & 0.1369 & 0.4046 \\ 0.4202 & 0.2097 & 0.1446 \\ 0.3055 & 0.6534 & 0.4506 \end{bmatrix}$$

Similarly, weight matrix for accessibility parameter is calculated as:

Weight matrix W=

$$\begin{bmatrix} \frac{(0.2744 + 0.1369 + 0.4046)}{3} = 0.2719 \\ \frac{(0.4202 + 0.2097 + 0.1446)}{3} = 0.258 \\ \frac{(0.3055 + 0.6534 + 0.4506)}{3} = 0.4698 \end{bmatrix}$$

Therefore, W for accessibility is = $\begin{bmatrix} 0.2719 \\ 0.258 \\ 0.4698 \end{bmatrix}$

By Following the same steps for other two parameters (performance and design & content) we calculated the individual weight matrix for parameter performance and design & content:

W for performance is = $\begin{bmatrix} 0.1359 \\ 0.3562 \\ 0.0935 \end{bmatrix}$

W for design and content is = $\begin{bmatrix} 0.263 \\ 0.335 \\ 0.195 \end{bmatrix}$

A matrix is formed by obtaining weights in case of pairwise comparison matrix for three parameters as calculated above.

$[w]^T$ * [Accessibility Performance Design & Content]

$$=[0.6853 \quad 0.0935 \quad 0.2213] * \begin{bmatrix} 0.2179 & 0.1359 & 0.263 \\ 0.258 & 0.356 & 0.335 \\ 0.4698 & 0.0935 & 0.195 \end{bmatrix}$$

[Website 1 Website 2 Website 3] = $[0.2774 \quad 0.1471 \quad 0.2547]$

Deciding the rank according to the higher value of above matrix, hence ranking is $Website_1$, $Website_3$ and $Website_2$.

CONCLUSION

At each and every stage of life we need to take some decision for betterment purposes. Decisions vary from economical, social, technological and even political. It is important for an individual to make correct decisions. So we need to introduce a technique which can help to take wise decision when there are conflicting criteria in nature.

From this paper we came to know that AHP can be proved to be a best method for ranking the website based on various parameters. From this we came to know that AHP can be applied to wide-ranging fields. In standard AHP directly the numerical values of linguistic variables have been used for evaluation. If the environment of decision making takes place is fuzzy, then fuzzy numbers are considered for evaluation. Regarding some deviation of decision makers. For this type of situation, Fuzzy AHP can be able to deal with the problem.

In this paper we used FAHP method to evaluate website based on accessibility, performance and design & content. The pairwise is done between each alternative to alternative for each parameter and weights are obtained which decides the ranking of website.

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