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Grade 10 Namibian Learners' Strategies for Solving Algebraic Word Problems

Hesekiel K. Iilonga [a,*], Ugorji I. Ogbonnaya [a]

[a] *University of Pretoria, South Africa*

* *E-mail address: iihesekiel@gmail.com*

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Abstract

Solving mathematical word problems is a big challenge for many learners. One reason for the challenge could be the learners' use of inappropriate strategies in solving mathematical word problems. In Namibia, many examiners' reports show that learners do not attempt algebraic word problems fairly in examinations. This study investigated Grade 10 learners' strategies for solving algebraic word problems in the Ohangwena Region, Namibia. The study followed a qualitative approach. A sample of 351 Grade 10 learners from ten secondary schools participated in the study. Krulik and Rudnick's problem-solving strategies model was adopted as the framework that guided the study. Data was collected using the Algebraic Word Problem Solving Achievement Test and analysed using content analysis. The result shows that most of the learners could not use appropriate strategies to solve the given problems. Few learners employed one or two appropriate strategies in solving the problems. The strategies used by the learners to solve the algebraic word problems in the test include Computing or Simplifying (CS); Making a Table, Chart, or List (TCL); Making a model or a diagram (MD), and Guessing, Checking, and Revising (GCR). It is recommended that teachers model different strategies for solving mathematical problems for learners while teaching mathematics.

1. Introduction

Algebraic word problems are verbal descriptions of problem situations in which one or more questions are presented, and the questions can be answered by the application of mathematical operations to numerical data contained in the description of the problem (Verschaffel et al., 2000). The use of word problems has a long history in the early development of algebra (Walick, 2015). They are used to contextualise algebra and link classroom algebra to the real world.

Internationally, algebraic word problems are problematic for many learners (Cardellino & Woolner, 2020; Ferrucci et al., 2003; Lee et al., 2018; Takahashi, 2017). In Namibia, the examiner's reports of the Directorate of National Examinations and Assessment (DNEA) have consistently indicated algebraic word problems to be challenging to learners (DNEA, 2014; 2015; 2016; 2017; 2018). However, the examiner's reports did not specify the possible causes of the learners' problems in algebraic word problems. We observed that the causes of learners' difficulties in algebraic word problems have not been well-researched in Namibia. We believe that one of the reasons for learners' challenges in solving algebraic word problems is the use of inappropriate problem-solving strategies. Hence, in this study, we investigated the strategies used by Grade 10 learners in Namibia in solving algebraic word problems. The research was to answer the research question: What strategies do Grade 10 learners in the Ohangwena region of Namibia use when solving algebraic word problems?

Problem-solving strategies are the processes through which a person applies previously acquired knowledge, abilities, and comprehension to meet the needs of a new circumstance (Krulik & Rudnick,

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1996). It starts with the initial confrontation and finishes with a response verified against the current state of the issues. To solve a problem, the problem solver may employ different strategies or a combination of strategies (Verschaffel et al., 2020). Krulik and Rudnick (1996) identified eight strategies that can be used to solve mathematical problem-solving at the secondary level: (1) Using a formula (UF), (2), Computing or Simplifying (CS), (3), Making a Table, Chart, or List (TCL), (4) Making a diagram or model (MD), (5) Guessing, Checking, and Revising (GCR), (6) Pattern Searching (LP), (7) Consideration of a Simpler Case (SC) and (8) Elimination (E).

Using a Formula refers to the process of utilising a pre-made formula or an existing one, for example, inserting given values into a formula or selecting the appropriate formula to use. Grønmo et al. (2015) define using a formula as a problem-solving strategy that learners can use to solve mathematics problems involving geometry, percentages, measurement, or algebra, in which learners must select, recall or create the appropriate formula and substitute data from the problem into the appropriate variable fields of the formula to solve the problem (Tan, 2018). According to Kaur (2008, as cited in Ratnasari & Safarini, 2020), using an equation or a formula is when a learner or problem solver uses their knowledge of algebra to find a solution by choosing the right equation or system of equations from algebra.

Computing or simplifying, according to Krulik and Rudnick (1996) is a basic application of mathematical principles and the precise sequence of operations, as well as the use of other mathematical processes; for example, directly utilising arithmetic rules. According to Jarrett (1999), simplification is substituting a mathematical expression with an equivalent one that is simpler to understand (usually shorter).

Making a Table, Chart, or Listing refers to presenting information in a table, chart, or graph, or listing the alternatives, for example, to organise data (Krulik & Rudnick, 1996). Szabo et al. (2020) pointed out that some learners start solving problems by making an organised list, a graphical representation, or a table which is a systematic way to solve statement problems. Creating diagrams, lists, and charts during problem-solving helps problem solvers who are solving the problems to think logically and see what the problems look like in real life. Frequently, little or no calculation is required, as the problem solver merely needs to take data from the given problem and portray the information in a table, graphic, or list.

Making a model is using items, sketches or drawings, or algebraic statements to present the problem (Krulik & Rudnick, 1996). According to Ratnasari and Safarini (2020), drawing a diagram or a model is when a problem solver uses the strategy to illustrate the problem in pictures or diagrams. Creating graphical representations of mathematical problems learners are working on allows them to translate a mathematical word problem into a real-world scenario.

According to Krulik and Rudnick (1996), Guessing, Checking, and Revising refers to estimating the outcomes and checking their accuracy. If there is an error, the estimation is reorganised and rechecked, i.e., rechecking and trials. Ratnasari and Safarini (2020) defined the approach for the "Guess and Check" method as "guessing a solution and then plugging it back into the problem to see if the guess was accurate." This heuristic is a straightforward method of obtaining a solution, as learners only have to guess. As a result, there are two distinct sorts of guess and check. Ratnasari and Safarini further explain that the first type of guess and check is called systematics guess and check. It is the type of guess and check in which the problem solver can obtain the solution after a few repetitions, while the second type of guess and check is called unsystematic guess and check, and it may not result in the correct solution. However, Tostado-Véliz et al. (2019) pointed out that the strategy of guessing and checking is not necessarily an effective approach to obtaining a solution unless the first estimate results in a more accurate guess that yields a solution. Akyüz (2020) highlighted that if learners can additionally check that their guess matches the problem's conditions, they have mastered guess and check. Akyüz further emphasised that as questions get more complex, more strategies become more critical and successful and must be incorporated into the problem-solving process.

Looking for patterns is applying general characteristics of the given data to future situations or circumstances, such as a numerical sequence or series (Krulik & Rudnick, 1996). Rahayuningsih et al. (2020) define "looking for patterns" as a method for resolving problems by looking for recurring objects or numbers or a sequence of occurrences. Pattern recognition enables the determination of correlations from supplied data.

Simplifying the situation is when the problem is rewritten in a simpler manner (Krulik & Rudnick, 1996). The problem can be turned into a previously solved common problem, and working backward can be done, if possible, to find the solution when feasible. For instance, this can be accomplished by utilising smaller values, a more typical problem scenario, or dividing the problem into simpler problem scenarios. According to Bambi et al. (2019), it is helpful to divide problems that are too complicated to address in a single step into smaller problems and find a solution for each of these problems. Making mathematical problems more manageable by simplifying them in this way is often used together with various structured strategies (Gavaz & Yazgan, 2021).

Krulik and Rudnick (1996) say that eliminating means getting rid of impossible solutions based on the information given, getting rid of wrong answers and answers that might be wrong, or getting rid of solutions when the information and the solutions don't match up. The elimination approach is a method for solving problems in which many potential responses are eliminated one by one until only the correct answer is left (Jones et al., 2020).

Some studies investigated the problem-solving strategies used by students at various educational levels and in different contexts to solve algebraic word problems. For example, Sikukumwa (2017) investigated the strategies used by Grade 12 ordinary-level learners in the Kavango East region of Namibia to solve algebraic word problems and found that the learners utilised various strategies, including identifying, highlighting, reading, re-reading terms, visualizing problems, guessing and checking, stage procedures, pattern searching, and counting techniques. Chirove (2014) explored Grade 10-12 learners' strategies for solving nonroutine problems in South Africa and found that the learners used Making a Table, Chart, or List (TCL); Making a diagram or model (MD); Trial-and-error (TE); Using a formula (UF); Guessing, Checking, and Revising (GCR); Consideration of a Simpler Case (SC); Logical reasoning (LG); No logical reasoning (NLG); and Look for patterns (LP).

Mabilangan et al. (2011) studied the strategies adopted by students at a university high school when addressing non-routine problems and found that each student used at least four problem-solving strategies, with seven of the eight possible strategies being used at least once to address the twelve non-routine problems. "Making a Model or Diagram" was the most frequently used strategy by the students.

Yew and Zamri (2018) conducted a case study to investigate the problem-solving strategies of eight pre-service secondary school mathematics teachers from a public university in Peninsular, Malaysia, who were enrolled in a four-year Bachelor of Science with Education programme. The data was collected using a clinical interview technique. The study's findings indicate that the pre-service teachers used various strategies to solve the problem, with the majority of pre-service teachers using more than one distinct strategy to tackle the problem. Drawing a diagram was the most commonly utilised strategy, followed by trial and error, identifying patterns, utilising an equation, and then listing.

Duru et al. (2011) investigated pre-service primary school teachers' strategic preferences in solving word problems. The pre-service teachers were asked to utilise any strategy that they felt was suitable for solving the problems. The results showed that the student teachers used different approaches such as the algebraic method, arithmetic strategies, guess-and-check, looking for a pattern, and using a model to solve word problems. Ünlü (2018) investigated the strategies employed by pre-service mathematics teachers in solving a non-routine problem in Turkey using a sample of 104 candidates. The Problem-Solving Test was used to collect data. This study revealed that some mathematics teacher candidates used appropriate strategies in solving the problems while others could not. The studies reviewed here indicate that students at various levels of education can use different strategies to solve algebraic problems but often, most do not use the appropriate strategies.

2. Methods

This research employed a qualitative approach and descriptive research design. The participants were 351 learners from ten sampled secondary schools in the Ohangwena region, Namibia. All 351 learners wrote an algebraic word problem-solving test.

Data was collected using an Algebraic Word Problem-Solving Achievement Test. The instrument was developed and validated by the first author with the guidance of the Namibia Senior Secondary Certificate mathematics syllabus. The test comprised six algebraic word problems that were used to explore the learners' usage of problem-solving strategies to solve the test questions.

The Algebraic Word Problem-Solving Achievement Test was given to all the learners from the ten sampled schools to assess the problem-solving strategies they used when solving algebraic word problems.

I invigilated the test that was not given a time limit. The purpose of not providing time limits was to avoid rushing them to a solution but to let them show their work and strategies.

The algebraic word problem-solving achievement test was administered to the participants at the sampled schools after permission to conduct the study was granted by the Ministry of Education, Arts and Cultures and the schools' management. The test was administered to learners in the afternoon after normal schooling hours. The Grade 10 learners were first briefed about the purpose, and the ethics of the study was made clear to them before they started writing the test. The learners were instructed not to write their names or the names of their schools on the Algebraic Word Problem-Solving Achievement Test paper. Instead, they were provided with codes, one representing the learner and the other representing the school. Learners were told to take their time to answer the questions and that showing all their work could gain them more marks.

The analysis of the collected data was performed based on the solutions from Algebraic Word Problem-Solving Achievement Test to answer the research Question. Content analysis of the learners' test was done using Krulik and Rudnick's (1996) problem-solving strategies model.

To ensure the validity of the instruments, a curriculum expert, two mathematics educators (Head of Departments), and two mathematics subject advisors from two different regions were used to determine the content validity of the test. The experts evaluated the language used, the content covered with a comparison to the syllabus' learning objectives, and the marks allocated to each question. They judged the level of relevance of each question against the curriculum by using a three-point scale (1 = not relevant; 2 = useful but not relevant; 3 = relevant) adopted from Yusoff (2019). All six questions in the instrument gave a CVI value of 0.8, showing that the content validity indicated that the instrument offered an accurate measure of what they were designed to measure. To recommend an excellent questionnaire, a CVI of 0.80 or above is appropriate (Gilbert & Prion, 2016).

The research was limited to ten secondary schools in the Ohangwena region in Namibia, the generalisation of the result to other Namibian regions must be approached with caution. The research sample size was scheduled to be 360 learners, but only 351 learners participated. This reduction was owing to some learners dropping out during the data-gathering phase.

3. Results & Discussions

The learners' strategies were analysed and presented in the order of the test questions.

3.1. Strategies used to solve question 1.

The first question was "Frans and Meke are friends. Frans took Meke's mathematics test paper and will not tell her what mark she got. Frans knows that Meke doesn't like word problems, so he decided to tease her with word problems. Frans says: "I have 2 marks more than you do and the sum of both our marks is equal to 14." What are our marks?"

This problem can be solved with the strategies of Making a Model, for example, let Meke's mark be M, and Frans's mark be F, since Frans' mark is greater than Meke's mark by 2, then $F = M + 2$,

Since, $F = M + 2$ - (1)

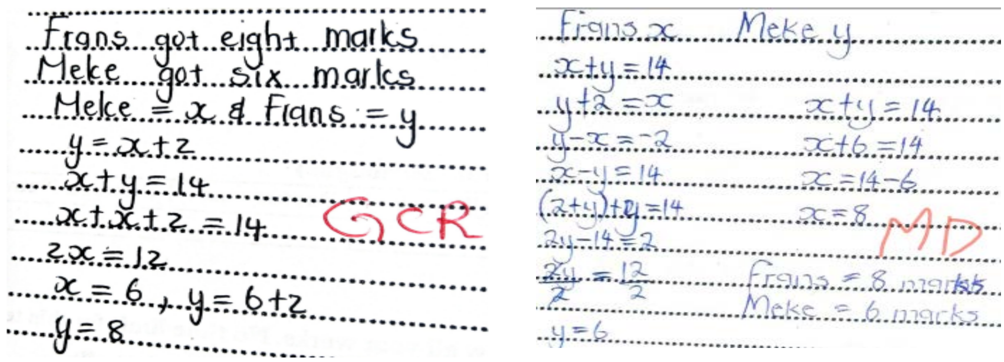
and Frans' and Meke's marks = 14, then,

$F + M = 14$ - (2)

Then substitute (1) into (2) to get $(M + 2) + M = 14$

However, some learners may opt to use of Guessing, Checking, and Revising strategy.

In solving the problem, 134 (38%) learners employed the MD strategy, 24 (7%) employed the CS strategy, and 12 (3%) of the learners used the GCR strategy. The strategy of TCL was used by 1 (0.3%) of the learners. Less than 1% of the learners combined different strategies of TCL/CS and MD/CS. From all the results, it can be seen that most learners did not use strategies to solve the problem in the test. Figure 1 is an example of learners who employed GCR and MD strategies.



(a) (b)

Figure 1. Examples of Learners who used GCR and MD Strategies in question 1.

Figure 1 shows two learners that used two problem-solving strategies to solve the given problem. The learner in (a) used the GCR strategy, the learner guessed the answer first and then checked if it made sense, while the learner in (b) used MD by translating the algebraic word problem into an equation that was used to solve the given problem.

3.2. Strategies used to solve question 2.

The second question was “A father is three times as old as his son, and his daughter is 3 years younger than the son. If the sum of their ages 3 years ago was 63 years, find the present age of each.” Learners could solve this problem with the following strategies: Making a Model or Diagram, by presenting the expression of the ages as follow:

Present age

Father = $3x$

Son = x

Daughter = $x - 3$

Then 3 years ago (-3)

Father = $3x - 3$

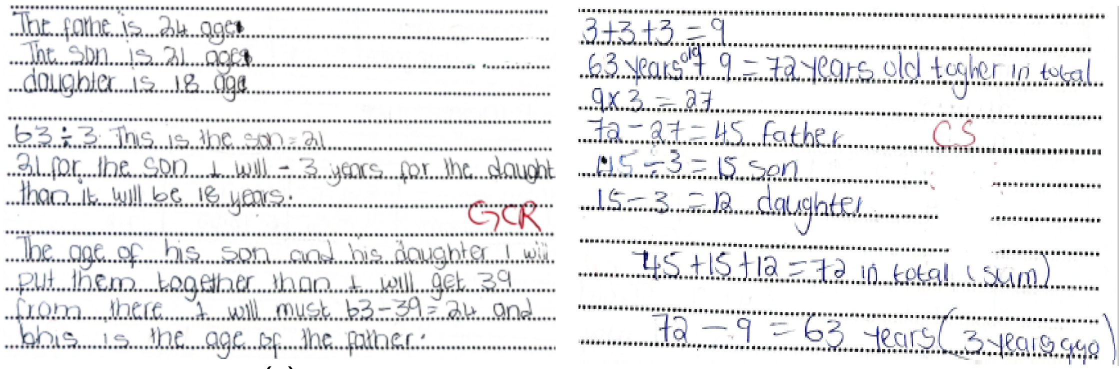
Son = $x - 3$

Daughter = $(x - 3) - 3 \; or \; x - 6$

Since the question asked the ages of three years ago, with a sum of 63 years. A learner could take $x - 3$ plus $x - 3$ plus $x - 6$ to give 63 years, which leads to an equation $(x - 3) + (3x - 3) + (x - 6) = 63$ that can be used to solve the problem.

Some learners may use Making a Model or Diagram strategy by starting with the translation of the problem into an algebraic equation. For example, $x - 3$ plus $3x - 3$ plus $x - 6$ to give 63 years, to give $(x - 3) + (3x - 3) + (x - 6) = 63$. The problem can also be solved using Guessing, Checking and Revising.

The results for Question 2, indicate that 106 learners (30%) employed MD, 62 (18%) employed CS, while GCR was used by 12(3%) of the learners (See Table 1). The combined use of MD/CS, strategies in Question 2 was less than 1%. Figure 2 shows examples of learners who used GCR and CS strategies in Question 2.



(a) (b)

Figure 2. Examples of Learners who used GCR, and CS Strategies.

Figure 2 (a) illustrates the implementation of the GCR strategy by a learner that did not result in the correct answer. Conversely, (b) depicts a learner who utilised the CS strategy.

3.3. Strategies used to solve question 3.

The third question was "A 1-litre bottle of mango juice costs N$2.00 more than a 1-litre bottle of strawberry juice. If 3 bottles of mango juice and 5 bottles of strawberry juice cost N$ 78.00, determine the price of each juice per 1-liter bottle." This problem can be solved by Making a Model. For example, Let Mango be x, and strawberry be y

This means that $x = y + 2$.

Hence, Mango $= y + 2$ (1)

while strawberry $= y$ (2)

The 3 bottles of mango juice can be represented as $(3x)$ and five bottles of strawberry is $(5y)$, the sum being N$78 can be expressed as:

$3x + 5y = 78$ (3)

If equation 1 is substituted into equation 3 then $3(y + 2) + 5y = 78$ hence $y = 9$ and $x = 11$.

Using the CS method,

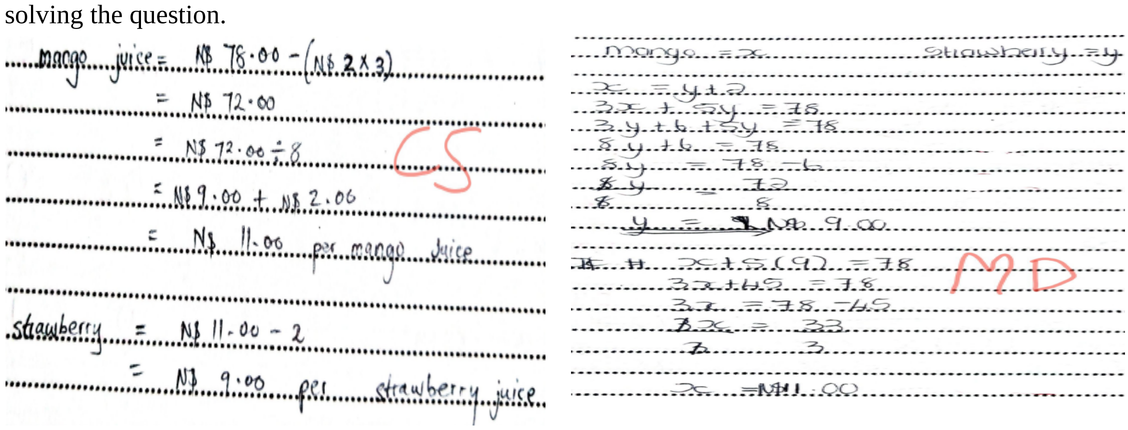
Strawberry $=$ $(78 - 6)/8$

Strawberry $=$ $N\$9$

Then Mango $=$ $N\$9 + N\$2 = N\$11$

Learners could also use the strategy of Guessing, Checking and Revising to solve the problem.

It was found that only 99 (28%) of the learners employed strategy MD in solving the question, while 35 learners (10%) employed CS. The TCL and GCR strategies were employed by 1 learner (0.3%) and 3 learners (1%) respectively. Figure 3 shows some examples of learners who used CS and MD strategies in solving the question.



(a) (b)

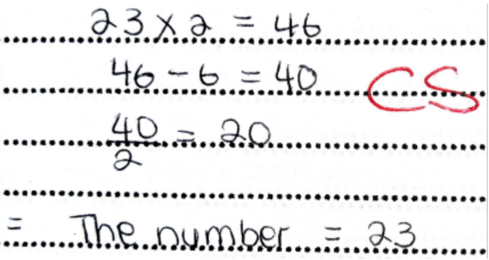
Figure 3. Examples of learner use of CS and MD Strategies in solving question 3.

In Figure 3(a) the learner used CS strategy, although with some mathematical errors, and the learner in (b) used the MD strategy.

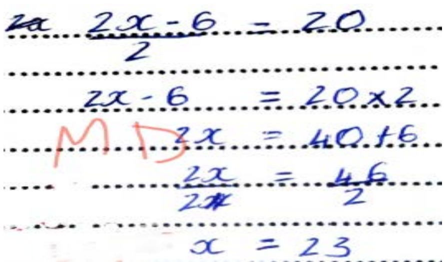
3.4. Strategies used to solve question 4.

The fourth question was "Natalia thought of a number. She doubled the number, then subtracts 6 from the result and divides the answer by 2. The quotient will be 20. What is the number?" This problem can be solved with the strategy of Making a Model or Diagram by deriving an equation from the given problem, for example,
Let the number be x.
Double the number = $2x$,
Subtracting 6 will lead to $2x - 6$.
Divide by 2 to get $(2x - 6)/2$.
Since it is equal to 20 then the following equation can be used to solve the problem
$(2x - 6)/2 = 20$

In solving the question, 161 (46%) learners employed MD strategy, 25 (7%) utilised CS strategy, 16 learners (5%) employed GCR strategy, while the TCL strategy was used by 1 learner (0.3%). Figure 4 shows an example of learners who used CS (a) and MD (b) strategies.



(a)



(b)

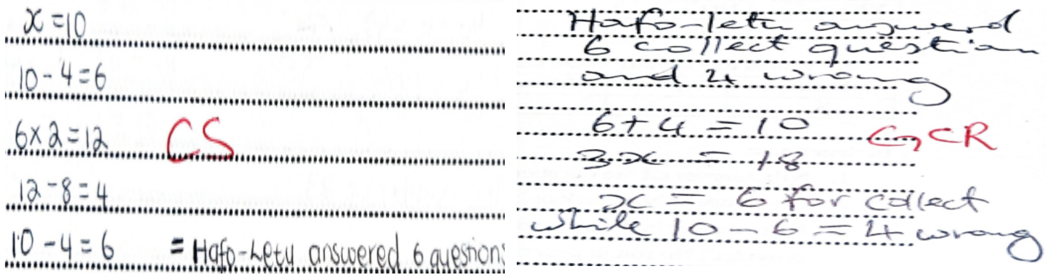
Figure 4. Examples of Learners who used CS and MD Strategies

3.5. Strategies used to solve question 5.

The fifth question was "In a physics quiz you get 2 points for each correct answer. If a question is not answered or the answer is wrong, 1 point is subtracted from your score. The quiz contains ten questions. Hafo-Letu received 8 points in total. How many questions did Hafo-Letu answer correctly?" This problem can be solved by Making a Model or Diagram, where learners have to translate an equation from the given algebraic word problem. For example, let the correct answer be x and the incorrect answer be y.
Incorrect answers added to correct answers is 10 which can be expressed as:
$x + y = 10$ - (1)

Then taking points for incorrect answers from points for correct answers that gave Hafo-Letu 8 points, and this is expressed by:
$2x - y = 8$ - (2)
Add equations 1 and 2 to get:
$(2x - y) + (x + y) = 8 + 10$ this equation can then be used to solve the problem.
The problem can also be solved using GCR.

In solving the question, it was found that MD strategy was employed by 39 (11%) learners, while 18 learners (5%) used CS, and three learners (1%) employed the GCR strategy. No learners used a combination of strategies to solve the question. Figure 5 are examples of learners' use of CS and GCR strategies in solving the question.



(a) (b)

Figure 5. Examples of learners' use of CS and GCR Strategies in solving question 5.

3.6. Strategies used to solve question 6.

The sixth question was "On a farm Tulukeni has goats and chickens. His son counted 70 heads and his daughter counted 200 legs. How many chickens and goats does Tulukeni have? This problem can be solved using Making a Model or Diagram strategy by either translating the word problem into an algebraic equation or by drawing a picture.

For example: Let the number of chickens be x and the number of goats be y

For heads: Since a chicken and a goat each has one head, then

$x + y = 70$ - (1)

For legs: Since every chicken has 2 legs, every goat has 4 legs. then

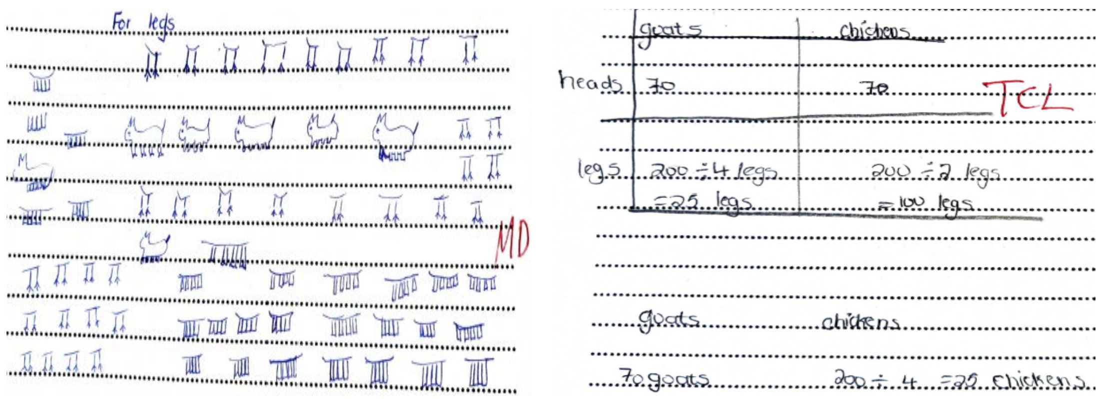
$2x + 4y = 200$ - (2)

Substitute equation (1) into equation 2 to get:

$2(70 - y) + 4y = 200$

The equations can be used to solve the problem.

It was found that 75 (21%) learners employed MD, 24 (7%) used CS, only two (0.6%) learners employed TCL strategy and 5 (2%) used the GCR strategy to solve the question. The use of combination of TCL and CS, and MD and CS in solving the question were below 1%. Figure 6 shows samples of learners' work who used MD and TCL strategies to solve the question.



(a) (b)

Figure 6. Examples of learners use MD and TCL strategies in solving question 6.

In (a), the learner made a model of goats and chickens drawing goats with one head and four legs and chickens with one head each and two legs. The learner in (b) shows the use of TCL by presenting the information in a table.

The strategies employed by the learners in solving the problems are summarised in Table 1.

Table 1. Learners' strategies in solving the algebraic word problems.

	Question													
	1		**2**		**3**		**4**		**5**		**6**		**Average**	
Strategy	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**	**n**	**%**
MD	134	38.2	106	30.2	99	28.2	161	45.9	39	11.1	75	21.4	10 2	29.1
CS	24	6.8	62	17.7	35	10.0	25	7.1	18	5.1	24	6.8	31	8.8
TCL	1	0.3	0	0	1	0.3	1	0.3	0	0	2	0.6	1	0.3
GCR	12	3.4	12	3.4	3	0.9	16	4.6	3	0.9	5	1.5	9	2.6
TCL/CS	1	0.3	0	0	1	0.3	0	0	0	0	2	0.6	1	0.3
MD/CS	1	0.3	1	0.3	2	0.6	0	0	0	0	3	0.9	1	0.3

The result shown in Table 1 is the summary of the strategies used by learners in solving the problems in the test. The frequencies and the percentage calculated are shown in Table 1. The MD strategy was the most used by the learners with an average percentage of 29% followed by CS strategy with an average of 9% usage, while the least used strategy was TLC with an average of 1%. On the other hand, the use of combined strategies was an average of 0.3%.

The result shows that the learners in this study used MD, CS, TCL, and GCR strategies to solve the algebraic word problems test. Since the test only had word problems, not all strategies could be used. The findings revealed that the most used strategy in almost all questions was MD, where learners translated the word problems into algebraic equations. This was followed by CS, where learners had to apply the arithmetic rules and use the order of operations, while GCR was infrequently used. The least used strategy was TLC. Very few learners were observed employing combinations of some of the strategies, such as TCL/CS, and MD/CS.

Some learners used more than one problem-solving strategy, while others used only one. The result is similar to that of Celebioglu et al. (2010). The results of this study show that the learners lacked the strategies to solve algebraic word problems. The findings of this study corroborated the findings of Elia et al. (2009). Solving non-routine mathematics problems (NRMP) like word problems necessitates adopting strategies (see, e.g., Elia et al., 2009; Polya, 1985; Schoenfeld, 2016), although those strategies do not guarantee a solution. Between 20% to 46% of the learners used the strategy of MD in all the problems except for Problem 5. Between 5% and 20% of the learners employed the CS strategy in solving the problems. The average usage of combined strategies was less than 1%.

4. Conclusion

The results of this study show that the learners lacked the appropriate strategies to solve algebraic word problems. Most of the learners were unable to solve the problems using appropriate Problem-Solving Strategies. They employed MD, CS, TCL, and GCR strategies to solve the test problems. The most used strategy was MD followed by CS. The least used strategies were GCR and TLC. Very few learners employed a combination of strategies, such as TCL and CS, and MD and CS. It is therefore recommended that mathematics teachers teach learners how to solve mathematics problems in general and algebraic word problems in particular, using different problem-solving strategies by modeling the strategies for learners.

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