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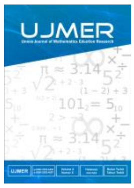

Didactical Situations of Students' Mathematical Reasoning Based on the Learning Obstacle on Quadrilateral Areas

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Abstrak

This study aimed to: (1) identify some learning obstacles experienced by students related to the topic of rectangular areas, (2) find out the didactical design of mathematical reasoning to develop a didactical situation, (3) describe the didactical situation of mathematical reasoning in learning process, and (4) increase mathematical reasoning ability through the learning of didactical design. The method employed in this study was a qualitative method with Didactical Design Research (DDR) approaches consisting of three formal stages. The results showed that (1) there were four types of learning obstacles experienced by students about the concept of quadrilateral area, (2) the didactical design that has been prepared facilitated the students construct the area of quadrilateral through the triangle areas approach, (3) the didactical situation developed consisted of 12 didactical situations along with predictions of student's responses and anticipation, (4) students' learning obstacle could be minimized and students' mathematical reasoning abilities had been increased. Thus, it could be concluded that the didactical design of mathematical reasoning could be used as a guideline for educators to design a didactical situation in mathematics learning activities.

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INTRODUCTION

Mathematical reasoning abilities of junior high school students are still under the low category. The reasoning and character of students are still limited because of the learning activites applied in schools still unable to empower the potential of students optimally (Santyasa et al., 2015; Sulistiawati et al., 2015). In the other hand, a reasoning ability is necessary especially for didactical situations to support the mental activities that lead to the formation of new mental objects (Suryadi, 2010). It means that a reasoning ability is an important domain in learning mathematics, especially related to geometry topics.

Geometry topics were identified as a difficult material for most teachers and students (Adolphus, 2011). Students also have difficulties in understanding the basic concepts of geometry and they actually learn geometry without understanding the basic of terminology (Halat et al., 2008). In addition, 7th grade secondary school students have a number of misconceptions and lack of knowledge on the topic of geometry, because the geometry topics are more complex than numerical or algebraic operations (Özerem, 2012). Thus, geometry becomes the most common teaching and learning problems in mathematics.

These difficulties arise due to the instant delivery of mathematical concepts as a readily-made product so that students' knowledge of the mathematical context becomes limited. If the mathematical concept is directly given in the form of final results, it can cause students to experience the difficulties in learning mathematics (Sarah et al., 2017). Therefore, mathematics should not be seen as a product but as an activity. So the focus of learning mathematics is not merely on the final results, but also on the process of forming mathematics itself.

Two fundamental aspects of mathematics learning activities according to Kansanen (2003) are the relationship between students and material (didactical relationships), and the relationship between students and teachers (pedagogical relationships). But, the didactic and pedagogical relationships cannot be viewed partially because the both of them can occur simultaneously (Suryadi, 2010). So, it is necessary to add an anticipatory relationship between the teacher and the material referred to as a didactic and pedagogical anticipations (ADP). Thus, the didactical situations can be understood as a pattern of interactivity between students, teachers, and material in one learning environment.

The pattern of interaction between students, teachers, and material needs to be developed in a learning plan. Learning devices must be made in accordance with the responses and characteristics of students so that learning activities are more effective, it is called the didactical design (Fauzia et al., 2017). In didactical design, teachers are required to predict the various student responses from each didactical situation created, accompanied by anticipation of the responses given by students (Rizqiyani et al., 2017). So that when learning takes place, all student responses can be anticipated by the teacher well. Therefore, teachers need to increase their knowledge through repersonalizing and reconstructing the materials in order to be able to design a learning process which is appropriate with the flow of students' thinking.

The aspect that must be considered in developing the didactical and pedagogical anticipations is the existence of learning difficulties. Brousseau (2002) offered the classification of learning obstacles in three types comprising: 1) ontological obstacle (due to limited learning development); 2) didactical obstacle (due to the learning system); and 3) epistemological obstacle (due to students' knowledge is limited to certain contexts). In the other side, identification of learning obstacles can be used as a reference for the teacher to determine the prediction of student responses based on the given didactical situations, because students have experienced certain obstacles or difficulties historically.

Based on the explanation above, this study intended to: 1) identify the learning obstacle experienced by students related to the topic of quadrilateral area (parallelogram, rhombus, kites, and trapezoid); 2) find out the didactical design of mathematical reasoning to develop the didactical situations; 3) describe the didactical situations of mathematical reasoning in learning of quadrilateral areas (parallelogram, rhombus, kites, and trapezoid);

4) increase mathematical reasoning ability through the learning of didactical design.

METHOD

This research was a qualitative research using Didactical Design Research (DDR) approach which consists of three stages, these were: 1) the analysis of the didactical situations before learning; 2) metapedadidactical analysis; 3) retrospective analysis (Suryadi, 2013). The focus of this research was to develop didactical design and investigate the didactical situations of mathematical reasoning related to the quadrilateral areas (parallelogram, rhombus, kites, and trapezoid).

The forming of didactical design was preceded by the repersonalization and reconstruction of the material carried out by reviewing various literatures such as textbooks, some books related to the concept of geometry and various formal evidences in conctructing the formula of quadrilateral areas. In addition, the didactical design was prepared based on the result of identification of learning obstacle experienced by students regarding the prerequisite material for the area of quadrilateral.

Identification of learning obstacle was carried out by testing the diagnostic test of mathematical reasoning ability to students 7th grade of SMP Negeri 7 Cirebon as the research subjects. Then interviews were conducted with several identified subjects who had difficulties in answering questions. The interviews activity was conducted to find out the detail of obstacles or difficulties experienced by students in answering the diagnostic test. Based on the data that obtained from the results of repersonalized and learning obstacle identifications, the didactical situations about the area of parallelogram, rhombus, kites, and trapezoid was designed then, accompanied by predictions of student responses and didactical anticipations of the response given. The series of these activities are the early stage of the DDR approach, namely the analysis of didactical situation before learning.

The next stage was metapedadidactical analysis conducted by analyzing the didactical situations that developed in the classroom during the implementation of didactical design, analyzing the learning situations as a students' response to the didactical situation developed, and analyzing interactions which have an impact on both didactic and learning situations. The final stage of this study is to analyze the suitability between student response predictions, which is formed in didactical design with the results of metapedadidactical analysis, confirming the accuracy of didactical anticipation with predictable student responses or new responses that arise during implementation.

Data collection techniques used in this study were observation, interviews, and documentation studies. The instruments utilized in this study consisted of diagnostic tests for early and final mathematical reasoning abilities, and didactical design of mathematical reasoning in the form of worksheet as material for student discussion. The diagnostic test instrument for early mathematical reasoning abilities contains the prerequisite material, while the diagnostic test instrument for the final mathematic reasoning ability contains the topic of quadrilateral areas.

RESULTS AND DISCUSSION

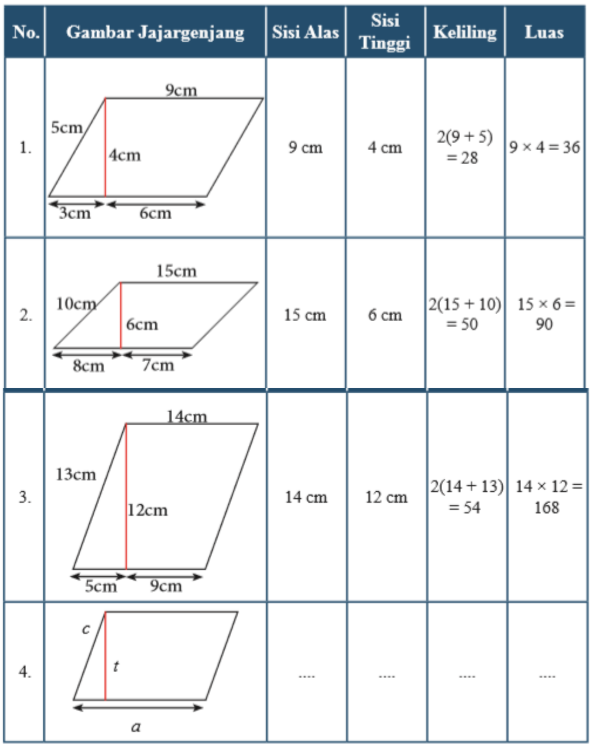
Learning Obstacle Findings

Based on the results of the students' answers to the diagnostic test of early mathematical reasoning abilities and the results of interviews with several students, several learning obstacle that was experienced by students were found as follows:

1) A total of 29 students experience learning obstacle about concept image in determining the base and height of a triangle
2) Three students experience misconception in using the concept of point and line, all of students experience misconception about algebra and comparison.
3) A total of 15 students experience learning obstacle in combining information from the mathematical problems given, referred to vertical mathematization
4) A total of seven students experience several errors about the procedure which is used to solve the problem given.

Repersonalization

A repersonalization study of some literatures (textbook and some books related to the concept of geometry) aims to be able to reconstruct the material about the area of pararellogram, rhombus, kites, and trapezoid. The book exploited by teachers and students 7th grade of SMP Negeri 7 Cirebon is "Buku Matematika SMP/MTs Kelas VII Semester 2" which published by the Ministry of Education and Culture. The concept of parallelogram, rhombus, kites, and trapezoid areas in the book are explained through a concrete approach to abstracts. The sample can be seen in Figure 1.



No.	Gambar Jajargenjang	Sisi Alas	Sisi Tinggi	Keliling	Luas
1.	9cm, 5cm, 4cm, 3cm, 6cm	9 cm	4 cm	2(9 + 5) = 28	9 × 4 = 36
2.	15cm, 10cm, 6cm, 8cm, 7cm	15 cm	6 cm	2(15 + 10) = 50	15 × 6 = 90
3.	14cm, 13cm, 12cm, 5cm, 9cm	14 cm	12 cm	2(14 + 13) = 54	14 × 12 = 168
4.	c, t, a	....	....	....	....

Figure 1. Construction the Formula of Parallelogram Circumference and Areas
(Source: Buku Matematika SMP Kelas VII, page: 224)

Based on Figure 1, it can be seen that the procedure to determine the area of parallelogram in these book is through a concrete example repeatedly. Then from some of these examples, students are asked to conclude in general by changing the known elements with a certain symbol, so that a formula of the area is obtained. The procedure presented basically used the formula of parallelogram area, so that the implemented activity is not constructing the formula of parallelogram area. Moreover, the construction of the formula's area through several examples can bring up the concept image in the students' thinking flow, so that if the student is given a picture of another level that is not in accordance with the sample, then the student will experience the difficulties. The concept image is activated when the concept image raised, then at different times students are given the different-looking images that are essentially same, a students' cognitive conflict will arise (Tall & Vinner, 1981).

The deepening of the material in the book was developed through the discussion activities which support students to practice their reasoning skills. This was identified in the "Let's reason" section, which presents several problems leading the students to construct the formula of parallelogram, rhombus, kites, and trapezoidal areas. The first problem is that the students are asked to find the formula of parallelogram, rhombus, kites, and trapezoidal areas by using the square area approach. However, this procedure does not apply to all forms of parallelograms. The steps of proving the parallelogram areas using the rectangle areas approach is only valid for parallelogram as illustrated in Figure 2, yet is invalid for the parallelogram as illustrated at Figure 3 (Moise, 990),.

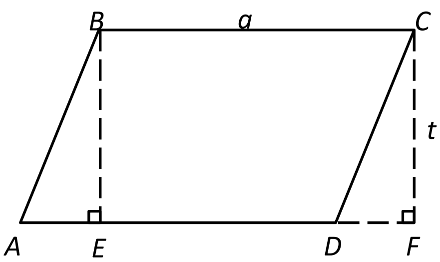


Figure 2. Sketch of proving the parallelogramareas based on quadrilateral areas

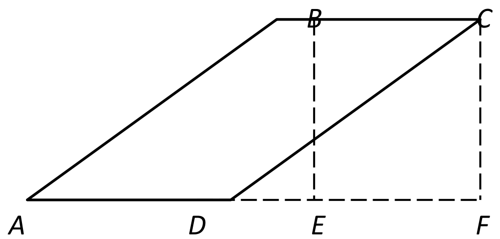


Figure 3. Construction sketches of parallelogram areas that cannot be proven through the quadrilateral areas

Thus, it can be concluded that the construction of parallelogram, rhombus, kites, and trapezoidal areas through area of quadrilateral approach is not generally applicable. The second problem is that students are asked to determine the area of parallelogram, rhombus, kites, and trapezoid by cutting the plane into several parts (at least two parts). This problem is not explained the terms of cutting the specified plane. It allows students to cut parallelogram, rhombus, kites, and trapezoidal shapes in very diverse forms. So that the process of construction the formula of areas becomes complicated and requires a long time.

The composition of the material presented in the book related to the topic of plane begins with quadrilateral and then continues with triangular material. It is true that we got the formula of triangular area from the formula of rectangular area. But, a triangular area cannot be expressed as the union of a finite number of the rectangular areas. So, the area of polygon is the sum of a finite number of triangular regions (Moise, 1990). It shows that the triangular concept should be delivered before the quadrilateral concept. In the other words, triangular concept can be the one of prerequisite material that must be managed by students as a provision to learning about the quadrilateral concept.

Didactical Design

The didactical design of mathematical reasoning about the subject of parallelogram, rhombus, kites, and trapezoidal areas was organized based on the results of the repersonalization of the material and learning obstacle findings. Because the area of polygon can be obtained from a finite number of triangular area (Moise, 1990), it means that the area of quadrilateral can also be obtained from a finite number of triangular area. It underlies the researchers in compiling a didactical design of mathematical reasoning in the form of constructing the area of parallelogram, rhombus, kites, and trapezoid using a triangular area approach. The procedure is by cutting parallelogram, rhombus, kites, and trapezoid into a finite number of triangular regions that only intersect in its edges and vertices.

Since there will be many triangular regions obtained, then the researcher limits only two formed triangular regions. Thus, what the students have to do is cutting the quadrilateral based on one of the diagonals, so that two triangles are obtained. Next, the construction of the prallelgram, rhombus, kites, and trapezoidal areas is obtained by summing the area of two triangles. Didactical situations at didactical design are arranged with the main objectives of learning is construction of parallelogram, rhombus, kites, and trapezoidal area through the triangular areas approach. Didactical design is presented in the form of worksheet consisting of 12 didactical situations. The didactical situations 1, 4, 7, and 10 are related to the shape and elements of parallelogram, rhombus, kites, and trapezoid. Didactical situations 2, 5, 8, and 11 is related to the construction of the area. Didactical situations 3, 6, 9, and 12 are concerning mathematical problems regarding the concept of parallelogram, rhombus, kites, and trapezoidal area.

Didactical Situations in Learning

Most students experienced learning obstacle in the form of concept image in determining the base and height of triangle in didactical situation 2. This response is in accordance with the predicted response, so that the anticipation is given agrees with the planned anticipation. The anticipation is given by providing assistance for students regarding the base and height of the triangle. Because the possibility of learning obstacle is predicted to incessantly to appear in any didactical design implementation, the researcher anticipates it by giving apperception about how to determine the base and height of the corresponding triangle.

Apperception about the base and height of the triangle was given since the first meeting, and the apperception was given again at the second meeting. This apperception has implications to decrease the students' learning obstacle in determining the base and height of triangle, it was seen at the next meetings. This is in accordance to the research results of Puteri (2018) that apperception is a psychological interpretation of the mind which is combined with students' observations and experiences, and it is believed to be one of the strategies that make the learning process would be succeed.

Another learning obstacle that arises during the didactical design implementation is students' misconceptions regardingthe algebraic concept. Student difficulties related to algebraic concepts are often found in the form of difficulties that arise during the implementation of the procedure. For example, students cannot reduce the factors from algebraic forms, and do the algebraic multiplication and addition operations. These obstacles were found in the didactical situation related to the construction of the area and the didactical situation regarding the mathematical problems about the concept of quadrilateral area. This is accordance with the predicted response.

In general, the anticipation given to minimize the learning obstacle about the algebraic concepts is by guiding students to conducting algebraic operations, helping students to recall how to operate algebraic forms by grouping the same variables, or by using examples of certain symbols which is associated with the daily life concept. Mathematical representations and symbols can form a very important semiotic system in students (Bussi & Bazzini, 2016). The anticipation given is quite effective in minimizing the learning obastacles about the algebraic concepts. It is evident that during the didactical design implementations, where students' knowledge of algebraic concepts are increasing and the difficulties about the algebraic concept are diminishing.

Students' misconceptions regarding the concept of mathematical comparison were also found when didactical design was implemented. Some of these misconceptions are because the students forget about the concept of comparison. Some other students have limited knowledge related to the concept of comparison, so students do not know the steps to be taken when they faced a didactical situation about the concept of comparison. This difficulty is found in didactical situations 3, 6, 9, and 12. Anticipation that given for the response is to guide the students regarding the concept or mathematical comparison. Guidance about the concept of comparison which is given in didactical anticipation is associated with the concept of algebra as comparisons expression. Using standard algebraic symbolism can express generality (Radford, 2000). However, student difficulties related to the concept of comparison cannot be minimized properly.

Some students have difficulty in making conclusions about the activities that have been carried out on worksheet. The difficulties that was experienced by some students are verbal difficulties in composing sentences about the results obtained. Researchers have predicted this response, so that the anticipation of the response has been prepared. The anticipaton given by asking some reflective questions to students that aims to guide students to be able to explain a new knowledge obtained by students based on the activities that have been done. The anticipation was chosen based on Prabawanto & Mulyana (2017) that giving questions can help students who experience difficulties during learning activities.

The anticipation is a process of transfer or application of schemata into a situations before new situations actually occur (Lim, 2006). The didactical anticipation that was carried out in this study is the anticipation that was given to students' responses to the didactical situation. The tecaher's activity (in this case the researcher) in providing the didactical anticipation is only limited to be a facilitator who bridges the students' thinking process, not be done by transferring knowledge totally to students. So that the learning of mathematics developed is as the result of the students' work and ideas, not as the observation result of experimental lessons.

Most of students' responses for each didactical situation provided are in accordance with student response predictions designed before learning activities, so the anticipation that needs to be given has been prepared. There are several responses that occurred outside of predictions, such as errors to do the number counting operations. However, this response can be overcome by spontaneous anticipation so that the didactical situation can still run smoothly.

The pretest and posttest results that were tested to 30 students of 7th grade of SMP Negeri 7 Cirebon showed that the mathematical reasoning abilities of all students are increased after learning using didactical designs. Recapitulation results of the calculation about the index gain of students' mathematical reasoning abilities can be seen in Table 1.

Table 1 Recapitulation of Gain Index Results

Gain Criteria	Students amount
Low	3
Medium	24
High	3

CONCLUSION

Learning obstacle was found from the results of diagnostic test of mathematical reasoning abilities is categorized didactical obstacle and epistemological obstacle. The didactical design developed can minimize students' learning obstacle, as seen from the decreasing of learning obstacle experienced by students in each learning activity. Moreover, the didactical design of mathematical reasoning can improve students' mathematical reasoning abilities, especially about the concept of parallelogram, rhombus, kites, and trapezoidal areas. The didactical situation is dynamic, because when students respond to a certain didactical situation then proceed with providing the necessary didactic or pedagogical anticipatory actions, there will be a new didactical situations and these conditions develop throughout the learning process. Therefore, the theory of didactical situations can continue to be studied along with the evolving individuals as well as the extensive of mathematical knowledge and the evolving of educational system. The recommendation for further research is that a didactic situation study can be carried out based on learning trajectory.

REFERENCES

Adolphus, T. 2011. "Problem of Teaching and Learning of Geometry in Secondary Schools in Rivers State, Nigeria". *International Journal Emerging Sciences,* 1(2): 143 – 152.

Brousseau, G. 2002. *Theory of Didactical Situations in Mathematics.* New York: Kluwer Academic Publishers.

Bussi, M., & Bazzini, L. 2016. "Research, Practice and Theory in Didactics of Mathematics: Towards Dialogue between Different Fields". *Educational Studies in Mathematics,* 54(2-3): 203 – 223.

Fauzia, T.A., Juandi, D., Purniati, T. 2017. "Desain Didaktis Konsep Barisan dan Deret Aritmetika pada Pembelajaran Matematika Sekolah Menengah Atas". *Jurnal Pendidikan Matematika Indonesia,* 1(1): 1 – 10.

Halat, E., Jakubowski, E., Aydin, N. 2008. "Reform-based Curriculum and Motivation in Geometry". *Eurasia Journal of Mathematics, Science & Technology Education,* 4(3): 285 – 292.

Kansanen, P. 2003. "Studying-the Realistic Bridge Between Instruction and Learning. An Attemp to a Conceptual Whole oh the Teaching-Studying-Learning Process". *Educational Studies,* 29(2/3): 221 – 232.

Lim, K. 2006. "Student's Mental Acts of Anticipating in Solving Problems Involving Algebraic Inequalities and Equations". *Dissertation.* San Diego: University of California.

Moise, E. 1990. *Elementary Geometry from an Advance Standpoint: Third Edition.* USA: Addison-Wesley Publishing Company, Inc.

Puteri, L.H. 2018. "The Apperception Approach for Stimulating Student Learning Motivation". *International Journal of Education, Training and Learning*, 2(1): 7 – 12.

Prabawanto, S., & Mulyana, E. 2017. "Developing Lesson Design to Help Students' Triangle Conseptual Understanding. *International Conference on Mathematics and Science Education,* 895: 1 – 7.

Radford, L. 2000. "Signs and Meaning in Students Emergent Algebraic Thinking: A Semiotic Analysis". *Educational Studies in Mathematics*, 42: 237 – 268.

Rizqiyani, R., Fatimah, S., Mulyana, E., 2017. "Desain Didaktis Bangun Ruang Sisi Datar untuk Meningkatkan Level Berpikir Geometri Siswa SMP". *Jurnal Pendidikan Matematika Indonesia,* 1(1): 22 – 30.

Santyasa, I., Warphala, I., Tegeh, M. 2015. "Validasi dan Implementasi Model-Model *Student Centered Learning* untuk Meningkatkan Penalaran dan Karakter Siswa Sekolah Menengah Atas". *Jurnal Pendidikan Indonesia,* 4(1): 512 – 527.

Sarah, S., Suryadi, D., & Fatimah, S. 2017. "Desain Didaktis Konsep Volume Limas pada Pembelajaran Matematika SMP Berdasarkan

Learning Trajectory". *Jurnal Pendidikan Matematika Indonesia,* 1(1): 31 – 41.

Sulistiawati, Suryadi, D., & Fatimah, S. 2015. "Desain Didaktis Penalaran Matematis untuk Mengatasi Kesulitan Belajar Siswa SMP pada Luas dan Volume Limas". *Kreano,* 6(2): 135 – 146.

Suryadi, D. 2010. "Penelitian Pembelajaran Matematika untuk Pembentukan Karakter Bangsa". *Makalah.* Seminar Nasional Matematika dan Pendidikan Matematika di Universitas Negeri Yogyakarta. Yogyakarta, 27 November 2010.

Suryadi, D. 2013. "Didactical Design Research (DDR) dalam Pengembangan Pembelajaran Matematika". *Prosiding*. Seminar Nasional Matematika dan Pendidikan Matematika. Cimahi: STKIP Siliwangi.

Tall, D., & Vinner, S. 1981. "Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity". *Educational Studies in Mathematics,* 12: 151 – 169.

Özerem, A. 2012. "Misconception in Geometry and Suggested Solutions for Seventh Grade Students". *International Journal of New Trends in Arts, Sports & Science Education,* 1(4): 23 – 35.