Misconception as a critical and creative thinking inhibitor for mathematics education students

Rochmad*a, Muhammad Kharis*a, Arief Agoestanto*a, Muhammad Zuhair Zahid*a, Mashuri*a

*a Universitas Negeri Semarang, D7 Building First Floor, Sekaran Campus Gumngpati, Semarang 50229, Indonesia

* E-mail address: rochmad@mail.unnes.ac.id

Abstract

The accurate understanding of critical thinking and mathematical creativity in solving the current problem is still difficult to standardize. These two thinking skills are indispensable to anyone who is studying mathematics, especially for Undergraduate Mathematics students who are studying Linear Algebra. However, the difficulties in critical thinking discourage students to think creatively and mathematically. In linear algebraic subject matter, many problems require critical reasoning. It goes without saying that the difficulties in various reasoning aspects critically cause other difficulties in developing creative thinking aspects. Further, mathematical critical thinking skills in solving problems require a background in understanding the concepts related to the problem faced. In addition, the failure to understand and connect between concepts in solving linear algebra problems makes it worst and difficult to critically and creatively think.

© 2018 Published by Mathematics Department, Universitas Negeri Semarang

1. Introduction

In solving mathematics problems, students are required to understand the concepts which are related to the problems encountered. Students who lack of understanding the concepts will be hampered in developing their critical thinking skills in solving the problem. While students who are stuck in critical thinking will be hampered in developing their creative thinking skills. Mathematics education experts attempt to define concepts from different points of view. The concept is a tool used to organize knowledge and experience into various categories constructed by making connections between new information and conceptual networks or existing mental structures (Arends, 2008; Woolfolk & Margetts, 2013; Carpenter et al., 1988; Zahid & Sujadi, 2017). Gagne, as quoted by Nasution (2000) suggests that if one can deal with objects or events as a group, class, class or category, then he has learned the concept. Concrete concepts can also be obtained through observations in which it can be shown "what is the object". In consequence, it causes in the use of an inductive mindset in constructing concepts which are based on observations on specific cases given. As Slavin (2005) argues that concepts are generalized abstract ideas of specific examples.

A learner at a higher level may construct abstract concepts, for instance concepts in the form of definitions, such as the definition of "solution of an equation system", the definition of "vector space of a non-empty set", and the definition of "linear transformation of a vector space to another vector space". A new concept can be learned and then stored in a person's mind in long term memory. It will be better embedded in a longer time if the concept can be attributed to the concept which possesses and has already existed in his mind (Rochmad, 2010).

Besides, various definitions of critical thinking also have been delivered by many experts. According to Van de Walle (2007), critical thinking is a directional and clear process used in mental activities such as problem solving, decision making, analyzing assumptions, and conducting scientific investigations. By using critical thinking skill, it allows students to systematically study the

To cite this article:
problem, deal with challenges in an organized way, work on problems in various ways, design original solutions, and develop or more detail in their thinking. In other words, in this circumstance, it is highly necessary to think critically.

Regarding to preliminary research, this study discusses the deliberation of critical thinking which becomes the cause of the delay in critical thinking in solving algebra problems in the Linear Elementary 2 course in Bachelor Degree of Mathematics Education study program of Universitas Negeri Semarang.

2. Method

This study is a qualitative research which took 36 participants of Elementary Linear Algebra 2 subject as research subject. In collecting the data, this study used written test, observation, and interview method. Interview was used as clarification of student answers to their written answers, as well as triangulation which focuses to find out the connection between conceptual error and critical thinking.

To obtain data of the relationship between conceptual ability and critical thinking ability, a written test was done with the following questions.

1. a. Write the complete definition of linear transformation.
   b. Investigate whether \( W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ad - bc = 0 \right\} \) is a subspace of \( M_{2 \times 2}(R) \).

2. Given that set \( S = \{v_1, v_2, v_3, v_4\} \) with \( v_1 = (1,0,1,1), v_2 = (-3,3,7,1), v_3 = (-1,3,9,3), \) and \( v_4 = (-5,3,5, -1) \). Find the subset of \( S \) which forms the basis for space spanned by \( S \). What is the dimension?

3. Review B base = \( \{p_1, p_2\} \) and base \( B' = \{q_1, q_2\} \) with \( p_1 = 6 + 3x, p_2 = 10 + 2x, q_1 = 2, \) and \( q_2 = 3 + 2x \).
   a. Find the transition matrix from B to B'.
   b. Calculate the coordinate matrix [p]\( B' \) with \( p = -4 + x \).

4. a. Write the complete definition of linear transformation.
   b. Investigate whether \( F: P_2 \rightarrow \) which is defined as \( F(a_0 + a_1x + a_2x^2) = a_0 + a_1(x + 1) + a_2(x + 1)^2 \) is a linear transformation.
   c. Let’s say T is the multiplication by the matrix \( \begin{pmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{pmatrix} \) look for T nullity.

5. a. Write the definition of a matrix diagonalizable.
   b. Investigate whether the matrix \( A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \) can be diagonalized. If yes, then find the matrix \( P \) and determine \( P^{-1}AP \).

The analysis is based on misconception indicators as follows: (1) inaccurate concepts definition, (2) improper or false the use concepts, (3) classifying incorrect examples of concepts, (4) misinterpretation of concepts with the meaning of the concept, (5) confusion because does not master the supporting concept yet; and (6) improperly linking the concept. In addition, critical thinking aspects which are observed include the ability: (1) to think in understanding and clarification; (2) to think in conducting assessment problem; and (3) to make inferences in problem solving. According to Perkins & Murphy (2006), critical thinking skills are often cited as aims or outcomes of education. So that the learning process in the school should be planned to help learners improve their critical thinking skills. Above all, in this study, critical thinking indicators refer to those which are proposed by Perkins and Murphy (2006) namely clarification, assessment, inference, and strategies.

3. Results & Discussion

Firstly, the analysis was done toward the result of 36 students’ works on Linear Elementary Algebra 2. Based on the analysis results, it was found that the achievement index (IP) of 22 students can be categorized thoroughly in the course. From the obtained data, there are 4 students who get the value of 86 above with the IP acquisition of A, 2 students who get the value from 81 to 85 with IP acquisition of AB, 7 students with the value from 71 to 80 with IP acquisition of B, 4 students who get the value of 66 up to 70 with the IP acquisition of BC, and 4 students who get the value from 61 to 65 with the IP acquisition of CD, 6 students with value from 51 until 55 with IP acquisition of D, and 6 others get IP acquisition of E with value less than 51. Overall, the average value of students’ works result in linear elementary algebra 2 is 61,33. If the value is converted into IP scoring system then

\[ 58 \]
obtained IP of C, so that it can be categorized completely.

Furthermore, a related-qualitative analysis of conceptual difficulties and critical and creative thinking skills was conducted. In solving students’ algebra problems, it is involved the understanding and mastery of algebraic concepts which was going to be used. However, the difficulties in understanding concepts or connecting between concepts hampers in critical and creative thinking. Nevertheless, the main conceptual difficulties directly impact on the difficulty in critical thinking. The following information relates to some misconceptions of students in solving problems and their relation to critical thinking.

Based on the analysis of the written test results of the students in defining the subspace of a vector space; defining a linear transformation; and defining the diagonalizable matrix results in the following causes of misconceptions as follows.

3.1. Do not know the concept in question
When the students were asked to write the definition of linear transformation; they did not answer at all. For example for question 4a: "Write down the definition", yet not write anything; it indicates that the student does not know the concept. From this ignorance, the student probably forgets how the concept of linear transformation is. Also, they did not answer when they were asked to write the definition of a matrix which is can be diagonalized (question 5a): "Write the definition of a matrix which can be diagonalized"

3.2. The concept was incorrectly answered.
3.2.1. Confusion: answering the concepts that he feels to know or memorize, yet the answer is out of expectation.

Based on the data in Figure 1, from question 1a, student has to answer the question relates to a rule which can be used to indicate a subspace of a vector space; in fact, he answers the definition of vector space, indeed the answer is wrong. This student’s confusion is also predicted caused that he lacks of mastery of other concepts such as indicated by writing \( \{v_1, v_2, v_3, \ldots, v_n\} \) which contains \( \mathbb{R}^n \). The vector space concept error that students think if it is linear based. Thus, they suffer from misconceptions due to confusion: about the concept which is being asked, about the concepts that are supposed to support it; in relating one concept and others. As a result of this misconception, his confusion continues. He cannot work correctly with question 1b, which does not use the concept which he wrote at 1a.

3.2.2. The existence of overlapping knowledge and unable to sort it out.

Firstly, students have to understand what is being asked, write down what will be defined that is vector subspaces (usually called subspaces only), yet they remember about other concepts that encompass it namely 10 vector space axioms, consequently, he is carried toward the concept of vector space axioms. The Figure 2 is as an illustration of student misconceptions.

Figure 1. Student’s confusion in answering

Figure 2. Overlapping Misconceptions

The student's mindset is first when entering the subspace sphere of vector space \( V \); they have already focused their thinking on added ”+” and the multiplication results scalar \( \alpha \) with vectors; but they enter the realm of vector space axioms; cannot sort it out so it does not return to the subspace sphere. It is also illustrated in figure 3.

Figure 3. Students have difficulty connecting between concepts
In answering this question, at first student’s thought was M is outside V (also outside W). After he understood the problem he made the mathematical symbols of the vector space and its subspace represented by W. He made V as vector space and W as the subset of non-empty V. He had known that he had to find the rules (definitions or concepts) that can be used to show that W is a subspace of vector of V. The position of M is at 1 that is in the set of W. He wanted to answer the question: how the theorem which can be used to determine that W is a subspace of V. Then he remembered that the vector space must have two operations: addition and multiplication operation with scalar. Above all, misconception occurs when student was reminded of 10 vector space axioms; it means that his thought is in position 2 out of W into V and tried to find a match for the definition.

Basically the third line written by the student, \(\alpha(a + b), \forall a, b \in V; \alpha \in R\), is a rule (definition) to show that W is a subspace of V. The overlap concepts affect the student being unable to sort them out of 10 vector space axioms (that only 1 and 6 axioms and if it is filled, it indicates that W is a subspace vector of V), as the result misconception occurs. The rules obtained were used to solve the question number 1b, in which it went without saying resulted in a wrong solution.

According to Smolleck & Hershberger (2011), the term of misconception is used to describe situations in which student’s ideas about concepts are different from scientists. The difference between theoretical concepts and the imprecise notions of the scholarship leads to misconceptions. Meanwhile, according to Luz, et al (2008), misconceptions are understood as ideas that differ from those which are received by experts, yet constantly held by students as a result of repeated experiences with their daily phenomena. The use of wrong concepts stored in their minds which affects the occurrence of mathematical misconception.

Moreover, concepts in mathematics are abstract ideas that can be used, enable and facilitate people in grouping an object or event into the sample or not. In mathematics learning, including linear algebra, students should understand the concept first; and sometimes the concept is hierarchically arranged. However, difficulties in understanding concepts (misconceptions) will hamper their critical and creative thinking. According to Urban (2005), to test the traditional creative thinking ability, all this time they are only given a quantitative information about creativity which is obviously less precise. Indeed, qualitative aspects need to be put forward in testing students’ critical and creative thinking skills. Further, the analysis of creative thinking is based on indicators of creative thinking, as follows: (1) clearly; (2) flexibility; (3) originality; and (4) elaboration. While concepts are the building blocks of thinking, the basis of the higher mental processes id to formulate principles and generalizations. To solve a problem, a student has to know the relevant rules which are based on critical and creative thinking aspects.

Regarding to above explanation, this study is concerned with the effect of creative activities on high thinking skill level. Students who are taught and given creative activity (instruction with creative activity) have a higher thinking skill better than those who are taught without creative activity (instruction with no creative activity). However, the final test results of both groups are not significantly different there was no significant difference between pre-test and post test of the two groups (Ramirez & Ganaden, 2008).

Regarding to explanation above, the participation of the undergraduate students of Mathematics Education is very important in the formation of creative young generation, capable of producing something for themselves, others, and their environment. Creative is also intended for prospective mathematics teachers to do learning to solve various problems which fulfill various aspects of creative thinking. According to Storm (Sharwa, 2014), the end of creative thinking is a major concern in the world. The role of learning in developing students’ thinking skills, such as creative thinking, is an important aspect that contributes to the success of mathematics education. According to Sharma (2014), in education, creativity should include a variety cognitive and skills-based knowledge, as well as the development of students’ interests, values and beliefs in creative activities.

To cultivate critical mathematical thinking skills, math learning is needed which involves students’ thinking in every learning process. As Duron et al. (2006) argue that it would be difficult to cultivate critical thinking skills when only using teacher-centered learning. A suitable lesson to develop students’ critical thinking is learning that uses a student-centered approach.

Another opinion from Jacob and Sam (2008) in the same issue that is the process of critical thinking of students is the stages experienced by students to solve open problems. This study refers to Jacob & Sam (2008) who define 4 stages of
critical thinking process, as follows: (1) clarification which is a phase stage in which student formulate problem correctly and clearly; (2) assessment which is the stage in which students find the important questions in the problem; (3) inference which is the stage in which students make inferences based on information that has been obtained; (4) strategy which is the stage in which students think openly in solving the problem. According to Fascione (2011), someone who has critical thinking ability can be indicated through ability of (1) interpretation, (2) analysis, (3) evaluation, (4) inference, (5) explanation, and (6) self-regulation.

According to research conducted by Recio & Godino (2001), it can be assumed that there are still many college students in the first semester who think as concrete as in operation phase with inductive reasoning and less able to learn mathematics by using deductive mindset. As Recio and Godino explain that the ability of critical and creative thinking of undergraduate students of Mathematics Education is low. Based on the results of preliminary studies, it is pointed out that the students’ lack of criticism is caused by the inaccuracy in changing from written language into the language of mathematics.

Though Winn (2004) argues that teachers should teach critical thinking. The disposition of critical thinking and problem-solving skills become essential to daily life. Winn states that few teachers use and discuss strategies which lead to building students’ creative thinking. To understand a topic, students must be able to think freely and apply the skills obtained from learning skills (Saurino, 2008). For example, class writing activity is one way to understand the concepts and structure of mathematics (Consiglio, 2003).

Again, Facione (2011) argues that critical thinking as a skill with the self-realization of self-regulation in giving reasoning considerations to the evidence, context, standards, methods, conceptual structures by which a decision made about what is believed and distrusted. A broader understanding of critical thinking encompasses the characteristics of critical thinking which involves inductive and deductive reasoning, reflective thinking, dialectical thinking, and problem solving (Chan, Dixon, Sullivan, Tang, & Tiwari, 1999).

4. Conclusion

Based on the description of analysis, it can be concluded that generally, the undergraduate students of Mathematics Education in following the Linear Elementary Algebra 2 course have to get mastery in learning. Some students have difficulty in critical thinking. This difficulty makes them difficult to think creatively. These difficulties are caused by lack of understanding about the underlying concept of the problem, or difficulty in connecting between mathematical concepts.

References


