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Students critical thinking skills toward concepts differences in finding area of a plane region and definite integral

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Abstract

This study aimed to describe students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral. This study used an exploratory test survey method with test instruments. Data were taken from 40 students of the mathematics department at a university in Central Java. The results showed that students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral were in the medium category. The students' critical thinking skills towards the concepts differences in finding area of a plane region and definite integral were medium (47.5%), with clarification by 57.5% (medium), assessment by 40.0% (medium), inference by 65.0% (medium), and strategies by 27.5% (low). These weaknesses are expected to be followed up by conducting learning that can show the linkages between the concepts and with various ways.

1. Introduction

Mathematics is applied directly or indirectly in human's life. It has an essence role in life. Mathematics has developed since the beginning of human existence and will continue to grow in the future for the benefit of humankind. Mathematics was developed in scientific research, and the result was the development of science and technology for humans. Mathematics and other sciences together have many benefits in facilitating human life, namely technology to support the activities of human life. Mathematics also exists in every human thought processes such as in the process of logical, systematic, critical, and other thinking. Formally, mathematics lessons are given at school.

Mathematics learning at schools has a goal to improve students' ability to think logically, analytically, systematically, critically, and creatively. Students are expected to have thinking skills that will help them make strong decisions to acquire new knowledge quickly, such as critical thinking skills that are urgently needed in the 21st century (Lau, 2011; Kharbach, 2012; Fuad et al., 2017). In elementary, secondary, and higher education learning, students are given a variety of mathematical material. Calculus is one of the mathematical material which is the development of arithmetic, algebra, and geometry. Calculus is given to students in secondary and higher education. Like mathematics, calculus learning has the aim of increasing students' ability to think logically, analytically, systematically, critically, and creatively.

Calculus has special characteristics, namely learning about change. These characteristics make calculus very useful for solving problems in various fields of human life, such as in physics, biology, economics, medicine, and others. The main focus of the calculus is divided into two, namely differential calculus and integral calculus. In differential calculus learning, the discussed matters are the real number systems, functions, limits, continuity, derivatives, and derivatives applications. Meanwhile, in integral calculus learning, the topic discussed are indefinite integrals, sigma notation and Riemann sums, definite integrals, and integrals applications.

In addition to its usefulness in human's life, calculus has a big challenge in the learning process, which is a lot of difficulties experienced

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by students in calculus learning. Some of these difficulties are in drawing graphics functions, solving infinity problems, determining what to prove, making paths or proof algorithms, and exploring the problems given, especially the problems of applying differential calculus and integral calculus.

The following are various difficulties which are common problems in calculus learning; (1) the concept of infinite numbers (Tall, 2001), (2) drawing graphs of derivative values (Pichat & Ricco, 2001), (3) algebraic manipulation (Aspinwall & Miller, 2001), (4) the concepts of limits of functions (Tall & Vinner, 1981; Williams, 1991; Cornu, 1991; Szydlik, 2000; Juter, 2005; Naidoo & Naidoo; 2007; Susilo, 2011; Syaripuddin, 2011; Denbel, 2014), (5) the concepts of limits and continuity of functions (Bezuidenhout, 2001; Karatas et al., 2011), (6) understanding the concept of derivative of functions (Tarmizi, 2010; Tall, 2010; 2012; Pepper et al., 2012; Hashemi et al.; 2014), (7) the concept of integral of functions (Orton, 1983; Tall, 1993; Kiat, 2005; Metaxas, 2007; Yee & Lam, 2008; Mahir, 2009; Rubio & Gomez-Chacon, 2011; Usman, 2012; Salazar, 2014; Zakaria & Salleh, 2015; Serhan, 2015; Yudianto, 2015; Ferrer, 2016), and (8) understanding calculus tasks (Kremžárová, 2011). It goes without saying that these various types of difficulties experienced by these students will have an impact on learning outcomes and also on students' ability to think logically, analytically, systematically, critically, and creatively.

Integral calculus specifically studies a function by discussing antiderivatives, indefinite integrals, Riemann sums, fundamental theorem of calculus, definite integrals, and applications of integrals. There is a relationship between finding area of a plane region and definite integral. It is almost similar, yet different. Finding the area of a plane region is one form of application of definite integral.

Some students mistakenly work on the problem about finding the area of a plane region with integral applications. One cause is that students have difficulty to understand the different concepts in finding the area of a plane region and definite integral. In finding definite integral or $\int_a^b f(x)dx$, generally $\int_a^b f(x)dx$ gives the signed area of the region trapped between the curve $y = f(x)$ and the x-axis on the interval $[a, b]$, meaning that a positive sign is attached to areas of parts above the x-axis, and a negative sign is attached to areas of parts below the x-axis. In symbols, $\int_a^b f(x)dx = A_{up} - A_{down}$ where A_{up} and A_{down} are as shown in Figure 1 (Varberg et al., 2007). A_{up} and A_{down} are areas and nonnegative number.

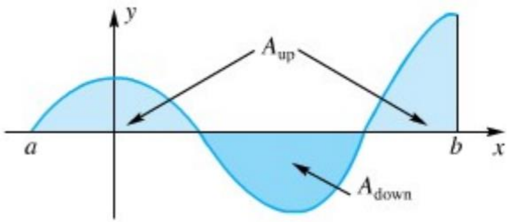

Figure 1. The region of $\int_a^b f(x)dx$

In finding the area of a plane region, let R the region trapped between the curve $y = f(x)$ and the x-axis on the interval $[a, b]$, R must be investigated, whether (1) R a region above the x-axis or (2) R a region below the x-axis. If R a region above the x-axis, then area $A(R)$ is given by $A(R) = \int_a^b f(x)dx$. But if R a region below the x-axis, then area $A(R)$ is given by $A(R) = -\int_a^b f(x)dx$, its cause $\int_a^b f(x)dx$ is a negative number and therefore cannot be an area. In the case of R, as shown in Figure 1, the areas $A(R)$ is given by $A(R) = A_{up} + A_{down}$, with A_{up} and A_{down} are areas and nonnegative number.

The difference between the results in finding the area of a plane region and definite integrals is showed in the following example.

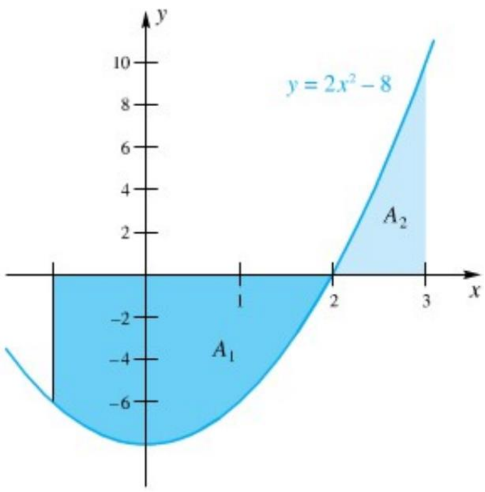

Figure 2. The region of $\int_{-1}^3 (2x^2 - 8)dx$

Figure 2 suggests that $\int_{-1}^3 (2x^2 - 8)dx = -A_1 + A_2 = -\frac{54}{3} + \frac{14}{3} = -\frac{40}{3}$, where A_1 and A_2 are the areas of the regions below and above the x-axis. This result is different from finding the area of a plane region that is let R the region trapped between the curve $y = 2x^2 - 8$ and the x-axis on the interval $[-1,3]$. Figure 2 suggests that:

$A(R) = -\int_{-1}^{2}(2x^2 - 8)dx + \int_{2}^{3}(2x^2 - 8)dx$

$= A_1 + A_2 = \frac{54}{3} + \frac{14}{3} = \frac{68}{3}$.

The description of students' understanding of concepts differences in finding the area of a plane region and definite integral can be viewed from students' critical thinking skills. This review of critical thinking skills needs to be used to find out whether students are able to see the concept's difference in finding the area of a plane region and definite integral after they have gathered evidence. For more, critical thinking skills can be used to investigate how students make an analysis or clarification of the data provided (clarification), give an evaluation or evaluate by giving reasons or examples (assessment), make conclusions or inferences (inference), and make problem-solving strategies (strategies) (Perkins & Murphy, 2006).

Based on the preliminary description, this research aimed to describe students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral.

2. Methods

This study used an exploratory survey method with a questionnaire as the instrument. The stuy subjects were 40 students who have taken integral calculus courses in the mathematics department at a university in Central Java in the academic year 2018/2019. Several questions were needed in order to reveal the description of students' understanding regarding finding the area of a plane region and definite integral. Test questions were compiled in the beginning with a statement that the finding results area of a plane region and definite integral are not the same. Then, students were asked to do the work by drawing graphics, making a clarification with calculations, evaluating the truth of the statement, giving reasons, and showing calculation strategies.

The questionnaire was specifically designed so that respondents gave an overview of their understanding of the concepts differences in finding the area of a plane region and definite integral based on indicators of critical thinking skills (clarification, assessment, inference, and strategies). Respondents' answers were classified into groups of true answers, false answers, and blank answers. By calculating the mean and standard deviations, the low, medium or high categories were obtained. By transforming in percentages form, the researchers obtained the description of students' understanding of the concepts differences in finding the area of a plane region and definite integral.

The following were questions that have been developed. "The region R bounded by $f(x) = x - 2$, the segment of the x-axis, the line $x = -2$, and $x = 3$. The area of the region R is not equal to $\int_{-2}^{3}(x - 2)dx$. Sketch the geometric situation, calculate it, then show the calculation strategy and the truth of the statement."

3. Results & Discussions

A total of 40 students have participated in the study and have completed the tests given according to the indicators of critical thinking skills. The following figures are two examples of student works.

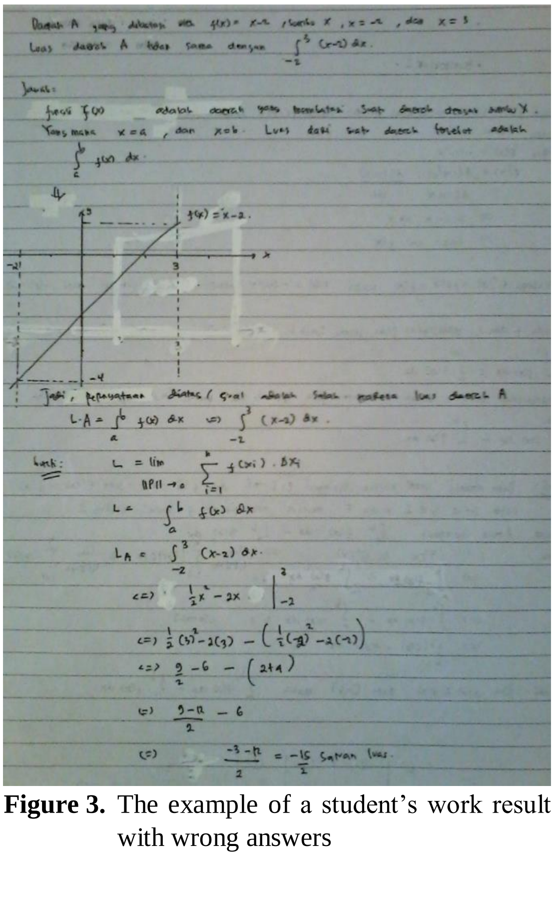

Figure 3. The example of a student's work result with wrong answers

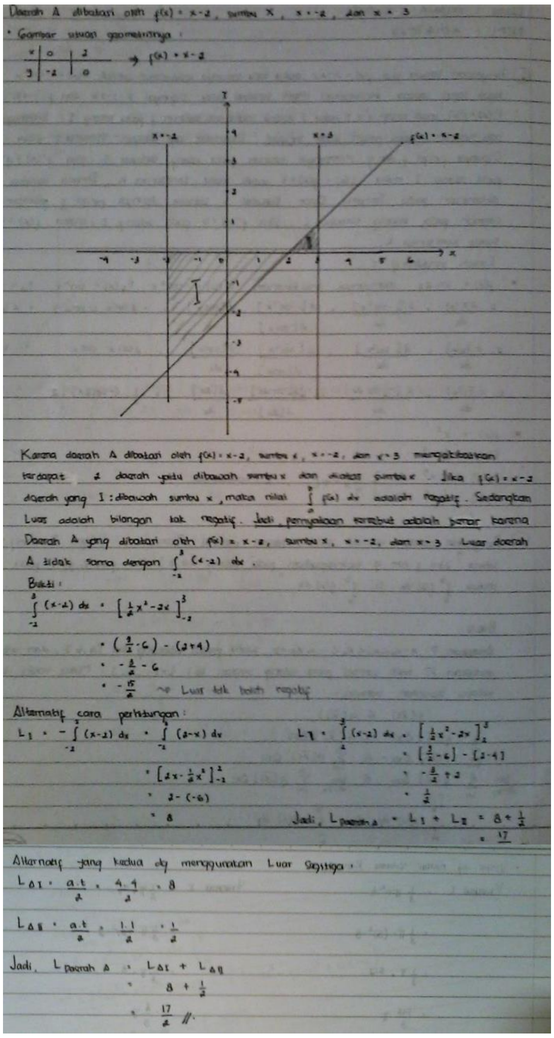

Figure 4. The example of a student's work result with correct answers

The students were asked to give clarification whether the statement was true or false, an assessment/evaluation with reason, then provided their inference and strategy. The recapitulation of students' work results is presented in the following table.

Table 1. Students' critical thinking skills towards concepts differences in finding the area of a plane region and definite integral.

Stage of critical thinking	Answers			Category
	True	False	Blank	
Clarification	23	17	0	Medium
Assessment	16	11	13	Medium
Inference	26	6	8	Medium
Strategies	11	13	16	Low
Average	19	12	9	Medium

Based on Table 1, it can be seen that the average of correct answers in the indicators of students' critical thinking skills towards the concept's difference in finding area of a plane region and definite integral was 19 (47.5%), with clarification by 23 (57.5%), assessment by 16 (40.0%), inference by 26 (65.0%), and strategies by 11 (27.5%). On the other hand, the average of wrong answers were 12 (30.0%) and the average of blank answers were 9 (22.5%).

Based on Table 1, it can be seen that the average of students' critical thinking skills towards the concepts differences in finding area of a plane region and definite integral were 47.5% or in medium category, with the ability to provide clarification by 57.5% (medium), the ability to carry out assessment by 40.0% (medium), ability to give inference by 65.0% (medium), and strategies ability by 27.5% (low). The description of students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral in detail according to the indicators of critical thinking can be seen in Figure 5.

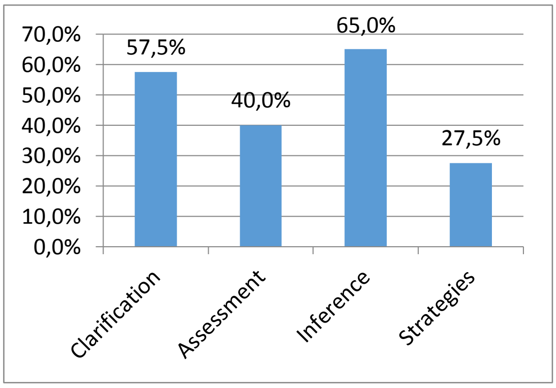

Figure 5. Achievement of each indicator of students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral.

From Figure 2, it was known that clarification and inference indicators got higher achievement compared to assessment and strategies indicators. Assessment indicators refer to the ability to provide explanations in the form of reasons or supporting evidence. This shows that students do not have a good understanding of definite integrals concepts. Also, it shows that students have limitations in understanding definite integral concepts (Serhan, 2015). Regarding these theories, in this study, students were quite able to build clarification through the results of their integral calculations but not enough to understand the differences in concepts between finding the area of a plane region and definite integral. This has implications to strategies indicators which resulted low achievement. Many students solved integral problems using only the formulas provided. Students rarely associate interrelated concepts, in this case, definite integral concepts to find the area of a plane region (Yudianto, 2015), it is almost similar, but different. This study showed that students have difficulty in relating the concepts of limit, area, and integral. This is because students consider the concept of limit, area, and integral not as a sustainable concept (Tasman et al., 2018).

Some of these weaknesses are expected to be followed up by conducting learning that can show the linkages between the concepts of limit, area, and integral (Tasman et al., 2018). For more, learning that shows the concepts differences in finding the area of a plane region and definite integral and with varied ways is needed. Hence, it can increase students' understanding of integral concepts (Attorps et al., 2013, Prasetia, 2016)

4. Conclusion

Integral calculus specifically studies a function by discussing antiderivatives, indefinite integrals, Riemann sums, fundamental theorem of calculus, definite integrals, and applications of integrals. Based on the results and discussion in this study, it can be concluded that students' critical thinking skills towards the concepts differences in finding the area of a plane region and definite integral are in the medium category. The students' critical thinking skills towards the concepts differences in finding area of a plane region and definite integral were medium (47.5%), with clarification by 57.5% (medium), assessment by 40.0% (medium), inference by 65.0% (medium), and strategies by 27.5% (low). These weaknesses are expected to be followed up by conducting learning that can show the linkages between the concepts and with varied ways to improve students' understanding of integral concepts.

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