PRISMA 3 (2020): 310-316
PRISMA, Prosiding Seminar Nasional Matematika
https://journal.unnes.ac.id/sju/index.php/prisma/
ISSN 2613-9189

How to Develop the Algebraic Thinking of Students in Mathematics Learning

Anifa Mirza Liadiani[a,*], Alifah Kanza Widayati[b], Gilang Kusuma Lestari[c]

[a,b,c] *Departement of Mathematics FMIPA Universitas Negeri Semarang, Gunungpati Semarang*

** Email Address: anifamirzaliadiani@students.unnes.ac.id*

Abstract

The aim of this article is to explain how to develop the algebraic thinking of students in mathematics learning. Algebraic thinking is ability of students to analyze, make generalization, solve problems, predict, justify, and prove as well as model some situations. It is an essential and fundamental element of mathematical thinking. But apparently, based on the results of previous literature studies, obtained some facts that algebra is one of the materials that is difficult to be mastered by students. This article is written based on several scientific journals. This article is going to explain some strategies for developing algebraic thinking skills. The strategies are learning using 7E learning cycle model (elicit, engagement, exploration, explanation, elaboration, evaluation, and extend), learning using technology (video-game; software that are spreadsheet, geogebra, group explorer, pearson's, MyMathLab, digital mathematics environments, SimCal), learning using manipulatives, learning using multiple representation that is realistic approach (orientation, exploration, internalization, and evaluation), and also through government regulation that is early learning.

Keywords:

Algebraic Thinking, Strategies

1. Introduction

Algebraic thinking is an essential and fundamental element of mathematical thinking. But apparently, based on the results of previous literature studies, obtained some facts that algebra is one of the material that is difficult to be mastered by students. Secondary school students have difficulties and misconceptions in learning algebraic subjects. One fact is that only 22% of California's eighth-graders demonstrate proficiency in an equivalent algebra course. In addition, sixth, seventh and eighth grade students do not have a good understanding of interpretation of equivalence equations and variable concept**s.** (Nurhayati *et al*, 2017)

Algebraic thinking learning in upper elemetary grades is not included in the curriculum, but it is implied in some basic mathematical competencies that use algebraic thinking processes. The study of algebraic thinking skills in elementary schools, especially upper elementary grades, is needed to make the design of learning tailored to the elementary school mathematics curriculum. the study reveals that the average student's ability to think algebra in algebraic thinking is 55.5%. It means that based on the indicators of algebraic thinking, the algebraic thinking ability of upper elementary grades is less good. Also found obstacles to algebraic thinking that occur in students are obstacles in describing, conveying, and making generalizations about geometric and numerical patterns, and obstacles in showing and analyzing patterns and functions, using words, tables, and graphs, barriers in showing a mathematical relationship by using equal signals, as well as obstacles in modeling the problem situation with objects and using representations, such as graphs, tables, and equations to draw conclusions. Based on the obstacles of thinking thought that have been found, such have been generated Hypothetical learning. (Pratiwi *et al*, 2017).

To cite this article:
Liadiani, A. M., Widayati, A.K. & Lestari, G.K. (2020). How to Develop the Algebraic Thinking of Students in Mathematics Learning, *Prosiding Seminar Nasional Matematika* 3, 310-316

How can we help students, if we do not know what the difficulties faced by each student. Each student has different characteristics and different ways of thinking. As teachers, we should be able to anticipate the diversity of student characteristics. We should be aware of the level of algebraic thinking of students as early as possible, so that we can help them to bridge each level well. Students can increase their level, if they are aware of their positions and try to bridge to the next level of course with the help of teachers and friends. So that students can increase their algebraic thinking level with meaning. (Maudy, 2018).

Teachers are needed to know students' algebraic thinking skills, especially in math problems. Teachers must understand the way students think in algebra. (Kusumaningsih *et al*, 2018)

2. Literature Review

The following will be explained the basic theories needed in this article, those are algebraic thinking, component of algebraic thinking, phases of algebraic thinking, levels of algebraic thinking of students.

2.1. *Algebraic Thinking*

Algebraic thinking is ability of students to analyze, make generalization, solve problems, predict, justify, and prove as well as model some situations. According to Ntsohi (2013), algebraic thought is the use of symbols and mathematical tools to analyze different conditions by representing information mathematically regarding words, diagrams, tables, graphs and equations and using mathematical findings such as calculating unknown values, proving and determining relationships between functions. Students' algebraic thinking and reasoning are analyzed with respect to their ability to use mathematical symbols and tools to analyze situations with a mathematical problem (1) extracting information, (2) representing information using a variety of forms, and (3) interpret and apply the ideas mathematics into a new situation (Nilklad, 2004). Manly & Ginsburg (2010) states there are three big ideas in algebra, ie, variables, symbolic notation and multiple representations. (Toheri, 2017)

2.2. *Component of Algebraic Thinking*

Algebraic thinking is arranged into two main components, the development of tools to think mathematically and study of the basic ideas of algebraic (Kreigler, 2007). Tools to think mathematically defined analytically habits of mind that are structured around three topics: problemsolving skills, representation skills, and quantitative reasoning skills. Fundamental ideas in algebra represent the content domain in which mathematical thinking tools developed. The fundamental idea explored into 3 parts: algebra as generalized arithmetic, algebra as a language, and algebra and functions as a tool for mathematical modeling. According to Usiskin (1995); Kaput, Hegedus & Lesh (2007) that the algebra is the language of symbols to describe and overcome real problems, patterns, generalizations, making predictions, making sense of the world, the study of functions, relations, multi-representation, modeling to generalize. (Toheri, 2017)

NCTM (2000) states that there are some fundamental components of algebraic thinking. Fundamental components include: 1) understanding patterns, relationships or relationships, and functions; 2) represent and analyze mathematical situations and structures using mathematical symbols; 3) using mathematical models to represent and understand quantitative linkages; 4) analyze changes in a variety of contexts. (Toheri, 2017)

Herbert & Brown (1997); Wagner & Kieran (1989); Dindyal (2003) states there are three components associated with the thought of algebra, namely: 1) the use of symbols and algebraic relations, 2) the use of forms of representation are different, 3) usage patterns and generalizations.

2.3. *Phases of Algebraic Thinking*

Matos & Ponte (2009) stated that there are five phases of algebraic thinking are interrelated; 1) generalizing and formalizing patterns and constraints, 2) manipulating the formal, 3) examine the structures of abstract, 4) review of functions, relations and joint variations , 5) use different language in a mathematical model and control some of the phenomena. (Toheri, 2017)

2.4. *Levels of Algebraic Thinking of Students*

We should be aware of the level of algebraic thinking of students as early as possible, so that we can help them to bridge each level well. Students can increase their level, if they are aware of their positions and try to bridge to the next level of course with the help of teachers and friends. So that students can increase their algebraic thinking level with meaning. (Maudy, 2018)

Based on the theoretical review, the 2013 curriculum, and the research results of previous researcher, five levels of student algebraic thinking are concluded as follows.

Level 0: Using ways of arithmetic, such as counting and operation with extensive object.
Level 1: Indication of factual generalization.
Level 2: Understanding one unknown and operating it. The generational activities of algebra involve the forming of the expressions and equations that are the intensive objects of algebra. Contextual generalization indicated.
Level 3: Viewing the relationships between variables. The transformational activities includes, for instance, collecting like terms, solving equations, simplifying expressions, working with equivalent expressions and equations, and so on. Viewing the relationships between variables.
Level 4: Using parameters and variable in generational activity. Symbolic generalization indicated.
Level 5: Treatment with parameters.
Level 6: Performing analysis with algebraic structures. (Maudy, 2018)

3. Discussion

There are 5 strategies that can develop the algebraic thinking of students in mathematics learning. Those are learning using 7E learning cycle model (elicit, engagement, exploration, explanation, elaboration, evaluation, and extend), learning using technology (video-game; software that are spreadsheet, geogebra, group explorer, pearson's, MyMathLab, digital mathematics environments, SimCal), learning using manipulatives, learning using multiple representation that is realistic approach (orientation, exploration, internalization, and evaluation), and also through government regulation that is early learning. The explanation is as follows.

3.1. *Learning Using 7E Learning Cycle Model*

Rahmawati *et al* (2018) stated that in order to improve student's algebraic thinking ability and student's mathematical dispositions are needed learning model that is able to construct students' understanding in understanding and solving algebaraic problem. And able to encourage positive attitudes: more diligent, persistent and never give up so that the algebraic thinking ability and mathematics dispositions is better.

Student centered learning approach encourages students to build knowledge, attitudes and behavior (Yulianingsih & Hadisaputro, 2013). Learning model and teachers are the main factors that can affect student learning outcomes. One of learning models that has a constructivist paradigm is Learning Cycle 7E model (Elicit, Engagement, Exploration, Explanation, Elaboration, Evaluation and Extand). Learning Cycle 7E is one of student centered learning model which has stages of activities organized in such a way that students can proficient the competencies that must be achieved in learning by playing a role.

This results are indicated by: (1) validator's assessment of learning tools is good; (2) Learning quality is good; (3) students' positive response for learning reached more than 70%; (4) students who accomplish minimum completeness are more than 75%; and (5) the mean of student's algebraic thinking ability in the Learning Cycle 7E model is better than the mean of student's algebraic thinking ability in the expository learning model.

Based on the above conclusion, it is given several recommendations as follow. (1) Learning Cycle 7E model is considered quality and can improve student's algebraic thinking ability. So, this learning model can be applied in teaching and learning process; (2) Student response to learning model of Learning Cycle 7E in good criterion, it is recommended for students to choose effective way of learning and according to their mathematics dispositions to improve algebraic thinking ability; and (3) The results show that students with low and medium category of disposition mathematics are only able to achieve several ability of algebraic ability. Its recommended for students with low and medium category of disposition mathematics can be given more guidance and motivation in order to solve algebra problems.

3.2. *Learning Using Technology*

The use of worksheets, exercises and factual questions that are the main materials for learning algebra has not been successful to deepen students' thinking. Students also face problems when they deal with algebraic operations involving objects, variables, functions, and invariable relations or structures, as well as procedures. They often face difficulties in grasping the nature and the concepts of algebra. Various methods could be applied to minimize these problems, including the use of technology to overcome limitations of nontechnology learning environment that, sometimes, ignore the linking between geometry and algebraic manipulations.

In non-technology learning environment sometimes students are unable to receive feedbacks on time and to learn anywhere and anytime at their convenience. Moreover, teachers frequently ask closed questions that lead the learning process to consist of only algebraic procedures and unable to enhance students' thinking in mathematics. In learning of mathematics, the students faced difficulties in visualizing what they have learnt without use of supporting tools. The tools, in the forms of multiple representations, namely graphing symbolic manipulation and numerical computations, which may be provided by software technology, play dual roles as both a visualization tool and a computational tool.

One technology application that is most popular in learning algebra is the spreadsheet, applicable for learning of basic of generalization. Generalization is one of the characteristics of algebraic thinking. It involves the growth of patterns and the sums of consecutive numbers. Technology serves as a scaffold in the learning process. GeoGebra being known as a dynamic mathematics software is used to improve the concept of algebra and visualization which requires increasing the number of three-dimensional objects. Meanwhile, Group Explorer is freely available software that supports students' visualization with various features, which consists of basic algebraic structures for undergraduate students in mathematics. There are also several online algebra learning software such as Pearson's MyMathLab and Digital Mathematics Environments (DME). Pearson's MyMathLab is software based on instructional
content consisting of worked example problems, videos, homework, quizzes, and ests. Similarly, DME is a learning management system that provides online algebra tasks. DME algebra consists of two modules of linear equations and quadratic equations that provide the students with the opportunities to practice algebraic skills. On the other hands, SimCal is software that is related to coherent curriculum that is able to transfer students' mathematical constructs into various representations. Meanwhile, multimedia learning is learning through visual and audio channels which promote spatial skills consisting of signal principle that connects the graph and equations assisted by colors.

Generally, the overall results show that the software used in learning algebra yielded positive impacts. The software, namely, GeoGebra, Pearson's MyMath Lab, SimCal and Group Explorer were easy to use and have been implemented in the learning environment. Although the use of Pearson's MyMathLab does not substantially contribute towards powerful algebraic thinking, it enables the students to gain some procedural fluency. For example, students were assisted in squaring binomial, solving proportions, finding equivalent equations, finding function values of radical functions, rationalizing two term denominators and writing the equation of a circle with a given centre and radius, compared to problems that involve graphing and polynomial. The use of technology for learning algebra in enhancing algebraic thinking, despite its benefits, causes some obstacles in the learning process of the students. They, notably, showed a lack of symbol sense, failure to use variables for the same elements and a lack of algebraic flexibility of pseudo-structural thinking. From the review, it is suggested that, GeoGebra, with its many benefits, be used to explore the characteristics of algebraic thinking.

This was evident in the improvements achieved in their mathematical skills and understanding of the concepts. Digital technology is able to enhance mathematical reasoning through numerical, algebraic and graphically. However, the usage and effect of the technology should be studied in-depth to include the characteristics of algebraic thinking in middle schools. In comparisons to the other software used in the study, GeoGebra is the free open source dynamic software were used both at the school and tertiary level. GeoGebra consist of algebra display, spreadsheet view and geometry view are specially designed to connect geometry and algebra. More importantly, coordinates can be entered directly as well as equations. It is used as visualization of concepts, connecting computer algebra system with other mathematical concept as well as educational materials. The strength of Geogebra for learning algebra should be explored in particular examining algebraic thinking.

Algebra as one of the basic branches of mathematics has the ability to promote algebraic thinking which involves problem-solving, representation, and reasoning skills. Skills learnt in algebra are based on the idea that an equation can be manipulated by performing the same operation on both sides to form another equation that has the same value but is written differently. Edwards found that the most difficult aspect in developing algebraic abilities of students in the earlier stage of middle school is to master some important concepts of algebraic notation, variables, functions and properties of numbers. These four basic concepts have been classified as major algebraic ideas and need to be developed to build a solid foundation of problem solving in the classroom. (Siew, 2016)

In response to difficulties in learning algebra, Ferrucci and Sinclair claim that the modelling approach is a 'powerful tool' to improve problem solving skills among middle school students. According to Ferrucci, the modelling approach emphasizes pictorial representation to analyse and represent quantity relationships in algebra problems, such as linear relationships. This approach helps students to visualize abstract mathematical relationships in the form of a model in which students can gain a deeper understanding of the concepts and skills to manipulate symbols as variables. (Siew, 2016)

According to Sensory Stimulation Theory, more learning will occur if more senses are stimulated simultaneously. Sensory stimulation is achieved through colours, level of sounds, visual of facts and media consumption. Video games are electronic, interactive games known for their vibrant colours, sound effects, and complex graphics. Video games stimulate learning as they allow players to experience novelty, curiosity and challenge. Through video games, students learn perceptual skills, learn actively and carry out repetitive practice. Some previous research findings showed that video games help in improving students' basic mathematics abilities, achievement, concepts, and problem solving skills. According to Devlin, video games promote students' innovative mathematical thinking skills. As a result, past research has suggested that playing video games benefited learners by enhancing their cognitive level. (Siew, 2016)

On the other hand, Vygotsky's Zone of Proximal Development suggests that children could perform difficult tasks successfully with the help of adults or peers who are more skilful. Video games allow players to go beyond the 'Zone of Proximal Development' because of the desire to solve a challenging problem on their own. Players could look for some guidance if they need help to proceed with the game. This will encourage players to succeed through continuous exercise either progress to the new level or repeat it. Realising the advantages of video games in learning, this study sought answers on the impact of video games in developing algebraic abilities of students. (Siew, 2016)

3.3. *Learning Using Manipulatives*

Chan (2015) reported using manipulatives such as pattern blocks increased the understanding of algebraic patterns in early grades. A sequential development using representations from concrete to visual and then to symbolic supported students in identifying patterns and making generalizations. Perhaps these approaches to teaching could have made the difference in students understanding of the algebra topic. These findings support previous findings by Blanton and Kaput (2004) in which they reported that students in early grades could understand covariation and functional thinking in algebra. (Ayieko, 2018)

3.4. *Learning Using Multiple Representation*

Multiple representations are widely used in mathematical research to improve students' mathematical concepts. Multiple representations consist of various formats of representation that can be used in the learning process. The multiple representation strategies consist of orientation, exploration, internalization, and evaluation. According to Kohl and Finkelstein (2006) and Knight (2013), representations consist of (1) verbal representation, (2) diagrams, (3) graphical representations, and (4) mathematical representation, required when students solve quantitative problems using equations that match the information obtained. Learning by the multiple representation strategies with realistic approach affected the ability of students' algebraic thinking. The multiple representation strategies with realistic approach were able to improve the algebraic thinking. The ability of algebraic thinking of student on the course subject of polyhedron using multiple representation strategies with realistic approach provided an algebraic thinking capability better than the scientific approach. Ainsworth (2008) states that multiple representations are highly relevant and necessary in learning to build and develop an understanding of the concept of the situation in depth scientifically. Kohl's research, et al. (2008) shows that students who learned through multiple representations complete a set of mathematics tests better than those who learned through few representations. (Kusumaningsih, 2018)

3.5. *Early Learning Algebra*

Symbolic notation in mathematics is taught in the algebra strand of mathematics in middle school and is introduced by some teachers in the early elementary school years (Blanton & Kaput, 2005). The teaching and learning of algebra is a challenge for teachers and students (Kieran, 2007). Across nations, content guidelines and expectations differ on grade levels for teaching algebra and the depth of coverage. Therefore, a comparative analysis of the opportunities to learn algebra before high school, and relating these learning opportunities to students' algebra mastery is a step towards demystifying the learning of this component of mathematics. (Ayieko, 2018)

4. Conclusion

A number of researchers have shown that video games have an effect on student's attitudes to mathematics learning. For example, Fengfeng found that games promoted a more positive attitude to mathematics learning. Kebritchi et al.'s study indicated that students showed increased focus and desire to learn when playing DimensionM™ math video games because they liked to succeed at the games' missions. Students were increasingly aware of the relationship between mathematics and real life while mathematics phobias were reduced. What is true for mathematics in general is especially true for algebra since its understanding assumes knowledge of the specialised processes and language nuances associated with symbolic representations. The fact that video games enhance students' mathematical learning and develop more positive attitudes in them toward mathematics is confirmed by Kebritchi et al.'s study. Kebritchi et al. found that a web based computer game (ASTRA EAGLE), was able to enhance 5th grade students' mathematics performance and attitudes toward mathematics. In addition, the Fennema-Sherman (1976) attitudes toward mathematics questionnaire contains scales of confidence, usefulness, fun, teacher's expectation and attitudes, and has been used by researchers to investigate the correlation between its scales and algebra achievement.

The study reveals that learning algebra using DragonBox 12+ as supported by the six dimensions of Relevance, Embedding, Translation, Adaptation, Immersion and Naturalisation, can impact positively on students' algebraic thinking. This study also shows that DragonBox 12+ can help students to develop a more positive attitude to algebra learning. Students become more confident in learning algebra and find algebra useful in their daily lives.

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