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Analysis of students error in global meta-level algebraic thinking on problem proving on CORE learning assisted by scaffolding

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Abstract

The aims of this study are to identifies the quality of mathematics learning used CORE model assisted by scaffolding, examine whether students' ability in global meta-level algebraic thinking pass the minimum completeness criteria, describes the type and the causes of students' error in global meta-level algebraic thinking, and type of scaffolding for lower group students. This is a mixed-method research. The population of this research are students of class XI-IPA at SMAN 1 Bojong. The sample of this research are students of class XI-IPA 3 as many as 32 students. The research subjects were taken 9 students, each of 3 students from the upper, middle, and lower groups. Data collection methods used documentation, interview, and test. The validity of the data is done by triangulating the result of test and interviews. Data is analyzed by doing data reduction, data presentation, and verification. The results of this research showed that (1) the quality of mathematics learning used CORE model assisted by scaffolding in very good criteria; (2) students' ability in global meta-level algebraic thinking on CORE learning assisted by scaffolding pass the minimum completeness criteria; (3) upper group students tend to make some error in the process skill and encoding steps, middle group students tend to make some error in the process skill steps, while lower group students tend to make some error in the transformation and process skill steps; (4) the cause of the error are lack of students' accuracy in writing problem information modeling, inaccurate in calculation process, not able to complete the proving process, and inaccurate in writing down the variables in conclusion sentences; (5) scaffolding reviewing is given for transformation and encoding errors, scaffolding explaining, reviewing, and restructuring are given for process skill errors.

1. Introduction

According to Permendiknas Number 22 of 2006, one of the goals to be achieved in mathematics learning is to use reasoning on patterns and properties, do mathematical manipulation in making generalizations, working on proof or explain mathematical ideas and statements. Based on these objectives, it can be said that knowledge of how to prove is needed in learning mathematocs. However, all this time students still think that mathematics is just a scence of calculation. This based on the fact that the scoring system on the final math test is a student's skill in solving calculation problems. This view is unconsciously separating mathematics froms mathematical proof which is an important component of mathematics. According to Hanna, as quoted by Abdussakir (2014), mathematical proof is the core of mathematical thinking, so it can be said that someone has not studied mathematics except if he has learned what and how the mathematical proof is. However, mathematical proof is a difficult mathematical concept for students to study and compile it. Faruq's research (2014) shows that the ability to compile mathematical proof in high school students is still low. Students are used to using formulas in analyzing matehmatical problems,

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so sometimes students work on problem proving like a question that must be known how is the results. The failure to fulfill this standars does not only occur at the high school level. Based on the research of Nurrahmah & Karim (2018) and Junaedi (2012), it shows that college students still having difficulty in understanding and compiling proof. This shows that the experience of students in high school in compiling proof will have an impact on proof ability when they attend college. Form the result of these research can be seen that the ability to prove need to be studied at the high school level.

Algebra is one of the fields of study in mathematics. Algebraic abilities is very important because algebra will be the main requirement when students learn a material tha tinvolves the form of algebra in the next stages. In the learning process that requires mastery of the basic concepts of algebra, algebraic thinking skills is needed. Kieran (2004) stated that algebraic thinking is thought process that involve ways of thinking using algebraic symbols, such as analyzing relationship between quantities, noticing structure, studying change, generalizing, problem solving, modelling, justifying, and proving. Algebraic thinking is important because it can expand the thinking needed to solve concrete problems using abstraction and operate on mathematical entities logically (Windsor, 2010).

According to Kieran (2004), in solving algebraic problem students do several activities, that is generational activities, transformational activities, and global meta-level activities. The generational activities are algebraic abilities which include the forming of the expressions and equations. Then, transformational activities are algebraic abilities related to factoring, simplifying expressions, working with equivalent expressions and equations. While global meta-level activities are capabilities that involve algebra is used as a tool in solving algebraic problems, modeling, generalizing, justifying and proving.

Algebraic thinking in proof and reasoning must be introduced to children since elementary school. This is accordance with the rules of the NCTM 2000 Principle and Standars for School Mathematics, which state that proof and reasoning must be introduced from elementary to high school. Thus learning about mathematical proof needs to be applied in high school.

All this time, algebraic thinking skills of high school students are still in the low category. This is strengthened by data from TIMSS in 2011. Based on these data, the percentage of algebraic thinking skills of Indonesian students is below the International average in all aspect, and algebra is the content that has the lowest score that is 22%. This also happended in Senior High School 1 Bojong. Based on results of the National Examination in 2018, it was shown that the percentage of students in Senior High School 1 Bojong about the ability to solve problems related to algebra is still below the district, province, and national averages.

To find out the causes of student's low algebraic thinking skills in proof problems, an algebraic error analysis can be done. By doing an error analysis, we can find the cause and the type of the error. The analysis method that will be used in this study is the Newman error analysis. According to the Newman, as quoted by White (2009), to solve the problem statement there are five sequential steps that must be passed by students, that is (1) reading, (2) comprehension, (3) transformation, (4) process skill, and (5) encoding. If one of the stages is not fulfilled, it will cause errors that make the student's answer are incorrect.

Seeing the importance of students to master algebraic thinking skills and the difficulties of students in solving algebraic problems, it is necessary to have learning with a learning model to minimize errors made by students in solving math problems. One learning model that can be used is the CORE learning model. According to the Curwen et al. (2010), CORE model is a discussion model that can influence the development of knowledge and reflective thinking which has four stages of teaching, that is Connecting, Organizing, Reflecting, and Extending. The research related to algebraic thinking skills and CORE model, one of them is research conducted by Yumiati in 2014. The results showed an increase in algebraic thinking ability of CORE learning groups of students who were higher than conventional learning groups where the achievement of algebraic thinking ability of students in the CORE learning group was 33.94%, meanwhile the achievement of algebraic thinking ability of students in the conventional learning group students was 26.67%.

Besides determining the appropiate learning design, the teacher can also provide assistance to students who have difficulty. With this assistance, the teacher is expected to be able to overcome the problems faced by students, especially students in the lower group. According to the Vygotsky, this assistance is called scaffolding. Veeramuthu (2011) stated that scaffolding is support in learning and solving the problems. Wood, Bruner, & Ross, as quoted by Anghileri (2006), assistance is given in the early stages of learning, then the assistance is finalli removed when children can learn independently.

Based on the background described, the formulation of the problem to be studied in this study are as follows: (1) how the quality of mathematics learning used CORE model assisted by scaffolding on students' ability in global meta-level algebraic thingking of problem proving; (2) whether students' ability in global meta-level algebraic thingking of problem proving pass the minimun completeness criteria; (3) what is the type of students error in global meta-level algebraic thingking of problem proving based on Newman's procedur; (4) what is the cause of students error in global meta-level algebraic thingking of problem proving based on Newman's procedur; and (5) what is the type of scaffolding is given to lower group students to reduce students error in global meta-level algebraic thingking of problem proving.

2. Methods

The method use in this study is mixed method. The research design used is a concurrent embedded design, which is a research method that combines qualitative and quantitative research methods by mixing two methods out of balance. This research emphasizes more on qualitative methods than quantitative methods (Sugiyono, 2016). This research was conducted at senior high school 1 Bojong in the academic year of 2018/2019. The population of this research are students of class XI-IPA at SMAN 1 Bojong which consisted of 5 classes. While the sample of this research are students of class XI-IPA 3 as an experimental class given the CORE learning models. In quantitative research, the sample was selected using a simple random sampling technique. The research subjects were taken 9 students, each of 3 students from the uuper, middle, and lower groups. The selection of subjects for qualitative data in this study used purposive sampling technique which was chosen based on certain considerations, that is to find out the type and the causes of student's error in global meta-level algebraic thingking.

Data collection methods for qualitative data are observation, documentation, and interviewies, while for quantitative data is test method. This research uses participatory observation method, where the implementation of the researcher is directly involved with the activities of people who are being attended or used as a source of research data (Sugiyono, 2016). In this research the documentation in the form of audio recordings from interviews, test results for student's global meta-level algebraic thinking, and others. The interview method used is a semi-structured interview. Semi-structured interviews are free interviews where researchs do not use interview guidelines that have been systematically and completely arranged for data collection. The interview guide used is only in the form of outlines of the problems to be asked (Sugiyono, 2016). The test used in this study consisted of a global meta-level algebraic thinking ability test on problem proving. The test is in the form of a description test and consists of 4 question. The tests is used to find out student's algebraic thinking ability and the type of student's error in global meta-level activities. Before the test is conducted, the test is tested first on the trial class to determine the validity of the test which includes validity, reliability, level of difficulty, and the discrimination power of the test.

After the data is obtained then the data is analyzed quantitavely and quantitavely. Quantitative data analysis was used to know the completeness of students' ability in global meta-level algebraic thingking in CORE learning assisted by scaffolding. Quantitative data analysis technique include normality test and learning completeness test. The learning completeness test uses the average test with t test to find out the student's ability in global meta-level algebraic thingking on problem proving pass the minimun completeness criteria or not, and test of proportion with z test to find out the percentage of completeness of the experimental class students which has been specified that is a minimum of 75% of the number of students in the class get a scores ≥ 70 or not. While qualitative analysis is used to describe the quality of mathematics learning used CORE models assisted by scaffolding, describes the type and the causes of students error in global meta-level algebraic thingking of problem proving, and describe the type of scaffolding for lower group students to achieve minimal completeness. Data is analyzed by doing data reduction, data presentation, and verificatiton.

3. Results & Discussions

3.1. Data Quality of CORE Learning Assisted by Scaffolding

In this study, to find out the quality of mathematics learning in the CORE model, researchers looked at three stages of learning based on Suryosubroto's opinion (2009), that is (1) learning plan, including syllabus and lesson plan; (2) implementation of learning, including teacher activities and student activities; and (3) evaluation of learning outcomes, including quiz and tests of global meta-level algebraic thinking on problem proving. The results of the assessment of the quality of the CORE learning model for each aspect are presented in Table 1. below.

Table 1. Results of the Assessment of the Quality of the CORE Learning Model Assisted by Scaffolding

Learning Stages	Component	Percentage of Value	Criteria
Planning	Syllabus	94%	Very good
	Lesson Plan	94%	Very good
Implementation	Teacher Activities	87.5%	Very good
	Student Activities	76.67%	Very good
Evaluation	Quiz	79.98%	Very good
	Test	74.94%	Very good
Average		84.52%	Very good

Based on the Table 1. above, it can be seen that the syllabus and lesson plan in this research are suitable for use in CORE learning models assisted by scaffolding, and the planning of the learning has been well implemented. The results of the assessment of teacher activities and student activities showed that the teachers and students had implemented the CORE learning model assisted by scaffolding very well. The results of the quiz and the algebraic thinking test indicate that the students scores have passed the minimun completeness criteria, so that the evaluation of learning outcomes with the CORE model has been well implemented.

3.2. Test Results of Global Meta-Level Algebraic Thinking Ability

After carrying out learning in the experimental class and do an assessment with description test instruments as many as 4 items, data obtained the value of algebraic thinking ability on problem proving with an average of 74.94, the standard deviation is 11.10, the highest score is 95, and the lowest score is 48.

Data analysis of algebraic thinking test result include normality test and learning completeness test. The normality test was carried out using SPSS with the Shapiro-Wilk test. From the normality test using the Shapiro-Wilk test, it was found that data on global meta-level algebraic thinking abilities were normally distributed. The results of the test of student's ability in global meta-level algebraic thinking are said to reach completeness if the average student learning outcomes have reached the minimum completeness criteria that is 70, and if the percentage of students in the experimental class who get a score ≥ 70 is at least 75%. The calculation results show that the average score of student's algebraic thinking is 74.94. Furthermore, based on the results of the calculation of the right-side average test with $\alpha = 5\%$ obtained $t_{count} = 2.77$ and $t_{table} = 1.698$. Because $t_{count} > t_{table}$, it can be concluded that the average student learning outcomes have reached the minimum completeness criteria, that is 70. In addition, it was found that 87.5% of students in the experimental class scored ≥ 70. Based on the results of the calculation of the right-side proportions test with $\alpha = 5\%$ obtained $z_{count} = 1.69$ and $z_{table} = 1.64$. Because $z_{count} > z_{table}$, it can be concluded that the percentage of students in the experimental class who obtained a score ≥ 70 had achieved proportional learning completeness.

3.3. Research Subjects

After a global meta-level algebraic thinking ability test was carried out, then the results of the global meta-level algebraic thinking ability tests were divided into three groups, that is the upper group, the middle group, and the lower group. After obtaining the results of grouping global meta-level algebraic thinking skills, then from each group 3 subjects were chosen, so that in this study there were nine research subjects. The nine subjects of this study can be seen in Table 2. below.

Table 2. Research Subjects

Global Meta-Level Algebraic Thinking Ability		
Upper	Middle	Lower
$S > 86.04$	$63.8 \leq S \leq 86.04$	$S < 63.8$
(SA-1)	(SM-1)	(SB-1)
(SA-2)	(SM-2)	(SB-2)
(SA-3)	(SM-3)	(SB-3)

Note: S = score

3.4. Analysis of Upper Group Students Error

All research subjects from the upper group made mistakes on the questions given, although not all the questions they were doing were wrong. The following is the tendency of mistakes made by upper group students.

Table 3. The Tendency of Mistakes Made by Upper Group Students

Subject	Accumulated Errors					Tendency
	R	C	T	PS	E	
SA-1	0	0	1	2	2	Process skill and encoding
SA-2	0	0	0	3	3	Process skill and encoding
SA-3	0	0	1	4	4	Process skill and encoding

Based on the Table 3. above, it can be seen that upper group students tend to make mistakes in the process skill and encoding steps. At the process skill step, algebraic thinking ability that are not mastered, that is on the indicators able to use algebra as a mathematical proof tool. While at the encoding step, students write conclusions based on incorrect results of proving process.

3.5. Analysis of Middle Group Students Error

All research subjects from the middle group made mistakes on the questions given, although not all the questions middle group students. The following is the tendency of mistakes made by middle group students.

Table 4. The Tendency of Mistakes Made by Middle Group Students

Subject	Accumulated Errors					Tendency
	R	C	T	PS	E	
SM-1	0	0	1	3	0	Process skill
SM-2	0	0	2	4	3	Process skill
SM-3	0	0	2	4	0	Process skill

Based on the Table 4. above, it can be seen that middle group students tend to make mistakes in the process skill steps. At the process skill stage, algebraic thinking ability that are not mastered, that is on the indicators able to use algebra as a mathematical proof tool.

3.6. Analysis of Lower Group Students Error

All research subjects from the lower group made mistakes on the questions given. The following is the tendency of mistakes made by lower group students.

Table 5. The Tendency of Mistakes Made by Upper Group Students

Subject	Accumulated Errors					Tendency
	R	C	T	PS	E	
SB-1	0	1	0	4	3	Process skill
SB-2	0	0	4	4	0	Transformation and Process skill
SB-3	0	0	2	3	0	Process skill

Based on the Table 5. above, it can be seen that lower group students tend to make mistakes in the transformation and process skill steps. At the transformation step, algebraic thinking ability that are not mastered, that is on the indicators able to use algebra as mathematical modeling. While at the process skill step, algebraic thinking ability that are not mastered, that is on the indicators able to use algebra as a mathematical proof tool.

3.7. Data of Scaffolding Results to the Lower Group

The research subjects given the scaffolding were SB-1, SB-2, and SB-3. Scaffolding data is sourced from the results of a global meta-level algebraic thinking test on problem proving and interview. The process of giving scaffolding begins by showing the work results of a global meta-level algebraic thinking test and then giving scaffolding based on the difficulties experienced by the research subject. The following types of scaffolding are given to students in the lower group.

Table 6. Scaffolding are Given to Students in the Lower Group

Type of Errors	Scaffolding that is Suitable Given
Reading error	Not given scaffolding
Comprehension error	Not given scaffolding
Transformation error	Reviewing
Process skill	Explaining, Reviewing,
Error	Restructuring
Encoding error	Reviewing

Based on Table 6. above, it can be seen that the suitable scaffolding is given in the transformation step that is reviewing, the process skill step is explaining, reviewing, and restructuring, and the encoding step is reviewing. While in the reading and comprehension steps, students are not given scaffolding.

3.8. Discussion of Data Quality of CORE Learning Assisted by Scaffolding

Analysis of the results of the quality assessment of learning is done by data reduction, data presentation, and verification. In this study, data reduction was done by compiling a validation sheet for syllabus and lesson plans, teacher activity sheets and student activities neatly and processing them to obtain scores on the results of the assessment of each aspect of CORE model learning. The presentation of the data in this study was carried out by presenting data in the table to make it easier for researchers to present the results of the quality of the CORE learning model assisted by scaffolding. verification is done by considering the observations and suggestions or observers' notes on the instrument to obtain conclusions on the quality of the CORE learning model assisted by scaffolding. The results of the assessment of teacher activity in the implementation of the CORE learning model assisted by scaffolding showed that the final score was 87.5% with very good criteria. While the results of the assessment of student activity showed the final score was 76.67% with very good criteria. From the results of the assessment of teacher activities and student activities, it was shown that the teachers and students had implemented the CORE learning model assisted by scaffolding very well. Thus it can be concluded that the implementation of learning with the CORE model assisted by scaffolding has been done very well. The results of the quiz assessment at each meeting indicate that the average student gets a good score because 90.63% of students pass the minimun completeness criteria. While the results of the assessment of global meta-level algebraic thinking ability tests show that the average student gets a good score because 87.5% of students pass the minimun completeness criteria. Thus it can be concluded that the evaluation of learning outcomes with the CORE model assisted by scaffolding is well implemented.

Based on the description above, it can be concluded that quality of mathematics learning used CORE model assisted by scaffolding in very good quality.

3.9. Discussion of Completeness of Global Meta-Level Algebraic Thinking Ability on Problem Proving

Based on the results of the student's global meta-level algebraic thinking ability test on problem proving, it was found that the average meta-level algebraic thinking ability test results of proof problems had reached the minimum completeness criteria, taht is 70 and 87.5% of students in the experimental class scored ≥ 70. So that empirically students in the experimental class have achieved the specified classical learning completeness which is 75%. The statistical test used to test learning completeness is the right-side average test and the right-side proportion test. Based on the results of the average test and proportion test it can be concluded that the ability of students in meta-level global algebraic thinking on problem proving achieves the minimum completeness criteria.

This result is in line with the research conducted by Anggraeni (2015) and Safitri (2019). The results of Anggraeni's research (2015), show that students' problem solving abilities in cube and cuboids material using CORE learning can reach classical completeness, that is $> 75\%$ of students have reached the minimum completeness criteria, with a class average score is 75.30. The results of Safitri's research

(2019) show that the CORE learning model has a positive impact on improving student's mathematical reasoning ability. The application of the CORE learning model can improve student's mathematical reasoning ability from the first cycle by 47.06% and the second cycle by 82.35%.

There is a relationship between problem solving ability with algebraic thinking ability and between mathematical reasoning ability with algebraic thinking ability. On problem solving ability there are several indicators that have relevance to the algebraic thinking ability, that is (1) understanding the problem, namely being able to know what is known and what is asked, (2) planning a solution strategy that has relevance to ability to connect information in a problem to determine proving strategies, and make modeling on information, (3) implementing a solution strategy that has a relationship with the ability to do proof procedures according to the strategy chosen correctly, and (4) checking the results, that is re-check the results obtained. In mathematical reasoning ability there are several indicators that have relevance to algebraic thinking ability, that is (1) able to do mathematical manipulations that have relevance to the ability to do modeling to facilitate the proving process, and (2) draw a conclusion, compile evidence, provide reasons or evidence for the correctness of the solution that has relevance to the ability to do the proving process.

3.10. Discussion of the Types and Causes of Upper Group Students Errors

Based on the results of data analysis, upper group students tend to make mistakes in the process skill and encoding steps. Each subject made a process skill error in several questions. For question number 1 all subjects made mistakes. For question number 2, the subjects who made mistakes were SA-2 and SA-3. For question number 3 the subjects who made mistakes were SA-2 and SA-3. While for question number 4 all subjects made mistakes. Process skill errors are indicated by subjects who are unable to carry out the proving process correctly after determining the proving strategy. This error includes: (a) errors in calculating the multiplication of exponent numbers with different bases, (b) errors in calculating addition of fractions with different denominators, (c) errors in writing formulas so the proving results are wrong, and (d) not able to complete the proving process. The cause of this error is the inaccuracy of students in carrying out the calculation process, not carefully writing down the results of the calculation, and the inability of students to complete the proving process so that they often do not continue the proving process and immediately write the final results. While in the encoding error the subjects who made mistakes were SA-1, SA-2, and SA-3. SA-1 made an encoding error in questions number 1 and 4, while the SA-2 subject made a mistake in questions number 1, 3, and 4. Encoding error is indicated by the carelessness of the subject in writing conclusions so that the conclusions written are not in accordance with what will be proven (incomplete) and often conclusions are written based on the results of proving process that is not correct. The cause of this error is the lack of accuracy of students in writing down the variables in conclusion sentences.

This result is in line with research conducted by Renoningtyas (2016), which states that upper group students tend to make mistakes in the process skill and encoding steps. This error is caused by students not understanding the material given. This is consistent with the statement of Minggi (2016), which states that the causes of student difficulties in proving mathematics are due to a lack of understanding of mathematical proof and a lack of understanding of mathematical concepts and principles. Based on the description above, it can be taken the relationship between the type of student errors, that is in the process skill stage, which is not mastering the indicators able to use algebra as a mathematical proof tool.

The Agoestanto's research (2019), showed that upper group students made algebraic errors at the encoding step. The cause of the error is because students are not careful in writing concluding sentences, and incorrectly write down the mathematical units used. Based on this matter, it can be taken the relationship between the type of student's mistakes in the encoding step which is not able to write conclusions according to what has been proven.

In this research, another fact was found that the upper group students made mistakes in the transformation step. In the transformation step, the subjects who made mistakes were SA-1 and SA-3. SA-1 made a mistake in number 1. This is because SA-1 is not accurate when writing variables in formula $P(n)$. While SA-3 made a mistake in question number 3 because did not write it in the form of sigma notation. The cause of this error is the lack of student's accuracy in making an mathematical model.

3.11. Discussion of the Types and Causes of Middle Group Students Errors

Based on the results of data analysis, middle group students tend to make mistakes in the process skill steps. Process skill errors which is conducted by SM-1 can be seen in questions number 1, 2, and 4. While process skill errors which is conducted by SM-2 and SM-3 can be seen in questions number 1, 2, 3, and 4. Process skill errors are indicated by subjects who are unable to carry out the proving process correctly after determining the proving strategy. This error includes: (a) errors in calculating process, (b) errors in substituting the value of a sigma notation, (c) errors in writing formulas so the proving results are wrong, and (d) not able to complete the proving process. The cause of this error is the inaccuracy of students in carrying out the calculation process, not carefully writing down the results of the calculation, and the inability of students to complete the proving process so that they often do not continue the proving process and immediately write the final results.

This result is in line with research conducted by Agoestanto (2019), which states that middle group students tend to make mistakes in the process skill step. These errors include errors in the process of mathematical calculations to find solutions. The cause of the error is because students are not careful in carrying out the calculation process. Based on the description above, it can be taken the relationship between the type of student errors, namely in the process skill step, which is not mastering the indicators able to use algebra as a mathematical proof tool.

In this study, another fact was found that the middle group students made mistakes in the transformation and encoding step. In the transformation step, the subjects who made mistakes were SM-1 and SM-2. SM-1 made a mistake in number 4. This is because SM-1 is not accurate when writing variables in formula $P(n)$. While SM-2 made a mistake in question number 3 and 4. This is because SM-2 is not accurate when writing variables and x^{th} term in formula $P(n)$. This error includes: (a) errors writing variables in modeling, and (b) errors writing the form of sigma notation. The cause of this error is the lack of student's accuracy in making an mathematical model. While in the encoding error the subjects who made mistakes was SM-2. SM-2 made an encoding error in questions number 1 and 4, while the SA-2 subject made a mistake in questions number 1, 2, and 4. The conclusions written by SM-2 immediately answered the core of the problem. The cause of this error is that SM-2 wrote a conclusion sentence based on an inaccurate proof process.

3.12. Discussion of the Types and Causes of Lower Group Students Errors

Based on the results of data analysis, lower group students tend to make mistakes in the transformation and process skill steps. In the transformation step, the subject who made the mistake was SB-2 and SB-3. SB-2 made a mistake in number 1, 2, 3, and 4. This is because SB-2 is not accurate when writing variables in formula P(n). While SB-3 made a mistake in question number 1 and 4. This is because SB-3 is not accurate when writing variables and x^{th} term in formula $P(n)$. This error includes: (a) errors writing variables in modeling, and (b) errors writing the form of sigma notation. The cause of this error is the lack of student's accuracy in making an mathematical model. While in the process skill error the subjects who made mistakes was SB-1, SB-2, and SB-3. SB-1 made an process skill error in questions number 1, 2, 3, and 4. SB-2 made an process skill error in questions number 1, 2, and 3. While made an process skill error in questions number 1, 2, 3, and 4. Process skill errors are indicated by subjects who are unable to carry out the proving process correctly after determining the proving strategy. This error includes: (a) wrong in writing down the results of the calculation, (b) wrong in substituting the value of a sigma notation, and (c) not able to complete the proving process. The cause of this error is the inaccuracy of students in carrying out the calculation process, not carefully writing down the results of the calculation, and the inability of students to complete the proving process so that they often do not continue the proving process and immediately write the final results.

This result is in line with research conducted by Elisa (2016), which shows that lower group students tend to make mistakes in the transformation, process skill, and encoding steps. This error is caused by students not being able to plan work steps to solve the problem, calculation process errors, and not carefully write down the answers. According to the Crouch & Haines (2008), one aspect of student difficulties in making modeling is connecting real-world problems with mathematical models. Maas (2006) also states that the form of errors that occur when students do modeling problems include the difficulty of students to create a relationship between reality and mathematics, simplify and construct reality, and other problems related to mathematical solutions. Based on the description above, it can be taken the relation between the type of student errors, that in the transformation and process skills, which is not mastering the indicators to be able to use algebra as mathematical modeling and use algebra as a mathematical proof tool.

In this study, another fact was found that the lower group students made mistakes in the comprehension and encoding step. In the comprehension step, the subjects who made mistakes was SB-1. SB-1 made a mistake in number 3. This error is indicated by the incomplete subject in writing what will be proven. The cause of this error is the lack of accuracy of students in writing what will be proven, so that it is not written down. While in the encoding error the subjects who made mistakes was SB-1. SB-1 made an encoding error in questions number 1 and 2. Encoding error is indicated by the carelessness of the subject in writing conclusions so that the conclusions written are not in accordance with what will be proven (incomplete) and often conclusions are written based on the results of proving process that is not correct. The cause of this error is the lack of accuracy of students in writing down the conclusion sentences.

3.13. Discussion of Type of Scaffolding For Lower Group Students

The results of the error analysis and interviews that have been conducted with the lower group students on the global level-meta level algebraic thinking test on problem proving based on the Newman procedure then used as the basis for implementing scaffolding. The process of giving scaffolding in the interview session was carried out after seeing mistakes made by the lower group students. The types of scaffolding provided include: (a) explaining, (b) reviewing, and (c) restructuring. The following is a discussion of the type of scaffolding given based on the type of error made.

3.13.1. Reading Error

Based on the results of the interview, no lower group students made mistakes in the reading step, so that in this step no scaffolding was given.

3.13.2. Comprehension Error

Based on the results of the global meta-level algebraic thinking test and interview results, there were no students from the lower group who made comprehension error, so in this step no scaffolding was given.

3.13.3. Transformation Error

Based on the results of the global meta-level algebraic thinking test on problem proving, there is 1 student who made a mistake in the transformation step, that is SB-3. Based on the mistakes made, SB-3 was given a scaffolding reviewing, by asking the subject to re-chek the information that was known and then asked him to write down the modeling and formula $P(n)$.

This is in line with Putri's research (2017), which states that in the transformation step the form of scaffolding used is reviewing. Reviewing is repeatedly given to students to check the results of student work so that in the future students are more careful in doing modeling. Schukajlow et al. (2015), also stated that the use of scaffolding is useful for directing the way students work in solving modeling problems.

3.13.4. Process Skill Error

Based on the results of the meta-level global algebraic thinking test on problem proving, all lower group students made mistakes in the process skill step. Based on the mistakes made, the lower group students are given scaffolding explaining, by explaining the verification process correctly, then reviewing that is asking the subject to re-check the formula $P(n)$ then asking him to correct the error in formula $P(k)$, asking the subject whether there is a calculation error and showing the type of the subject error when writing the results of the calculation then asking him to make corrections. and restructuring, that is by giving questions and examples that can build students' understanding to direct the subject to find the final proof, asking the subject to see the final result (to be found), then asking him to connect with the results of calculations that have been done to find the final results of the proving process, and asking the subject to prove using existing modeling.

This is in accordance with the Fatahilah research (2017), which shows that the form of scaffolding given to students with process skill errors is (1) explaining, explaining the correct rules in make calculation operations (2) reviewing, asking students to correct the calculations with the answers written previously, and (3) restructuring, do questions and answers to guide students to get the right solution.

3.13.5. Encoding Error

Based on the results of the global meta-level algebraic thinking test on problem proving, there is 1 student who made a mistake in the encoding step, that is SB-1. Based on the mistakes made, SB-1 was given reviewing scaffolding, by asking the subject to look back on what was to be proved, then asked him to complete the conclusion sentence.

This is in line with the research conducted by Rahayuningsih & Qohar (2014), which states that the form of scaffolding for encoding errors is to review the work (reviewing).

Based on the results of data analysis, the form of scaffolding given to lower-class students SB-1, SB-2, and SB-3 has progressed. This can be seen from the results of remedial tests given to lower group students after being given a scaffolding. SB-1 initially tends to make mistakes in the process skill and encoding steps. After being given a scaffolding, SB-1 has not made a mistake again in the process skill and encoding steps so that there is an increase. SB-2 initially tends to make mistakes in the process skill step. After being given a scaffolding, SB-2 has not made a mistake again in the process skill step so that an increase also occurs. Although overall the SB-2 was able to solve the questions given, SB-2 still made a mistake in the encoding step. SB-3 which initially tends to make mistakes in the process skill step. After being given the SB-3 scaffolding, there was no mistake in the process skill step, so there was an increase.

Based on the assessment of the results of student’s work on remedial tests, it was obtained that the score data of the lower group students' algebraic thinking skills on problems proving had achieved minimal completeness. Other than that there was a change in the category of groups in the lower group students. The results of the assessment showed that the lower group students had increase ability in the global meta-level algebraic thinking on problem proving and can be categorized in the upper group.

The results of the scaffolding that given to students show that some students can correct their mistakes but there are still students who make mistakes, but these errors are fewer than the mistakes in the previous work. This can be caused when given scaffolding students are less focused. This is in line with the results of research conducted by Stillman et al., as quoted by Nuryadi (2018), who concluded that the final solution of students in solving problems was influenced by the scaffolding given. So that it can be concluded that the scaffolding given has succeeded in reducing errors made by students, especially lower group students.

4. Conclusion

Based on the results and discussion of this research can be conclude that (1) the quality of mathematics learning used CORE model assisted by scaffolding on student’s ability in global meta-level algebraic thingking in very good criteria; (2) students' ability in global meta-level algebraic thingking on CORE learning assisted by scaffolding pass the minimun completeness criteria; (3) upper group students tend to make some error in the process skill and encoding steps, middle group students tend to make some error in the process skill step, while lower group students tend to make some error in the transformation and process skill steps; (4) the cause of process skill errors in the upper group students is the inaccuracy of students in carrying out the calculation process, and the inability of students to complete the proving process so often they do not continue the proving process and immediately write the final results. While the cause of encoding errors is the lack of accuracy of students in writing variables in conclusions sentence so that the conclusions written do not match what will be proven. The cause of process skill errors in the middle group students is the inaccuracy of students in carrying out the calculation process, and the inability of students to complete the proving process so often they do not continue the proving process and immediately write the final results. The cause of transformation errors in lower group students is the lack of accuracy of students in writing variables and modeling on problem information. While the causes of process skill errors are student’s lack in carrying out the calculation process, not carefully writing down the results of the calculation, and the inability of students to complete the proving process so that they often do not continue the proving process and immediately write the final results; (5) the type of scaffolding that given to lower group students is based on the type of error made, scaffolding reviewing is given for transformation and encoding errors, scaffolding explaining, reviewing, and restructuring are given for process skill errors.

Suggestions that can be given relating to the results of this study are: (1) CORE learning model assisted by scaffolding has good quality, therefore CORE model can be used as an option in classroom learning; (2) mathematics teachers should ensure that students have completed material such as mathematical and algebraic modeling and their operations so that transformation and process skills error can be minimized; (3) mathematics teachers should familiarize students to complete the whole question from writing what is known until the conclusion. This is expected to minimize comprehension and encoding errors; (4) mathematics teachers should be able to provide scaffolding to students who need help to minimize errors in global meta-level algebraic thinking on problem proving.

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