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Analysis of mathematical representation ability based on students' thinking style in solving open-ended problems

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Abstract

This study aimed to know whether students' mathematical representation ability reach the classical completeness and to describe students' mathematical representation based on their thinking style, namely Concrete Sequential (CS), Abstract Sequential (AS), Abstract Random (AR), and Concrete Random (CR). This study used mixed methods as the research design. Population in this study was 10th graders in one vocational school in Pati. The sample was a class chosen randomly. The subjects of this research were 8 students consisted of 2 of every thinking style type. The methods of data collection of this research were questionnaires of thinking style, mathematical representation ability test, interviews, and documentation. The results showed that: (1) students' mathematical representation ability reached the classical completeness and (2) the students' mathematical representation ability based on their thinking style are (a) the CS students have moderate visual ability, excellent symbolic ability, and poor verbal ability; (b) the AS students have moderate visual ability, excellent symbolic ability and less verbal ability; (c) the AR students have moderate visual and verbal ability, and good symbolic ability; and (d) the CR students have moderate visual ability, excellent symbolic ability, and poor verbal ability.

1. Introduction

Education is an important factor for the progress of a country. Good quality and level of education will affect the intellectual abilities and civilization of a nation. In other words, a good education system will produce good human resources. Education is a conscious and planned effort to create an atmosphere of learning and learning process so that students actively develop their potential to have spiritual strength, self-control, personality, intelligence, noble character, and the skills needed by themselves, society, nation and state.

The purpose of national education according to law number 20 of 2003 is to develop the potential of students to become people of faith and devotion to God Almighty, noble, healthy, knowledgeable, capable, creative, independent, and become democratic and responsible citizens. The purpose of education is not only to develop cognitive aspects, but also to develop values of attitudes and behaviors so that students become intelligent, polite, and characterized human beings. Character values developed in learning are religious, honest, tolerance, hard work, creative, independent, democratic, curiosity, national spirit, love of the motherland, respect for achievement, friendly/communicative, love peace, love to read, care for the environment social care and responsibility.

The National Council of Teacher Mathematics (2000) states that there are five process skills that students need to have through mathematics learning that are covered by the standard processes, namely: (1) problem solving ; (2) reasoning and proof; (3) communication; (4) connection; and (5) representation. These five skills belong to high order mathematical thinking that must be developed in learning mathematics.

The representation referred to by NCTM is a method used by someone to communicate the answer or mathematical ideas in question. This implies that the learning process that emphasizes the ability of representation will train students in mathematical communication.

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Mathematical representations raised by students are expressions of mathematical ideas or ideas displayed by students in their efforts to understand a mathematical concept or in their efforts to find a solution to the problem being faced (Hutagaol, 2013). The term representation refers to the process or result (product) in the actions taken to capture a concept of mathematical relations in a mathematical form itself. Novira (2019) said that mathematical representations is the one of the basic skill student had to own.

Cai, Lane, and Jacabcsin (Sabirin, 2014) state that the various representations that are often used in communicating mathematics include: tables, pictures, graphs, mathematical statements, written texts, or a combination of them all. While, Mudzakir (Jaenudin, 2009) in his research groups mathematical representations into three main forms, namely:

- Visual representations in the form of diagrams, graphs or tables, and figures.
- Mathematical equations or expressions.
- Words or written text.

In the process of learning mathematics in class X of a vocatinal school in Pati is currently still applying the 2013 curriculum. Based on data from the school, the average of Computer-Based National Exam 2018 for math is 35.7 which has decreased over the past three years. The average of First Midterm test's Mathematics score class X AP 2018 is 65.8. The average value has not yet reached the completeness value applied by the school which is 70. Mrs. Sulistyowati, one of the mathematics teachers, said that the majority of students still experience difficulties in understanding the material. Students are also still confused when confronted with non-routine problems for example that are not found in books. This indicates that the level of mathematical representation of students is still low.

According to Juandi (2011) the low of students' representation can be caused by the unavailability of opportunities for students to develop their mathematical representation in deductive learning (delivery of material that directly applies or delivers formula) individually. It is possible that the mathematical representation of students is different from one another. This can be overcome by creating learning that supports the development of students' individual representations, one of which is the method of learning Problem Based Learning (PBL) aided by open-ended questions.

More than 20 years ago, many studies were conducted in implementing problem-based learning (PBL) in professional education and training (Albanese & Mitchell, 1993; Savin-Baden, 2000; McPhee, 2002; Gibson 2002; Pedersen & Liu, 2002; Wulandari, 2013; Farhan & Retnawati, 2014; Tandililing, 2015; Lestanti et al, 2016). This model allowed students to learn from each other by discussing a problem given. Students became more active in the class. When they met difficulty in understanding the problem, the teacher would give them explanation or hint as needed. Students could learn new things that couldn't be learned by themselves. Vygotsky (Rifa'i & Catharina, 2011) called it Zone of Proximal Development (ZPD). No wonder PBL became one of recommended learning model to be applied in 2013 curiculum learning.

Open-ended questions are questions that have multiple solutions or multiple approaches to a solution (Takahashi, 2008), while according to Becker and Shimada (1997), open-ended problems are questions that have a variety of answers. There are three types of open-ended questions, including (1) types of questions with multiple answers (problems with multiple solutions); (2) types of problems with many ways of solving a problem (problems with multiple solution methods); (3) types of problems with problems that can be developed into new problems (problem to problem). Open-ended problems with the type of problem with multiple solution is one type of problem that has more than one alternative solution/answer. Problems with multiple solutions type will be used to measure the level of mathematical representation of students by assessing the ability of visual, symbolic, and verbal representations.

Based on the description above, the purpose on this study are: (1) to determine whether students' mathematical representation ability reach the classical completeness; and (2) to describe students' mathematical representation based on their thinking style such as Concrete Sequential (CS), Abstract Sequential (AS), Abstract Random (AR), and Concrete Random (CR).

2. Method

This research method used in this study is a mix methods. The research design used is a sequential explanatory design, in which the researcher will collect and analyze quantitative data in the first stage, and is followed by the collection and analysis of qualitative data in the second stage to strengthen the results of quantitative research in the first stage. Quantitative data obtained from the score of material completeness test questions will be analyzed to determine whether students' mathematical representations ability have reached classical completeness, while qualitative data analysis is used to describe students' mathematical representation abilities, with various data collection procedures and in continuous time.

The main subject in this study was a class X AP student at one of vocational school in Pati. The selection of research subjects used a purposive sampling technique. Sugiyono (2016: 126) states that purposive sampling is a sampling technique with certain considerations. Researchers determine 8 students as research subjects for the analysis of the level of mathematical representation based on the classification of ways of thinking, where each type of thinking style is represented by two students. The choice of subject depends on the researcher herself, but Stewart and Shamdani (Moleong, 2007) suggest that adequate sampling can be made up of 20% of people.

Data collection methods used were the questionnaire method, test method, interviews, and the documentation method. The questionnaire method was used to collect data about students' thinking style. The test method is used to obtain data about the mathematical representation ability of students in quadratic equation and function material. Interviews in this study were conducted to determine students' mathematical representation abilities based on the tests provided. The documentation method is used to obtain data on student's work.

After getting the data in the form of the mathematical representation ability test's score and scores of students' thinking style, testing is conducted to test the truth of the proposed hypothesis. Classical learning completeness test that is the proportion test in the class that receives learning with the PBL. The data of students' thinking style was used to determine the research subject by choosing each two of Concrete Sequential (CS), Abstract Sequential (AS), Abstract Random (AR), and Concrete Random (CR) type. The mathematical representation ability test results and interviews were analyzed referring to the representation indicators shown in Table 1 as follows (Yudhanegara & Lestari, 2014).

Table 1. Forms of Mathematical Representation Ability

Representation	Indicator of forms
1. Visual representation: a. Picture	• Create geometric figures to clarify the problem and facilitate its resolution
2. Equation or Mathematical expression	• Make aequation or mathematical expression of other representation given • Problem solving involving mathematical expression
3. Words or Written text	• Write intepretation of a representation • Write the steps of solving a problem.

3. Results and Discussion

Questionnaire of thinking style instrument developed by John Parks Le Tellier was carried out in class X AP on Saturday, April 27, 2019. The instrument was filled in at 20 minutes of Mathematics with the permission of the subject teacher concerned. Questionnaire consists of 15 groups of words. From the questionnaire score thinking style is grouped into several categories based on the number of choices of answers circled by students in each column of the characteristics of thinking style. The highest score indicates the type of thinking stye that students have. Based on the data collected, the following are the results of grouping students according to the type of thinking style as shown in Table 2.

Table 2. Grouping Class X AP students' thinking style

Thinking Style					
CS	AS	CR	AR	CS, CA	CS, AS, AR
S-3	S-10	S-7	S-1	S-20	S-5
S-6	S-14	S-8	S-2		S-28
S-9	S-18	S-24	S-4		
S-11	S-26	S-25	S-12		
S-13			S-15		
S-19			S-16		
S-21			S-17		
S-27			S-22		
S-29			S-23		
S-30					

The distribution of the characteristic categories of students' thinking style is presented in Table 3 as follows.

Table 3. Distribution of Student's Thinking Style

Thinking Stye	Number of Student	Percentage (%)
CS	10	33,33
AS	4	13,33
AR	9	30,00
CR	4	13,33
CS,CR	1	3,33
CS,CR,AR	2	6,67

Based on Table 3, Concrete Sequential type dominated class X AP and then followed by Abstract Random. Abstract Sequential and Concrete Random came in third. The fourth was Concrete Sequential Concrete Random Abstract Random. And the last was Concrete Sequential Concrete Random.

After questioner given, learning activities carried out 4 times in class X AP class with 30 students. Learning is done to provide students with material about equations and quadratic functions. Learning was done using problem-based learning (PBL) which is carried out continuously at each meeting. PBL learning consists of the problem orientation phase, organizing students into groups, guiding individual or group investigations, developing and presenting work, and finally analyzing and evaluating problem solving processes (Arends, 2012).

3.1. *Classical Completeness*

Classical completeness is achieved if the classical absorption reaches a minimum of 75% (Depdikbud, 2013). Quantitative data were obtained from open-ended problem score types with multiple solutions that have been used to measure students' mathematical representation abilities. The test results are used to find out the number of students who have achieved classical mastery learning (at least 75% of the number of students in the research class). This analysis was conducted to prove the hypothesis that 75% of students 'mathematical representation ability reached the minimum completeness criteria value of 70, so that later it could be said that students' mathematical representation ability reached classical completeness. The hypothesis in this research was the proportion of the number of students who have achieved mastery learning in the material equation and function more than 74.5%.

The results of classical mastery learning calculations of students in the research class on the material equation and quadratic functions can be seen in table 4 as follows.

Table 4. The Result of One Sided Proportion Test

x	π_0	z_{score}	z_{table}	Conclusion	Meaning
28	74,5%	2,78	1,64	$z_{score} \geq z_{table}$	The proportion of the number of students who have achieved mastery learning in the equation and quadratic function material is more than 74.5% of all students in the research class

Based on the results of the z test, $z_{score} = 2{,}78$, with $\alpha = 5\%$ obtained $z_{table} = 1{,}64$. So $z_{score} \geq z_{table}$ then H_0 rejected, it means mathematical connection ability of the experimental class students achieve classical completeness. Based on the learning completeness test above the proportion of the number of students who have achieved mastery learning in the material equation and function more than 74.5%.

3.2. *Students' Mathematical Representation Ability Based-On Thinking Style*

The results of the classification of students' thinking ability characteristics in table 2 are used by researchers as consideration for selecting research subjects to be interviewed. From each of the characteristics of thinking style two students were randomly selected research subjects. Selected research subjects will be identified their mathematical representation ability based on thinking style in Table 5 below.

Table 5. Subject of Research To Be Interviewed

CS	AS	CR	AR
S-9	S-26	S-25	S-4
S-19	S-14	S-8	S-16

Based on the guideline for classifying the quality of students' mathematical representation abilities, the researcher identifies class X AP students into the level of mathematical representation based on the type of way of thinking presented in Table 6 below.

Table 6. Qualification Results Quality Mathematical Representation Ability based on Characteristics of Thinking Class X AP

Thinking Style	Representation		
	Visual	Symbolic	Verbal
CS	61,11%	96,67%	33,33%
AS	72,22%	96,67%	55,56%
AR	61,11%	80%	66,67%
CR	66,67%	96,67%	0

The visual representation ability of students in the Concrete Sequential (CS) type were moderate. Students of CS type think able to fulfill the indicator of visual ability on 2 number of questions correctly. They have no difficulty in the drawing process because instructions on the questions are considered clear and easy to understand. It's just that they are incomplete giving information on the graph and there is one student whose graphic drawing is less proportional. Symbolic abilities of CS students were in the excellent category. One student could answer all 5 symbolic questions precisely. While the other one could only solve 4 questions correctly. The later student do the calculation carelessly. Of course, it affected the rest of calculation. In the verbal ability indicator, CS students are only able to answer 1 question correctly. The most important indicator is writing interpretation of representation problem no. 1. CS students admitted that they were confused because they were not yet accustomed to writing the completion steps.

The ability of visual representation of AS students is in the moderate category. One of the AS students was only able to meet the indicator of visual ability on only one question number. While other students are not able to meet the visual ability indicator correctly and appropriately. This is because the graphics they make do not include complete information. Unlike the ability of symbolic representation, the ability of mathematical representation of symbolic aspects is excellent. One AS student is able to correctly fulfill the indicator of mathematical ability of symbolic aspects in all 5 questions available. While one other AS student is only able to answer 4 numbers correctly. This is due to the inaccuracy of these students while working in the middle of the calculation process. Next, the ability of verbal representation is in the lacking category. Each AS student is only able to fulfill the verbal representation ability indicator correctly only on 1 number. This is due to incompleteness when writing conclusions and one student is not accustomed to writing the answer conclusions and completion steps.

The ability of visual and verbal representations of students in the Random Abstract type is in the moderate category. One AR student is able to fulfill the indicator of mathematical representation ability in visual aspects in 2 of the 3 questions available, while other AR students are only able to answer 1 question correctly.

Based on the verbal aspect, both students are only able to answer 2 questions correctly, namely for indicators to explain the interpretation of a presentation. AR students like to write conclusion answers. Based on class activities, AR students tend to like to talk. It is not surprising that the verbal ability scores of AR students are highest among other types of thinking. The ability of mathematical representation of symbolic aspects of AR students is in the good category. AR students are only able to fulfill the indicator of symbolic ability correctly in just four questions. Both AR students just do not continue their work on one question.

The ability of the visual representation of students Concrete Random (CR) is in the sufficient category. One CR student is able to fulfill the indicator of visual representation ability on 2 questions, while the other students only have 1 question. Furthermore, the ability of symbolic representation is in the excellent category. One CR student is able to fulfill the indicator of symbolic ability in all 5 questions available, while one other student only 4 questions are fulfilled.

The ability of verbal representation of CR students is in the very least category. CR students are not able to meet the indicators of verbal presentation ability. They are still not accustomed to writing down conclusions and completion steps. CR students have the characteristics of trial and error. They mCRe a square equation with origin which produces the roots of the quadratic equation in the form of rational numbers.

4. Conclusion

Based on the results of research and discussion, it is concluded as follows. (1) The ability of mathematical representation of class X students of a vocational school in Pati on the subject matter of equations and quadratic functions in Problem Based Learning (PBL) learning achieves classical completeness. (2) The ability of mathematical representation of class X AP students of a vocational school in Pati in terms of the type of thinking is:

a. Students of the Concrete Sequential (CS) type have moderate visual ability, excellent symbolic ability, and poor verbal ability.
b. Students with the type of Abstract Sequential (AS) have moderate visual ability, excellent symbolic ability, and less verbal ability.
c. Abstract Random type students (AR) have the ability of visual and verbal representation in modearate categories and the ability of symbolic representation in good categories.
d. Type Concrete Random students (CR) have moderate visual ability, excellent symbolic ability, and poor verbal ability.

Based on the conclusions above, suggestions can be given as follows. (1) Teachers need to pay attention to the characteristics of students' ways of thinking and the level of mathematical representation in learning mathematics because there are differences in the way students solve problems. (2) The use of open-ended questions in the learning situation needs to be intensified so that it is expected to be able to encourage students to learn and hone their mathematical representation abilities. (3) Teachers need to provide motivation and direction for students so students solve mathematical problems in a variety of ways that can be found. (4) Further research is needed as an effort to improve students' mathematical representation abilities.

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