UJME 12 (2) 2023: 153-166
UNNES JOURNAL OF MATHEMATICS EDUCATION
https://journal.unnes.ac.id/sju/index.php/ujme/
ISSN: 2252-6927 (print); 2460-5840 (online)

Classification of figures in primary school based on van Hiele's model: Report of two cases

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ARTICLE INFO

Article history:
Received 16 February 2023
Received in revised form 24 March 2023
Accepted 31 August 2023

Keywords:
van Hiele levels;
Geometric figures;
Shape classification.

Abstract

In this paper, the results of an investigation based on an activity on the recognition of geometric shapes are reported, where different figures were presented that must be classified in order to identify the level of reasoning of each student. The present investigation follows a qualitative method and the data collection was carried out through clinical interviews applied to two children, an 8-year-old boy and a 10-year-old girl, who at that time were in the third and fourth grades of primary school respectively, in the State of Puebla, Mexico. Among the main results, it stands out that both children were located at level 0 of reasoning according to van Hiele's classification, since they recognize geometric shapes by their physical appearance, not by their properties and relationships.

1. Introduction

Primary education in Mexico consists of three cycles, which in turn are divided into two grades. The first cycle consists of the first and second (6-7 years). The second cycle consists of the third and fourth grade (8-10 years), while the third cycle consists of the fifth and sixth grades (11-12 years). In the New Educational Reform proposed by the Ministry of Public Education, it is established in the Expected Learning of the area of mathematics, within the axis Form, Space and Measure of the theme Geometric Figures and Bodies, that primary students of the first cycle build and describe figures and geometric bodies. Likewise, those in the second cycle are expected to construct and analyse geometric figures, in particular triangles and quadrilaterals, based on comparing sides, angles, parallelism, perpendicularity and symmetry (Ministry of Public Education [SEP], 2017).

A great challenge is evident for teachers, to enhance the development of geometric thinking from the consolidation of such learning in each of the cycles so that students are competent as they advance in academic level. However, the theory of official documents governing the educational process is usually utopian when it lacks real practical actions. Therefore, this work highlights one of the most solid contributions in Geometry, which has been achieved in the field of Mathematics Education, the van Hiele model (van Hiele, 1984; van Hiele-Geldof, 1984). It is a theoretical framework that provides a foundation for understanding the process of forming geometric concepts, which follows the students' progress with a hierarchically arranged series of levels characterized by an increase in abstractness (Anđelković, & Malinović-Jovanović, 2023). At each subsequent level, new knowledge is acquired, and in order to reach the next level, it is necessary to adopt the previous one. It depends solely on the understanding of a particular material and the perception of the whole concept, and not only on the acquisition of factual knowledge (Duranovic & Didic, 2023).

To cite this article:
De León-Zamora, W., Juárez-López., J.A & Juárez, E. (2023). Classification of figures in primary school based on van Hiele's model: Report of two cases. *Unnes Journal of Mathematics Education*, 12(2), 153-166. doi: 10.15294/ujme.v12i2.66357

According to Crowley (1987) the van Hiele model of geometric thinking can be used to guide instruction and assess students' abilities. Which means that, not only provides guidance in the teaching work so that students can access higher levels of reasoning, and thus materialize the written theory in the curriculum, but also can be used as a measurement tool to know the level of reasoning of students.

In this last use, the objective of our research is based, which is: To identify the level of reasoning in which two primary level students who belong to the second cycle are found. Taking into account that they successfully completed their first cycle, and, in addition, the expected learning of the preschool level according to the SEP (2017) is to reproduce models with shapes, figures and geometric bodies and build configurations with shapes, figures and geometric bodies. Under the hypothesis that most children in 3rd and 4th grade of primary school are at level 0.

1.1. van Hiele's Model

van Hiele's model describes how the way individuals' reason is modified through five levels of reasoning, and in turn, proposes how to organize teaching according to learning phases that facilitate progress in reasoning (Crowley, 1987). According to this model, the student, assisted by appropriate educational experiences, passes through these levels beginning with the recognition of forms as a whole (level 0, progressing towards the discovery of the properties of the figures and informal reasoning about these figures and their properties (levels 1 and 2), and culminating in a rigorous study of axiomatic geometry (levels 3 and 4) (Fuys, Geddes and Tischler, 1988, p.1). The first two levels of reasoning are described below, on which our attention is focused:

Level 0 Visualization: At this initial stage, students are aware of space only as something that exists around them. Geometric concepts are seen as total entities rather than having components or attributes. The geometric figures, for example, they are recognized by their form as a whole, that is, by their physical appearance, not by their parts or properties.

Level 1 Analysis: At level 1 begins an analysis of geometric concepts. For example, through observation and experimentation students begin to discern the characteristics of the figures. These pop-up properties are used to conceptualize classes of shapes. Therefore, it is recognized that figures have parts and are recognized by their parts (Crowley, 1987, p.2). Students who reason according to this level are able to consider all the forms included in a class rather than a singular form. Irrelevant characteristics (such as size or orientation) take a back seat (Godino and Ruiz, 2002, p. 499).

The theory, van Hiele's levels of geometric thinking have the following characteristics: they are sequential and hierarchical, so students must master lower levels to advance to higher levels; each level has its own language, symbols, and a network of relationships; and as they go through different levels, what is implied on one level becomes explicit on the next (Yi, Flores, & Wang, 2020).

According to Fuys et al. (1988), van Hiele points out that many failures in the teaching of geometry turn out to be a language barrier: the teacher uses the language of a higher level than that understood by the student. Progress from one level to the next depends more on instruction than on age or biological maturation. Along the same lines, Levenson, Tirosh and Tsamir (2011), believe that children can increase their math skills with proper instruction. For these authors, a minimum definition for preschoolers can be a disadvantage, since they observed that not all examples are recognized as such by preschoolers. They claim that the image of the figure promotes an immediate intuitive response, however, geometric concepts are abstract ideas derived from formal definitions. The interaction between the image and the abstract idea promotes both visual and attribute reasoning. The images of a concept are correct when they allow the student to discriminate without errors all the examples of that concept and when the associated properties are all relevant. These properties are not necessarily mathematical and may be physically irrelevant, especially in students who are in levels 0 and 1 of van Hiele (Gutiérrez and Jaime, 1996).

This shows a great challenge for geometry teachers. Therefore, this research aims to interpret from the Van Hiele model perspective, the way in which primary school students describe and classify geometric shapes.

2. Methods

The present investigation was qualitative (Hernández-Sampieri & Mendoza, 2018). The activity was applied to two students from a public elementary school in the State of Puebla, Mexico.

The task described in this research was proposed by Godino and Ruiz (2002) as an example that can be used for work in primary classrooms and corresponds to the first two levels of van Hiele. These authors mention that one of the first types of activities that can be proposed to children is to offer them the opportunity to find similarities and differences between a wide variety of forms. Therefore, this task consists of classifying forms for reasoning levels 0 and 1.

They were presented with 20 shapes cut out of cardstock, as shown in Figure 1, and the children were asked to select a shape at random and then find other shapes that were similar to the first in some respect. By asking them to form a subset of figures each time, the problem of trying to put each shape into a different category was avoided. In addition, students had to describe what trait the forms had to consider them similar.

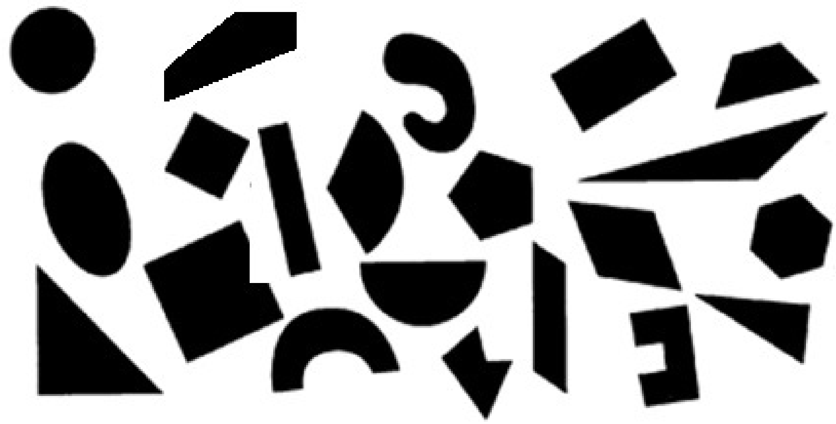

Figure 1. Variety of ways to work the task.

For the collection of the data, clinical interviews were used, in the sense mentioned by Ginsburg (1997). The clinical interview can be used to examine different aspects of the child's (or adult's) thinking, including understanding the basics of numbers, complex ideas about reality, moral judgment, and solutions to C.I. test items. The clinical interview can be used by the researcher interested in the exemplary child or by the physician trying to understand the child individually (Ginsburg, 1997, pp. 38-39).

The interviews allowed, through careful observation and interpreting what was observed in the dialogue with the students, the identification of their level of reasoning when developing the proposed geometric activity. The subjects were two boys, an 8-year-old boy and a 10-year-old girl, who were in the third and fourth grades of primary school respectively at the time. The aforementioned interview was semi-structured and had an average duration of 30 minutes. Therefore, he gave himself the freedom and autonomy to add other questions in order to deepen his answers.

The following is the script of the interview:

1. Why do you consider the figures similar?
2. How are the figures similar?
3. What characteristics do the figures have to consider them similar?

3. Results & Discussions

3.1. Interview with the first student

Below are the main results of the interview with the girl, who is the student number 1. She was identified with the (E) to the informant and with the letter (I) to the interviewer.

I: If you can see there are several figures, right?

E: Yes
I: Choose one, the one you want
E: (Select Figure 2)

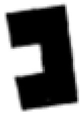

Figure 2

I: Now, you will find me from the rest other figures similar to the one you selected.
E: (Select Figure 3)

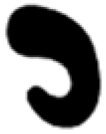

Figure 3

I: Why are they similar to you?
E: Because here you have the opening (points to the first selected figure) and here the same (points to the second selected figure)
I: Another similar one?
E: (Think and choose Figure 4)

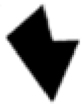

Figure 4

I: Why is it similar to the other two?
E: Because it still has the opening
I: Any others that we can place in the set?
E: I don't think so.

The interviewee, in this first set formed, classifies the figures without using the properties of the shapes or their names, but names them based on the global visual characteristics they have. For her, what defines a shape is its appearance, using terms like "opening." In this way, she selects three figures that possess this characteristic, no matter if the edges are straight or curved segments, she only distinguishes the shape as a whole. However, she did not select Figure 5 which also possesses the same feature.

Figure 5

I: Ok, now choose another figure... any
E: (Choose Figure 6)

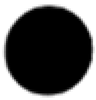

Figure 6

I: Of these that you have here (Point to the figures on the table) find one similar to this one (Point to Figure 6)
E: (Choose Figure 7)

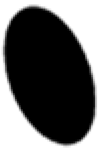

Figure 7

I: Why is that one (Figure 6) similar to this one (Figure 7)?
E: It has a circular shape, only it is a little more extended.
I: Which is more widespread?
E: It is (points to Figure 7)
I: Any other figures that we can place here (pointing to Figures 6 and 7)?
E: (Choose Figure 8)

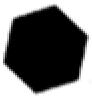

Figure 8

I: Why is this one (Figure 8) similar to these two (Figures 6 and 7)?
E: Because if we join everything or increase it more it will be equal to this (Figure 6)
I: If we increase it? How do we increase it?
E: mmm increase another piece that is circular (Make a circle with your fingers around the figure)
I: Any others that we can place in this set?
E: (Choose Figure 9)

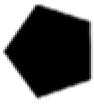

Figure 9

E: If we cut it in a circular way it is also
I: If we cut it?
E: Ujum
I: How would you cut it?
E: Make a small circle and cut it out (Make the circle inscribed in Figure 9 with your fingers)
I: Inside?
E: Yes
I: Ok, any other figures we can include?
E: I honestly think I don't think anymore.

In the second formed set, student 1 used the phrase "circular shape" to select common figures but included a pentagon and a regular hexagon because circular shapes could be generated through them. Which means that it does not take into account the specific properties of geometric figures but tries to change them and convert them into others. In this case she selected the four convex figures that most closely approximate the circle (Figure 6). In the hexagon (Figure 8), imagine a circumscribed circle, and in the pentagon, an inscribed circle, as approximations to the circle. Note that she did not consider quadrilaterals or triangles to

be "circular shaped" figures. From this it is concluded that student 1 understands geometric concepts as global entities, rather than having specific characteristics or attributes.
I: Ok, then... Let's choose another figure
E: (Choose Figure 10)

Figure 10

I: Which ones do you find here (point to the table full of figures) similar to this one (Figure 10)?
E: (Choose Figure 11)

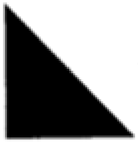

Figure 11

I: Why?
E: Because it has three sides (Figure 10) and is also (Figure 11)
I: Any others?
E: (Choose Figure 12)

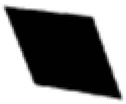

Figure 12

I: And that why?
E: We divide it in half and there are two small triangles
I: Another?
E: (Choose Figure 13)

Figure 13

I: And that why?
E: Maybe it has three sides
I: Are there any others we can include?
E: I don't think there's any more.
I: Safe?
E: Yes

In this third subgroup, student 1 identifies the first selected figure and chooses two more figures based on a characteristic of triangles: "has three sides". In the case of the rhombus, she says to select it because she says: "we divide it in half and there are two small triangles" that is, she considers that she can manipulate the figures, in this case, cut or divide them. The redeemable thing about his choice of the rhombus is that she explained, "it is composed of two triangles", which is why she considers it part of the set.

On the other hand, student 1 did not select Figure 14, which also has "three sides" without being a rectangle, so she implicitly also used as a selection criterion that the figures should have "straight sides". From this it can be affirmed that student 1 gave evidence of possessing van Hiele's level 1 because she begins to discern the characteristics or properties of the figures.

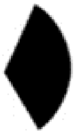

Figure 14

I: Well, choose another figure
E: (Choose Figure 15)

Figure 15

I: Are there any you can find similar to this one (Figure 15)?
E: (Choose Figure 14)
I: Why?
E: Because if we place it is here (place figure 14 on Figure 15), we mark it and cut it out another equal
I: Are there any others we can include here?
E: I don't think it's anymore.

In this fourth set, it is evident that it is based on the visual characteristics of the figure but does not relate it to the circle of its second set. In this case, select the two most similar figures only by their shape, not by the geometric concept they represent.

I: Ok, then choose another figure
E: (Choose Figure 16)

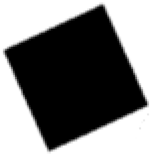

Figure 16

I: Of these (he points to the figures on the table) which ones resemble this one (Figure 16)?
E: This (Choose Figure 17)

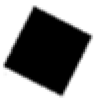

Figure 17

I: Why?
E: Because it has four equal sides
I: How do you know it has four equal sides?
E: Counting them... one, two, three and four (Count the sides of the figure with your finger)
I: Are they the same?

E: Only one is smaller and the other bigger
I: How do you know the sides are the same?
E: As long as it is always a square, this (Figure 17) and this (Figure 16) will be the same, only it is smaller (Figure 17)
I: Of the figures that are needed, which one could we place here? (Points to the formed set)
E: This (Choose Figure 18)

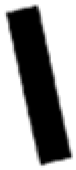

Figure 18

I: Why?
E: It has four sides, two small ones, two large ones, but it has four sides and is more extended.
I: And what are the sides of this figure like (Figure 18)?
E: They are two small sides and two larger sides
I: And why can we place it in the set?
E: On its four equal sides
I: Does it have four equal sides (Figure 18)?
E: No, he has two little ones, two big ones.
I: Of the missing ones, which one can we include?
E: I don't think any of them anymore.
I: None?
E: (Shakes her head)

In this fifth formed group, use the property "has four equal sides" when selecting Figures 16 and 17. However, afterwards it also selects the rectangle (Figure 18), not respecting the selection criteria it had previously established. This indicates a 0-level state of van Hiele's model, since, on the one hand, it identifies a characteristic of the first two chosen figures (the large square and the small square), but then no longer considers it and only takes care that the selected figure has four sides. That is, stop considering the characteristic of equal sides. Gutiérrez and Jaime (1996) state that this problem has been found in students of all educational levels, in that they can write the correct definition of a regular polygon but identify rectangles as such "because they have equal angles".

I: Then let's choose another figure
E: (Choose Figure 19)

Figure 19

I: Of these two that I have here (figures on the table), which one resembles this one (Figure 19)?
E: (Choose Figure 20)

Figure 20

I: Why?

E: Because it has both sides inclined
I: And why else? Just that or is there something else?
E: Because the top is also flat
I: And where would we place this (Figure 21)?

Figure 21

E: Because it has the sides inclined, aha the same inclined sides

In the case of the sixth subgroup formed, student 1 relied on the appearances of the figures to form the set, using phrases such as "inclined sides", without taking into account the number of sides of the figure or any other characteristic property of the trapezius or parallelogram.

Individual 1 divided the figures presented into six groups as shown in Figure 22.

Figure 22. Classification of reporting person number 1

In this way, we can conclude that student 1 classified the figures using more frequently their appearance and overall form, rather than their conceptual characteristics. It should be noted that in general, he did not use the names of the figures to form the subsets, with the exception of the triangle, a word he mentioned twice during the interview.

3.2. Interview with the second student

In this section we present the results obtained with the interview of the child, who is the student 2. The subject was identified with the letter (A) and the interviewer with the letter (I).
I: Look, we have several figures here at the table. Choose one, the one you like. One
A: (Choose Figure 8 which is a regular hexagon)
I: Now, of these that are here (he points to the figures on the table) he finds a similar one.
A: (Choose Figure 9 which is a regular pentagon)
I: Why are they similar?
A: Because they have equal sides
I: How do you know they have equal sides?
A: They look alike here and here (He points to one side of both figures).
I: So how are they similar?
A: On the sides
I: Why?
A: (Prolonged silence)

I: Do they look alike or don't they look alike?
A: If they look alike
I: In what?
A: On the sides
I: What do the sides have?
A: Equal parts
I: How do you share it equally? What is equal parts for you?
A: Have the same shape
I: Do they have the same shape?
A: No
I: So? Why are they similar to you? What do they have in common?
A: (Prolonged silence)
I: How are they similar?
A: On the sides
I: Why? What do the sides have?
A: Forms
I: What forms?
A: (Prolonged silence)
I: What do the sides have? What are the sides of these figures like?
A: Horizontal
I: How is it horizontal? Draw it with your hand
A: That (It points to one inclined side of the regular pentagon, then all its inclined sides and does the same with the regular hexagon)
I: Now is there another figure that looks like these two?
A: (Choose Figure 20 which is an irregular pentagon)
I: Why?
A: Because it has its horizontal sides (Points to the inclined sides of the figure)
I: Is there another figure that we can include in this set?
A: This (Choose Figure 12 which is a rhombus)
I: Why?
A: Because it has its horizontal sides (Points to the inclined sides of the figure)
I: Is there another one?
A: (Choose Figure 4, which is a concave pentagon)
I: Why?
A: It has horizontal sides, it is this, this and this (Points to all inclined sides of the figure)
I: Are there any others?
A: (Choose Figure 11 which is a right triangle)
I: Why?
A: Has horizontal sides (Points to two inclined sides of the figure)
I: Are there any other figures?
A: (Choose Figure 13, which is a scalene triangle)
I: Why?
A: Still has horizontal sides (Points to two inclined sides of the figure)
I: What does horizontal mean? What are they how?
A: Which is an almost lying line
I: Are there any other figures?

The second student also chooses Figure 19 which is a trapezoid, Figure 14 which is a circular sector, Figure 19, which is a parallelogram and Figure 10 which is a right triangle, with the same argument as above.

In the first part of this section of the interview, student 2 gives signs of knowing the concept of regular polygon, by selecting the hexagon and the pentagon, indicating as selection criterion "equal sides". However, it then selects a series of irregular polygons. It forms this first set, based on an erroneous criterion of horizontality, since it associates this term with the inclination of the sides of the figures. Without discriminating the geometric shape, it relates the figures according to their inclined sides, without realizing that the orientation of the sides of the figures in a plane is not a characteristic that defines them.

In this case there was an erroneous perception of the geometric figures, by attributing an orientation to their sides. This error has already been reported in various investigations, in which it is described that if the geometric figure is not in a certain position (for example, the right triangle resting on one of its legs, the isosceles triangle resting on its base or the rhombus resting on one of its vertices, among others), students fail to identify it (see for example Barrantes and Zapata, 2015; Gutierrez and Jaime, 1996; Valencia, 2018; Vinner and Hershkowitz, 1983). This is a characteristic of van Hiele's level 0, because appearance is the dominant factor at this level, this appearance can lead to attributing impertinent properties to forms (Godino and Ruiz, 2002, p. 499).

I: As we no longer find more of the set, now choose another figure from the table
A: (Select Figure 6 which is a circle)
I: Find a similar one
A: (Select Figure 15 which is half a circle)
I: Why?
A: Because it has a curved side
I: Of the missing ones, which ones can we include in this new set?
A: (Select Figure 7 which is an oval and then Figure 3 with the same argument)
I: Why?
A: Maybe it has a curved side
I: Is there another one that we can place here?
A: No

From the above, it can be perceived that the student uses the appearance of the figures to make the second classification, using words such as "curved side", without taking into account the name of the figures or their properties.

I: Now, choose a figure
A: (Select Figure 17, which is the small square)
I: Which of these (the table points to the missing figures) is similar to this one (Figure 17)?
A: This (Choose Figure 16 which is the large square) Because they are square, both
I: Are they both square?
A: Yes
I: Why? How do you know it's a square?
A: Because it has vertical sides
I: What do the vertical sides look like? Point them at me with your finger right there
A: (Points to one of the vertical sides of the square)
I: Are there more vertical sides or is it just that?
A: (Points to the other vertical side)
I: And what is this side (horizontal side) called then?
A: Horizontal
I: So what common characteristics do these figures have?
A: Horizontal and vertical sides
I: Which of these two can we include in this new set?
A: This (Choose Figure 18 which is a rectangle)
I: Why?
A: Because it has vertical and horizontal sides
I: And this (Figure 2) can we include?
A: Yes
I: Why?
A: Because it has sides: vertical – horizontal – vertical – horizontal (points with your finger the sides of the figure)
I: Right now, you told me that this way (an inclined side of a figure is shown) was horizontal.
A: (Smiles)
I: So? You told me it's almost tilted. Is it horizontal or vertical?
A: Horizontal

I: Are these (vertical sides) or these (horizontal sides) or do you have another name?
A: Has another name
I: Which one?
A: It can be vertical if I put it like this (place the rectangle resting on a smaller side)
I: And horizontal?
A: If placed like this (place the rectangle in an inclined shape)
I: What if we place it like this (place the rectangle resting on a larger side)? What's your name?
A:(Silence…)
I: Do you remember or not?
A: Not anymore

In the case of the third and last group created, student 2 began classifying by the name of the figure, alluding to the square, but then when he no longer finds it, he used properties such as vertical and horizontal sides. However, conceptual errors were perceived in these terms because it recognized verticality. However, he still showed confusion with horizontality, since he pointed out the inclined sides as if they were horizontal. Again, he mistakenly used the criteria of verticality and horizontality as a characteristic of squares, corroborating that the image of the rectangle concept that many primary students form is composed of a series of specific rectangles placed in standard position (the pair of horizontal longer sides) (Gutiérrez and Jaime, 1996, p. 4).

Student 2 divided the figures presented into three groups as shown in Figure 23.

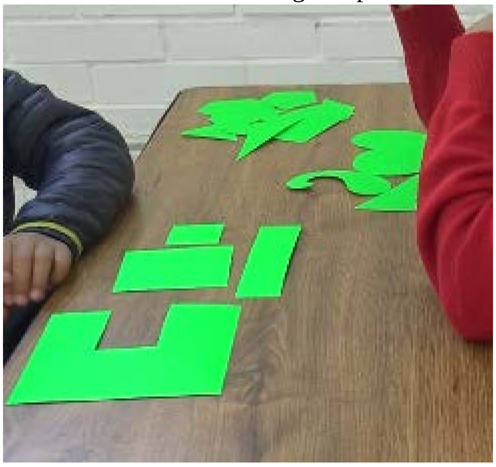

Figure 23. Classification of reporting person number 2

It is evident that student 2 classified the figures using their appearance and visual characteristics more frequently than their properties. It should be noted that, occasionally, he used the names of the figures to form the sets as in the case of the square, but concentrated more on the shape of the sides, whether they were vertical, horizontal or curved; even if he had no conceptual clarity of it.

4. Conclusion

According to Godino and Ruiz (2002), activities that correspond to van Hiele's level 1 reasoning focus more on the properties of forms and include some analysis of these properties. For example, at level 0, triangles may have been classified as "large" and "small," "pointed" or "non-pointed," or "square-cornered" and "no square corners." At level 1, the same set of triangles can be classified according to the relative size of the angles or the relative length of the sides. Consequently, both students could be placed at level 0 because they were not able to realize that a set of shapes belongs to the same class due to their properties, but used visual perception more, using terms such as "aperture" and focusing their attention on the shape of the sides whether curved, horizontal or vertical. These results are consistent with those from previous studies (Anđelković, & Malinović-Jovanović, 2023; FengSer, 2022; Md. Yunus et al., 2019; Wu, et al., 2015). Godino and Ruiz (2002) confirm the assertion of that, in general, children in preschool and up to second grade of primary school are at level 0, as well as that most children in third and fourth grade of primary school.

To develop geometric thinking, it is necessary that the teaching of geometric concepts includes a good amount of geometric experiences. Activities such as coloring, bending and building shapes; identify a shape in a drawing; and verbally describing geometric shapes using standard and non-standard language are appropriate for students at Level 0, as they support students' exploration of geometric shapes beyond simply naming them (Clements, 2003). Activities such as sorting and resorting to shapes by attributes and guessing a shape from given clues are considered appropriate for students in Level 1, as they guide them to explicitly recognize the properties of different geometric shapes and identify their relationships (Yi, Flores, & Wang, 2020). In this way the subject will experience the appropriate geometric thinking for that level and will be able to create in his own mind the types of objects or relationships that are the focus of attention of the thought of the next level (Godino and Ruiz, 2002).

References

Anđelković, S., & Malinović-Jovanović, N. (2023). Students'achievements in primary school mathematics according to the van hiele model of the development of geometric thinking. *Facta Universitatis, Series: Teaching, Learning and Teacher Education*, 155-167. https://doi.org/10.22190/FUTLTE221205011A

Barrantes, L. M., & Zapata, E. M. A. (2015). Obstáculos y errores en la enseñanza-aprendizaje de las figuras geométricas. *Campo Abierto. Revista de Educación [en línea]*, 27(1), 55-71. https://relatec.unex.es/revistas/index.php/campoabierto/article/view/1985

Clements, D. H. (2003). Teaching and learning geometry. In J. Kilpatrick, W. G. Martin, y D. Schifter (Eds.), *A research companion to principles and standards for School mathematics*, (pp. 15-78). Reston, VA: National Council of Teachers of Mathematics.

Crowley, M. L. (1987). The van Hiele Model of the Development of Geometric Thought. En M. Montgomery (Ed.), *Learning and Teaching Geometry, K-12. Yearbook of the National Council of Teachers of Mathematics*, (pp.1-16). Reston, Va.: National Council of Teachers of Mathematics.

Duranovic, M., & Didic, E. (2023). Prevalence and characteristics of geometric difficulties in elementary school children. *Asia Pacific Journal of Developmental Differences*, *10*(1), 5-26.

FengSer, P., Shaharom, M. S. N. B., Bajuri, M. R. B., & Chan, C. T. (2022). Pedagogical Strategies Using Van Hiele's Levels of Geometric Thinking in Learning Geometry Among Malaysian Year Five Students. Educational Leader (Pemimpin Pendidikan), 10, 13-29

Fuys, D., Geddes, D. & Tischler, R. (1988). The van Hiele Model of Thinking in Geometry among Adolescents. *Journal for Research in Mathematics Education*. Monograph, Vol. 3, The van Hiele Model of Thinking in Geometry among Adolescents, pp. i+1-196 Published by: National Council of Teachers of Mathematics Stable. http://www.jstor.org/stable/749957

Ginsburg, H. P. (1997). *Entering the child's mind. The clinical interview in psychological research and practice*. Cambridge University Press.

Godino, J. & Ruiz, F. (2002). *Geometría y su Didáctica para Maestros*. Manual para el Estudiante. Universidad de Granada.

Gutiérrez, A. & Jaime, A. (1996). Uso de definiciones e imágenes de conceptos geométricos por los estudiantes de Magisterio. En J. Giménez, S. Llinares, y M. V. Sánchez (Eds.). *El proceso de llegar a ser un profesor de Primaria. Cuestiones desde la educación matemática*. (pp. 145-169). Granada: Comares.

Hernández-Sampieri, R. & Mendoza, C. (2018). *Metodología de la investigación*. McGraw-Hill Interamericana.

Levenson, E., Tirosh, D., & Tsamir, P. (2011). *Preschool Geometry*. Sense Publishers.

Md. Yunus, A. S., Mohd Ayub, A. F., & Hock, T. T. (2019). Geometric Thinking of Malaysian Elementary School Students. *International Journal of Instruction*, *12*(1), 1095-1112.

Secretaría de Educación Pública (2017). *Aprendizajes claves para la educación integral, plan y programas para la educación básica*. Ciudad de México, México: Autor.

Valencia, P. (2018). Visualización geométrica en niños de enseñanza elemental. *Acta Latinoamericana de Matemática Educativa*, 31(1), 490-496.

van Hiele, P. M. (1984). The child's through and geometry. En D. Fuys, D. Geddes, y R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele* (pp. 243-252). Brooklyn, NY: Brooklyn College, School of Education. https://files.eric.ed.gov/fulltext/ED287697.pdf.

van Hiele-Geldof, D. (1984). The didactics of geometry in the lowest class of secondary school. En D. Fuys, D. Geddes, y R. Tischler (Eds.), *English translation of selected writings of Dina van Hiele-Geldof and Pierre M. van Hiele* (pp. 1-214). Brooklyn, NY: Brooklyn College, School of Education. https://files.eric.ed.gov/fulltext/ED287697.pdf.

Vinner, S. & Hershkowitz, R. (1983). On concept formation in geometry. *Zentralblatt für Didaktik der Mathematik*, 83(1), 20-25.

Yi, M., Flores, R., &Wang, J. (2020). Examining the influence of van Hiele theory-based instructional activities on elementary preservice teachers' geometry knowledge for teaching 2-D shapes. *Teaching and Teacher Education*, 91, 103038. https://doi.org/10.1016/j.tate.2020.103038.

Wu, D. B., Lee, D. C., Lin, S. H., & Ma, H. L (2015). A study of van Hiele of geometric thinking among 1 st through 6th graders. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(5), 1181-1196. https://doi.org/10.12973/eurasia.2015.1412a