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Analyzing the effect of using the geoboard for learning the Pythagorean Theorem

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Abstract

This research considered the need to implement didactic materials during learning the Pythagorean theorem as one of the topics present in mathematics curricula around the world. The objective was to analyze the effect of the use of the geoboard in the learning of the Pythagorean theorem with high school students. The contributions of the theory of figurative concepts and concrete didactic materials were considered. The method used was a case study design by applying a task with concrete material with ten students. Among the main results, the construction of figures on the geoboard was obtained by analyzing the students' mental images. It was also possible to demonstrate visual skills and reasoning development when implementing this material. In conclusion, the effect of use of the geoboard in the study of the Pythagorean theorem allowed the recognition of the difference in the unit of measurement between the legs and the hypotenuse. Also, the development of the skills allowed the students to build the concept of the relationship between the figures built within the square of the hypotenuse with the area of the squares corresponding to the legs.

1. Introduction

Traditionally, the teaching and learning of geometry have been developed mechanically, applying formulas and theorems without relating them to the student's previous knowledge, generating difficulties in their study. This is due to the complexity of mathematical concepts, which cannot be fully understood when they are presented for the first time (Mora, 1995). The concept of the Pythagorean theorem is among the most common and remembered mathematical content in secondary education (Gómez-Sánchez et al., 2020; Wares, 2017).

The theorem and its demonstrations are crucial topics in the school mathematics curriculum. Avila (2019) mentions that its demonstration initiates the deduction and reasoning of geometric problems in students, and its study has allowed identifying some difficulties in the learning processes. For example, they do not identify right triangles when the legs are not parallel to the edges of the paper (Troyano and Flores, 2016).

On the other hand, Conde-Carmona and Fontalvo-Meléndez (2019) evidenced how students had difficulty identifying and locating the values of the legs and hypotenuse or vice versa. The above could be due to the lack of knowledge of the relationship between the right triangle's sides, in addition to not considering that the modification in one of its legs implies a change in the other leg and the hypotenuse (Moreno et al., 2020).

For most students, applying the Pythagorean theorem is not exclusive to right triangles. They do not relate this theorem to its geometric meaning, applying the formula and conventional resolution techniques

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to decontextualized problems (Barrantes et al., 2018; Iglesias, 2017). It is one of the concepts least understood by students when interpreting its meaning, usefulness, and application in different contexts or situations (Gómez-Sánchez et al., 2020). The problem is due, in part, to its traditional teaching focused on memorizing the formula $a^2 + b^2 = c^2$ (Moreno et al., 2020).

The above difficulties may arise from the lack of knowledge of the characteristics of the mental entities used by geometry, specifically in this theorem. Fischbein (1993) worked on the Theory of Figural Concepts, where he defined geometric figures as mental entities characterized by their concept and figure. He also mentions the association or symbiosis between these aspects in geometric reasoning. To understand Geometry taking into account the mental entities, one must understand the relationship between these characteristics (Teófilo de Sousa et al., 2022). In the Pythagorean theorem, we work with the right triangle, the square, and their respective characteristics as mental entities.

In order to provide a possible solution to the difficulties mentioned above, learning based on Vygotsky's sociocultural approach is assumed, where students actively stimulate their mental processes through interaction with their peers, which occurs in different contexts and is channeled by the use of the language used (Salas, 2001). Including manipulative mathematical materials gives rise to sensory interaction between students and the concrete object, fostering conscious and unconscious mathematical thinking (Swan & Marshall, 2010). Likewise, implementing manipulative materials allows the stimulation and development of learning in the mathematics classroom.

In this study, the geoboard was incorporated as concrete didactic material through the construction of geometric figures, which allowed visualizing the demonstration of the Pythagorean theorem. In this sense, students must experience constructing a theorem by themselves and, therefore, that of their geometric knowledge, with very particular and different characteristics compared to other types of knowledge (Arrieta et al., 1997). Therefore, in the present research, we intend to analyze the effect caused by using the geoboard as didactic material for learning the Pythagorean theorem in high school students.

1.1. Concrete teaching materials for learning the Pythagorean theorem

Upon identifying the difficulties associated with learning the Pythagorean theorem, it is necessary to design and implement an activity in which concrete didactic materials are used to help reduce this problem in the classroom. These objects are used by the teacher and students in mathematics teaching and learning processes to achieve specific objectives (Villarroel & Sgreccia, 2011). These materials can help to build, understand or consolidate concepts, exercise and reinforce procedures, and influence attitudes during the student's learning process. According to Wares (2019), when students have objects they can manipulate, they communicate better with their teacher.

The activity design with concrete didactic materials for teaching and learning a topic allows interaction among students and the development of mathematical ideas that facilitate the learning of the mathematical object. For this reason, Villarroel and Sgreccia (2011) identified and characterized concrete didactic materials that can be implemented in the teaching and learning geometric content in secondary education. Among these didactic materials are the Dienes logic blocks, geometric puzzles, the tangram, and the geoboard, among others.

Research shows that using these materials allows learning from concretizing their physical expression (Avila, 2019; Iglesias, 2017; Troyano & Flores, 2016; Wares, 2019). The Pythagorean theorem is presented as an open situation that accepts several possibilities and ways of working (Barrantes et al., 2018).

1.2. The geoboard and its relation to learning the Pythagorean Theorem

The geoboard should be considered a necessary didactic material for teaching and learning secondary education mathematics (Arrieta et al., 1997). This object is composed of a flat base with embedded laces. Geometric figures can be built on them using colored rubber bands, as shown in Figure 1.

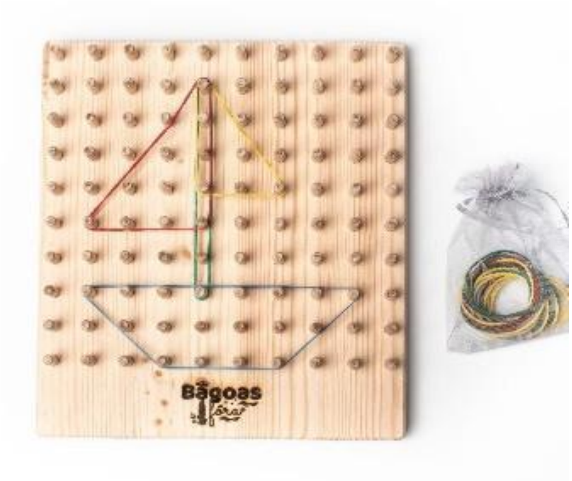


Figure 1. Geometric figures with rubber bands on the geoboard

The geoboard allows the development of visual skills through the construction of geometric figures. Communication; to interpret, name and define mathematical contents. Logical reasoning; to abstract characteristics and properties or to argue and explore figures; as well as application in other contexts (Villarroel and Sgreccia, 2011). For this reason, the implementation of this didactic material for learning the Pythagorean theorem in the classroom becomes relevant. The geoboard allows highlighting aspects derived from perimeter and area, for example, the unit of measurement is always present (Chamorro, 2005). The processes of triangulation and quadriculation are done without difficulty and the perpendicularity allows the Pythagorean theorem to emerge with ease (Mora, 1995).

1.3. Theory of Figural Concepts

For Fischbein (1993), geometry works with mental entities called geometric figures that simultaneously have conceptual and figural characters. This symbiotic relationship gives rise to figural concepts which intrinsically have a definition and an image. This same author defines the following entities:

Concept. They manifest an idea, a general ideal representation of a class of objects, supported by their common features.

Image. Refers to mental images as a sensory representation of an object or phenomenon.

Figural concepts. The figural concept is a mental reality. It is the construct worked by mathematical reasoning in the domain of geometry. It does not have concrete sensory properties such as color, weight, density, etc. but it has figural properties. Likewise, its figural construction is controlled and manipulated, at first without residues, by rules and logical procedures in the context of a specific axiomatic system. A figurative concept is also meaning. Its characteristic is that it includes the figure as an intrinsic property.

Figural concepts are a third mental entity different from image and figure. The objects of manipulation and investigation in geometric reasoning are mental entities called figural concepts. They show spatial properties such as shape, position, magnitude, and at the same time, they have conceptual qualities such as ideality, abstraction, generality, perfection (Fischbein, 1993).

For Teófilo de Sousa et al. (2022), the image that the student obtains of an object is the figural concept. This image is constructed based on the actions performed with the object. The school level, the age of the students, and the didactic aspects included in the geometry teaching process are taken into account. As students age, the association between the figurative and the conceptual is improved (Fischbein, 1993).

The teaching of geometry begins with the development of algorithms and techniques that favor concepts through formal definitions, demonstrations, and exercises that do not contribute to the cognitive development of the student's geometric reasoning (Teófilo de Sousa et al., 2022). For this reason, this study focuses on learning the Pythagorean theorem mediated by the geoboard, taking into account the integration process between figure and concept in students. Therefore, the present investigation had as objective to analyze the effect of the use of the geoboard in the learning of the Pythagorean theorem with high school students.

2. Methods

The present research was qualitative (Hernández-Sampieri & Mendoza, 2018) with a case study design (Galeano, 2012). The effect caused by the use of the geoboard as didactic material in the learning of the Pythagorean theorem was analyzed. The task was applied to a group of students of a public high school in a rural area of the State of Puebla, Mexico. There were ten participants, four females and six males, aged between 13 and 14.

The task was designed and validated by the research group and other collaborators. This task was organized in three moments and implemented during a 90-minute class session. The students were organized into working groups with the codes G1, G2, G3, and G4 of two (G1, G3) and three (G2, G4) members. The grouping was done in order to analyze their interactions and strategies carried out by them. In order to identify the participation of each student in the different groups, codification was used G1_1, ..., G1_3; G2_1, ..., G2_3; and so on.

First, students constructed horizontal, vertical, and diagonal segments by measuring their respective lengths, as Arrieta et al. (1997) suggested. Secondly, they calculated the area of a triangle by comparing the inscription of one of the squares formed with a leg in the square formed with the hypotenuse. Third, they performed the previous procedure with different measurements for each leg and again observed the relationship between the figures inscribed in the square formed with the hypotenuse and the squares formed with the legs.

Data were collected using worksheets, participant observation, and group interviews (Cohen & Manion, 2002). The first two researchers (R1 and R2) conducted the latter, video recordings, and photographs. The information was analyzed from the students' constructions on the geoboard by triangulating the video recordings, transcriptions, and photographs to gather the information obtained. These activities were done to demonstrate the geoboard's effect on learning the Pythagorean theorem implemented as didactic material.

3. Results & Discussion

The data collected from the activities implemented in the classroom are presented below. It is important to note that the results presented correspond to groups G1, G2 and G4. The results of the G3 group were not included in this report because they did not show an active commitment to the proposed tasks.

In the first activity, all groups constructed vertical, horizontal, and diagonal segments on the geoboard with colored rubber bands (Figure 2a). In addition, they measured their lengths, taking into account the given unit of measurement, without realizing that this unit is not the same as the diagonal (Figure 2b).

(a)

(b)

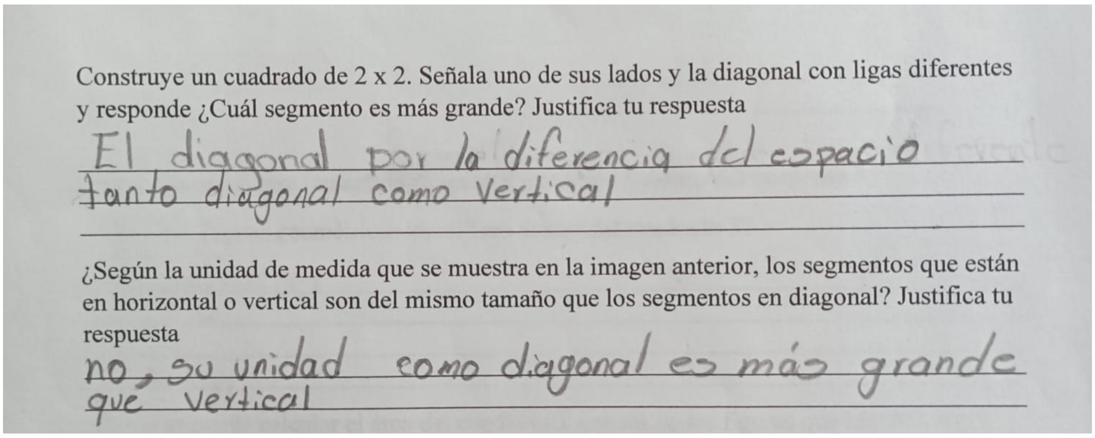
Figure 2. (a) Segments constructed by students on the geoboard; (b) Unit of measurement.

Then they built a 2x2 square (Figure 3), pointing out one of its sides and the diagonal of the polygon, to identify that the way to measure the mentioned segments is not the same. In this regard, within the G1 team, it was expressed:

Figure 3. Square of 2x2 built on the geoboard with its diagonal

R1: Of the segments you built, which segment is larger?
G1_1: The diagonal [sic] because it looks more stretched than the garter
R1: Why does it look more stretched?
G1_1: These points are farther apart (pointing to the points that are part of the diagonal) (Group 1, personal communication, May 19, 2022).

Likewise, team G4 took the unit of measurement into account in their analysis and answered the activity questions as seen in Figure 4:



Construye un cuadrado de 2 x 2. Señala uno de sus lados y la diagonal con ligas diferentes y responde ¿Cuál segmento es más grande? Justifica tu respuesta

El diagonal por la diferencia del espacio tanto diagonal como vertical

¿Según la unidad de medida que se muestra en la imagen anterior, los segmentos que están en horizontal o vertical son del mismo tamaño que los segmentos en diagonal? Justifica tu respuesta

no, su unidad como diagonal es más grande que vertical

Figure 4. Answer of G4 in activity 1

In addition, in the group interview, they further clarified their arguments by answering the following:
R2: Why do you think the diagonal is bigger?
G4_1: Because this space (referring to the separation between the points of the diagonal) is bigger than the unit.
R2: How did you realize that?
G4_2: The way you look at it, this one is bigger (the diagonal) because the garter stretches more (Group 4, personal communication, May 19, 2022).

The above shows how the work with the geoboard allowed a better visualization when constructing figures and their characteristics. In addition, it was possible to identify how the students could establish the difference when measuring the horizontal segment and the diagonal within the square. The teaching of geometry through concrete materials should be oriented to the development of specific skills. In this sense, Villarroel and Sgreccia (2011) mention some of these skills. For example, visual skills help to understand concepts by identifying figures and their properties.

In the second activity, the groups constructed a right triangle and its corresponding squares (Figure 5). Then, observing the figures, they calculated the area of the square built on the hypotenuse. In this part, within group G2 the following was expressed:

R1: How can you calculate the area of the square constructed with the hypotenuse by constructing a square inside it?
G2_1: [look at the figures built on the geoboard.
R1: What figures did you identify?
G2_1: A square and triangles.
R1: How many?
G2_1: Four triangles and a square.
R1: The square (inside - yellow), which one does it look like?
G2_1: The other two (built with the legs).
R1: What happens if we put the four triangles together?
G2_1: Another square is formed.
R1: Different or equal to the ones you have?
G2_1: Same (Group 2, personal communication, May 19, 2022)

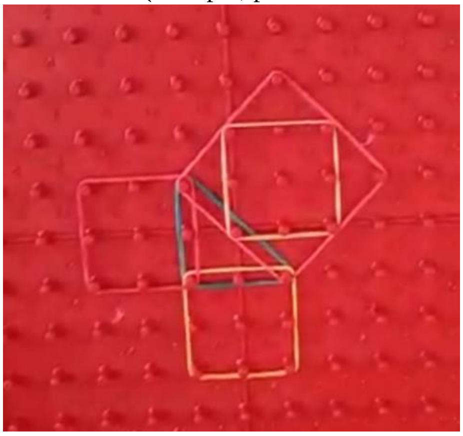


Figure 5. Construction of a right triangle with corresponding squares

Group G4 divided the squares constructed with the legs to cover the square corresponding to the hypotenuse (Figure 6), showing the following:
R1: how could you calculate the area of the square built on the hypotenuse?
G4_1: divide these (pointing to the squares corresponding to the legs) to form the large one.
R2: when you say divide them, what do you mean?
G4_1: make them in small parts (the small square) to accommodate it here (pointing to the large square).
R2: Is this square you made here (pointing to the square inscribed in the pink square) the same as any other?
G4_2: yes, to these (pointing to the squares corresponding to the legs).
R2: these triangles that are here, what happens with them?
G4_2: joining them together forms the other square. (Group 4, personal communication, May 19, 2022)

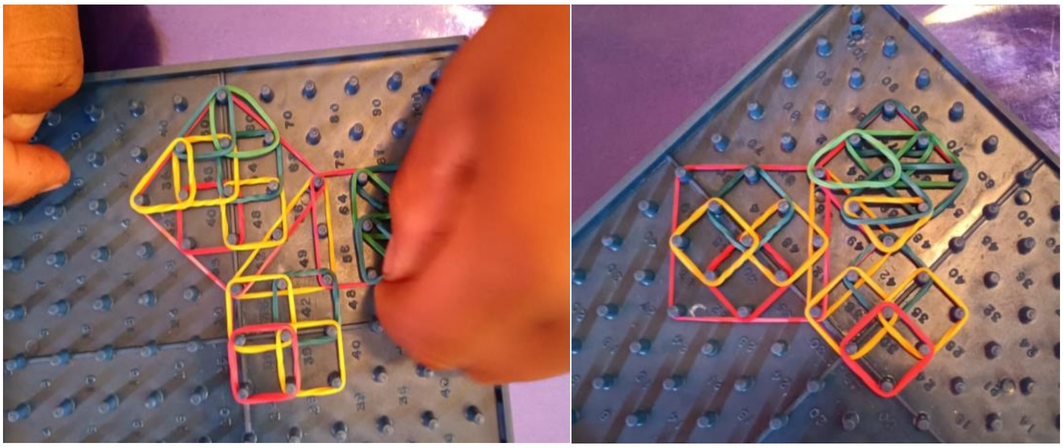


Figure 6. Division of the squares corresponding to the legs into smaller

At the beginning of this activity, it could be identified that the G2 team could not calculate the area of the square of the hypotenuse. However, after asking the students some questions about the figures they constructed, they could establish a relationship between the figures formed inside the square corresponding to the hypotenuse and those formed by the squares corresponding to the legs.

The G4 group divided the squares of the legs into smaller figures -squares and triangles- to cover the square of the hypotenuse. This action is explained by the mental entities that students form when imagining a geometric figure with spatial properties and conceptual qualities (Fischbein, 1993). This phenomenon is when we expect to obtain a quadrilateral by joining two equal right triangles. In this situation, the person generates a mental image based on his or her practical experience combining the concept and geometric figure.

After working with the geoboard, G4 was asked to explain their reasoning for the area calculation, followed by transcribing the theorem on their worksheet (Figure 7). Students expressed the following:

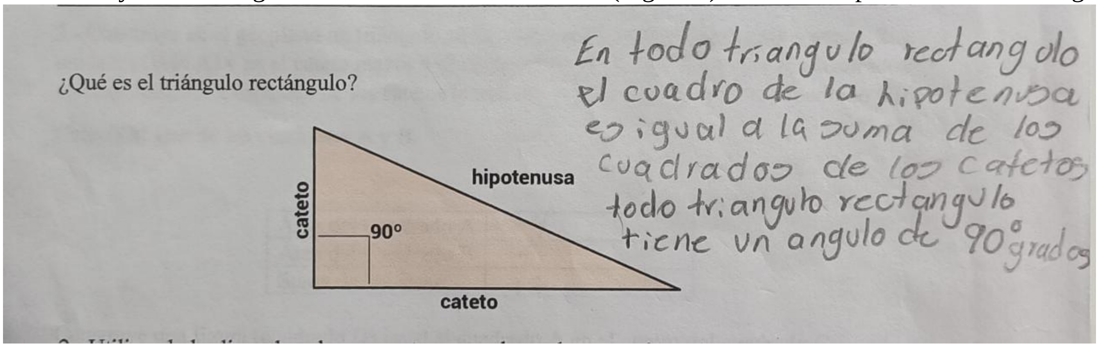


Figure 7. The idea on the Pythagorean theorem written by G4

R2: How would you do it to know the whole area of the pink square (square of the hypotenuse)?
G4_2: By adding the legs, the sum of the legs equals triangle 3.
G4_1: Square 3 (affirms).
G4_2: Aha, square 3.
R2: Which legs?
G4_2: This one and this one (pointing to the squares 1 and 2 of the legs), adding them together equals square 3. (Group 4, personal communication, May 19, 2022)

As could be observed in the explanations of G4, they establish a clear relationship between the figures constructed in the geoboard with the formal definition of the Pythagorean Theorem writing: *"In every rectangle triangle the square of the hypotenuse is equal to the sum of the squares of the legs"*. The objects or materials implemented didactically in the classroom have the potential to generate awareness and development of concepts and ideas related to mathematics (Swan & Marshall, 2010). As an effect of the implementation of the geoboard, students were able to build such a concept by developing visualization and reasoning skills. In addition, this material allowed to relate the shapes of the figures built within the square of the hypotenuse and their relationship with the area of the squares corresponding to the legs.

4. Conclusion

Taking as evidence the products elaborated by the students, as well as the answers given in the group interviews, it was possible to point out that the implementation of didactic materials for the teaching of Geometry allows for guiding the development of specific skills (Villarroel & Sgreccia, 2011). In this study, it was possible to identify several skills, such as visual and drawing, distinguished in constructing geometric figures within the geoboard. In addition, they served as scaffolding by allowing the recognition of the characteristics and properties of the figures involved. In addition, the manipulation in the use of concrete materials allowed evidence of the concepts and internal visual images that the students have when constructing their external representations, which serve as a study to analyze the geometric properties they dominate.

On the other hand, as a result of the group interviews, it was possible to identify the use of logical or reasoning skills, which was first manifested when the teams distinguished and argued the difference in the unit of measurement of the segments that are horizontal and vertical, with those that are built diagonally. This ability to distinguish allowed the students to develop conjectures about the geometric properties and concepts of their studied topics. In the case of the Pythagorean Theorem, students could recognize the properties of the measures of the legs and hypotenuse in the right triangle.

In a second moment, the students could recognize the geometric demonstration of the Pythagorean Theorem. This understanding was reflected when the teams pointed out that they could form the squares of the legs with the figures constructed within the square of the hypotenuse. With the latter, the capacity for abstraction and logical deduction was demonstrated by relating the geometric properties of this theorem with its formal demonstrations. In this way, some students constructed the definition of the theorem by relating the legs and the hypotenuse with the figures elaborated in the geoboard. Such evidence shows the two mental entities mentioned by Fischbein (1993): concept and figure and their symbiosis as figural concept.

Finally, it was possible to analyze those shapes and figures imagined and externalized by the students as part of their mental entities when making the constructions on the geoboard, establishing the relationship between their spatial properties to achieve progress in learning the theorem. In addition, the possibility is left for future research, for example, in learning the similarity of triangles by implementing the geoboard as concrete didactic material.

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