



## The APOS Content Basic Algebra Module to Improve the Learning Autonomy of Achieving Mathematics Connection Skills

Krisno Budi Prasetyo <sup>✉</sup>, YL. Sukestiyarno, Adi Nur Cahyono

Pascasarjana, Universitas Negeri Semarang, Indonesia

### Article Info

Article History:  
Received :  
5 April 2020  
Accepted:  
6 June 2021  
Published:  
30 December 2021

Keywords:  
Module, APOS  
theory, Independence  
Character,  
Mathematical  
Connection, Basic  
Algebra

### Abstract

This research aims to develop an APOS content-basic algebra for autonomous learning to achieve the mathematics connection skills that meet the instrument validity and implementation effectiveness criteria. The module included the basic concept of basic algebra as the learning reference for learners. Thus, learners could refer to the materials autonomously. The module was arranged systematically and complemented with detailed stages. They could facilitate the readers to act and to try the process of analogy. A similar stage also occurred on an *object*, the new problem, to create a theoretical framework in the form of a *scheme*. This research used a research and development design. The stages were defining the algebraic problems, designing the APOS-based module, creating a product draft, validating the module, and examining the effectiveness. The researcher conducted the field test with the learner representations from primary school, Junior High School, and Vocational High School. The research variables were learning autonomy and mathematics connection skills. The research used questionnaires and tests to collect the data. The data were then analyzed descriptively analytically. The experts' judgment obtained a score of 4.08, with a valid criterion. Then, the product was tested for the learners' representations. The results showed their learning autonomy achieved scores of 73 (the primary school learner), 75 (the JHS learners), and 81 (the VHS learners). On the other hand, the mathematics connection skill scores were 75 (the primary school learner), 80 (the JHS learners), and 85 (the VHS learners) It showed that learning with the APOS content module had positive responses and achieved the expected results (effective).

<sup>✉</sup>Correspondence:  
Gedung A Kampus Pascasarjana UNNES, Jl. Kelud Utara III  
Semarang 50237, Indonesia  
E-mail: krisnobp@gmail.com

## INTRODUCTION

The 2013 curriculum indicates the importance of paradigm change in mathematics learning. It shifts the teachers' roles to be learning motivators. Teachers are required to provide learning opportunities for the learners so that they can construct their knowledge from the given activities, such as mathematics-problem solving skill activities. Learners cannot only rely on listening and noting activities during the learning process as seen at typical schools. They need activities that could change their attitudes and behaviors during the learning process. Learning activity covers both physical and mental activities that are correlated to each other.

The learning materials provide freedom for the learners and facilitate them to construct their knowledge. Learning materials are any means to facilitate teachers and infrastructures to promote teaching-learning activities in the classroom (Depdiknas, 2006). One of the learning material types is a module. It facilitates learners to study autonomously in understanding the materials. Nurhasanah (2017) argued that modules could be designed and arranged systematically to allow learners to learn autonomously. The developed learning material in this research was the APOS-based reference module.

APOS theory is a constructivism-based theory. Arnon et al., (2014:17) explained that APOS was a model to describe how mathematics concepts could be learned. According to Kalapana (2014), constructivism refers to learners-active role emphasized perspective in constructing the knowledge and defining the existing information. An understanding of mathematical concepts on APOS theory is used as an elaboration about mental constructions of action, process, object, and scheme (Dubinsky & McDONald, 2002). Mental construction in this context refers to the interiorized action in the process. Then, it is summarized or encapsulated into objects. Then, the objects are de-encapsulated

or re-explained into processes. The action, process, and object could be organized into a scheme. Suryadi (2010:5) argued that the terms, action, process, object, and scheme, were essentially mental construction of individuals to understand a mathematics idea.

APOS theory implementation in this module considered the educational condition of Indonesia. This module was arranged systematically with inter-topic connections, the guideline to work, and examples. The next stage was inviting the learners to do the process. Thus, this module contained the patterned questions and exercises to make learners thinking. Then, the module invited the learners to do the objects. Therefore, the module contained supportive concepts to elaborate and analyze the problems. The learners could also do the scheme. They did it from what they learned. Therefore, the arrangement of the module had inter-related information to differ the concept matter and non-conceptual matter.

This research expected the APOS-based theory module could encourage the learners' autonomous character. This module guided the learners to develop their autonomy. Ranti (2017) argued that learning autonomy functions as skills to manage and manipulate knowledge in a learning process to monitor learning process improvement. Autonomous learners will take responsibility while learning. It is in line with Rini et al (2015). They found that learning autonomy was an individual's behavior. It was realized into creativity in learning, freedom, and belief to act based on the applied values, and responsibility in every learning activity.

The module also encouraged the learners' mathematics connection skills by habituating them to work on connective questions. Thus, learners with excellent autonomy could promote the connection. According to Resti Yulianti et al (2018), mathematics connection and autonomy were correlated. Autonomous learners could promote mathematics connection excellently.

Mathematics connection covers the skills to understand the mathematics concept, to explain the concept connection, and to apply the concept. Mathematics connection refers to a required skill for learners to master. Kenedi, et al. (2019) found that mathematics connection was a part of the knowledge network. It is connected with other knowledge of crucial concepts to understand and develop the connection among mathematics ideas, concepts, and procedures. Without mathematics connection, learners would memorize many concepts. It was in line with Rusmini & Surya (2017). They found that without mathematics connection, learners had to learn and memorize many concepts and procedures separately. It showed the importance of mathematics connection.

The learners' skills to connect the mathematics concepts and topics were still low. It was in line with a study by Saminanto & Kartono (2015). They found that the school learners' mathematics connection skills were low with a percentage of 34%. Therefore, it required a solution to develop the JHS learners' mathematics connection as demanded.

The APOS theoretical framework implementation was expected to habituate learners in solving problems. Thus, they could develop their knowledge in connecting the concepts. The clear concepts and the learners' habits to construct their knowledge based on APOS theoretical framework would facilitate them to learn autonomously. It meant the theory encouraged the learners' learning autonomy and mathematics connection skills. Therefore, this research developed the module with the APOS theory to facilitate learners in studying mathematics.

This research focused on the APOS theory implementation on the module with autonomous character and mathematics connection about algebra. This module could be used as a reference for learners to understand algebra at school so that learners at JHS and SHS could use the module as a reference in learning algebra.

This research aims to develop an APOS content-basic algebra for autonomous learning to achieve the mathematics connection skills that meet the instrument validity and implementation effectiveness criteria.

## **METHOD**

This research was a Research and Development based on Borg & Gall's stages. However, it was until the sixth stage, the limited field test stage. The applied sampling technique was purposive sampling. The subjects were five primary school learners, 10 JHS learners, and 8 VHS learners. The object was the APOS theory-based module for algebra material at schools. The techniques of analyzing the data: 1). Mathematics-module validity analysis, 2) The learners' practicability analysis.

## **RESULTS AND DISCUSSIONS**

The judgment done by the validators consisted of these indicators: The whole module, APOS theoretical construction, autonomy characters, and mathematics connection skills. The revisions were done based on the given suggestions. The results of school-algebra module validation are shown below.

**Table 1.** the Module Validation Results

Module Validation	Judgment				
	I	II	III	IV	V
The module validation by each validator	4.25	4.25	4.00	4.35	4.25
The module validation criteria based on every validator	Extremely excellent	Extremely excellent	Excellent	Very Excellent	Extremely excellent
Module Validation (VM)	4.08				
Module validity criteria	Excellent				

Table 1 shows the average score of each assessed aspect from five validators. It is higher or equal to 4 ( $\geq 4,0$ ), with the "valid" criterion). The average of the whole module aspect was 4.08, categorized as "valid".

**Table 2.** the Questionnaire Validation Results from Students' Responses

The judged aspects	The average	Categories
Primary Learners' Responses	82.14	Very Excellent
JHS Learners' Responses	80.64	Very Excellent
VHS Learners' Responses	83.82	Very Excellent

Table 2 shows the average of the learners' responses toward the module, 82.14%; the JHS learners' responses, 80.64%, and the VHS learners' responses, 83.82%. It meant the module implementation was excellent.

**Table 3.** the Questionnaire Results of the Learners' Autonomous Characters

The judged aspects	The average	Categories
Primary school learners	73.4	Excellent
The JHS Learners	75.1	Excellent
The VHS Learners	81.25	Excellent
The average	76.33	Excellent

Table 3 shows the average of the learners' responses toward the module, 82.14%; the JHS learners' responses, 80.64%, and the VHS learners' responses, 83.82%. It meant the module could encourage the autonomous character.

The autonomous learning character of primary and Junior High School learners was average during this module implementation. Their learning initiations were average while discussing the materials of the module. They read the materials seriously. The learners also noted

the important matters and asked questions when they encountered difficulties. The learners worked on the module questions while discussing them together. The findings on the autonomous learning need diagnosis showed the primary school learners prepared their stationary. On the other hand, the JHS learners prepared different learning resources to learn with the module. The JHS learners could develop and apply their learning strategies. It could be seen when they worked on their questions. They tried to complete

the tasks and submit them punctually. The JHS learners also revised their answers when they incorrectly did it. They could evaluate which materials they understood and not. It showed their evaluation aspect of autonomous learning.

The autonomous learning of VHS learners during the module implementation was excellent. They initiated their learning without other individuals' assistance. They also read and understood the module before discussing together. Most learners summarized the presented materials from the module. They did it to understand the materials. The VHS learners attempted to work on the module's questions autonomously. They seemed enthusiastic while discussing and paying attention to the researcher's explanations. They also paid attention when their friends explained and answered. Generally, the VHS learners could diagnose their learning needs excellently. This research found the learners could select their learning strategy and seemed enthusiastic to work on the given questions. They were not clumsy to ask questions while encountering difficulties in

understanding the materials or completing the questions. Most VHS learners could evaluate their learning results. They did it on their works if there were errors. The learners could differ between the mastered and the unmastered materials.

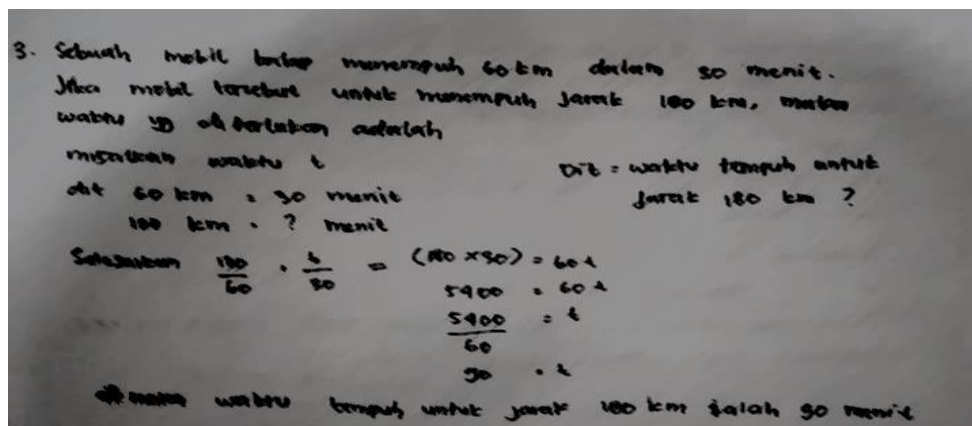
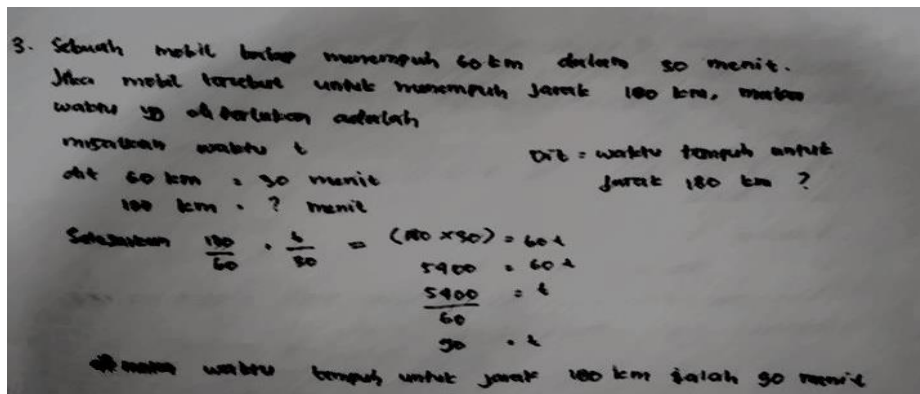
The learners' autonomous learning could not be separated from the module presentation based on APOS theory. It facilitated them to understand the stages from the action until the scheme to help them construct their thoughts. It indirectly developed autonomous characters. Hal ini sejalan dengan penelitian Dwi lestari, dkk (2018), yang menyimpulkan bahwa penggunaan bahan ajar yang berupa LKS berbasis teori APOS membuat siswa tertarik untuk belajar serta membantu siswa dalam memahami materi. Selain itu penelitian yang dilakukan Yerizon (2013) juga menyimpulkan hal yang sama yaitu penggunaan pendekatan Modifikasi APOS dapat meningkatkan kemandirian belajar. Jika kemandirian belajar meningkat, dapat dikatakan bahwa penggunaan pendekatan teori APOS memunculkan karakter kemandirian.

**Tabel 4.** Hasil Tes Kemampuan Koneksi Matematis

The judged aspects	The average	Kategori
Primary school learners	75	Cukup
The JHS Learners	80	Excellent
The VHS Learners	85	Excellent
The average	80	Excellent

Table 4 shows the average of the learners' mathematics connection skill test. The primary school learners obtained a percentage of 75%; the JHS learners obtained a percentage of 80%, and

the VHS learners obtained a percentage of 85%. It meant the module could encourage the autonomous character.



The Answer Analysis of the Connection Questions of JHS Learners

Figure 1 shows the learners could identify the mathematics principles and concepts interdisciplinary. They understood the problems and wrote the recognized parts of the questions. They did it to solve the questions. Then, they

could also use the inter-conceptual connections with the calculation procedure to solve problems of other lessons. They could also solve the questions excellently.

The Answer Analysis of the Connection Questions of VHS Learners

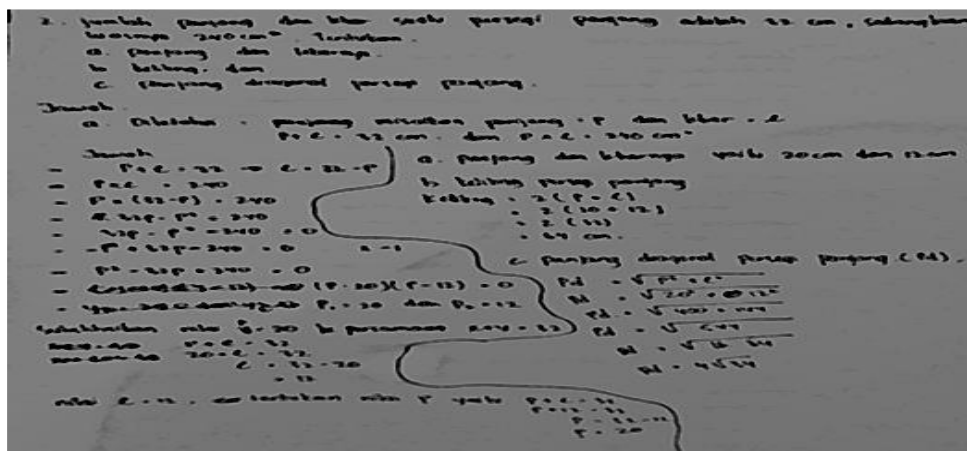


Figure 2 shows The learners' answers showed that they could apply the inter-conceptual connection of mathematics facts to the solved problems. It was seen from the learners' stages to write the recognized parts of the questions and arranged the solution stage. The learners could determine the connection between one mathematics principle to the other principles while determining a rectangular perimeter. Teachers also could apply the mathematics principle connection from one to the other. The researcher found it from the learners' stages to solve problems about the diagonal length of a rectangle.

While learning with the module, the learners had various learners' mathematics connection skills. During applying the module, the learners' mathematics connections were different based on the solutions shared by the answers. Several learners wrote the answers systematically from the recognized, the questioned, and the solution parts systematically. The challenging questions for the learners dealt with the indicators of inter-topic mathematics and mathematics with other science disciplines. They were also habituated to solve the questions. It facilitated them to connect their skills.

The APOS theory facilitated learners to construct the actions, processes, objects, schemes, and connections. The results were in line with the previous findings.

The produced module facilitated them to understand the stages from the action until the scheme to help them construct their thoughts. It indirectly developed autonomous characters. The finding was relevant with Dwi Lestari et al (2018). They concluded that APOS theory-based worksheet could facilitate learners to learn and understand the materials. The APOS theory-based algebra module demanded learners to develop their mathematics connection skills. This finding was relevant to Retno Marsitin (2017). He found APOS theory could improve the learner's mathematics connection skills and creative thinking skills.

## CONCLUSION

The product development obtained excellent validation results from the experts. The score of the validation was 4.08. It indicated the product met the instrument validity criteria. The product development also obtained positive responses based on the questionnaire given to the learners. Product

development could encourage the autonomous character. Based on the questionnaire result, the learners' autonomous average score reached 76 while the learners' mathematics connection score reached 80. The produced module could facilitate learners to understand the materials and enrich their learning experience. Teachers could use this module as a reference for their teaching when they needed it.

## REFERENCES

- Amon, et al. 2004. APOS Theory: A Framework for Research and Curriculum Development in Mathematics Education. New York: Springer.
- Balitbang Puskur Kemendiknas. 2011. Indikator-indikator Keberhasilan Sekolah dan Kelas dalam Pengembangan Pendidikan Budaya dan Karakter Bangsa. Jakarta: Depdiknas.
- Depdiknas. 2006. Standar Isi Mata Pelajaran Matematika SD/MI, SMP/MTs, SMA/MA (Permendiknas Nomor 23 Tahun 2006). Jakarta: Depdiknas
- Dwi Lestari, Darmawijoyo & Aisyah, N. 2018. "Pengembangan LKS Berbasis Teori APOS Materi Bangun Ruang Sisi Datar Konteks Rumah Adat Musi Banyuasin. *Jurnal Kreano*. 9(1), 1-9.
- Ed Dubinsky & Michael A.M. "APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research" in *The Teaching and Learning of Mathematics*. New York: Springer.
- Kalpana, T. 2014. "A Constructivist Perspective on Teaching and Learning: A conceptual Framework". *International Research Journal of Social Science*. 3(1), 27-29.
- Kenedi, Ary Kiswanto, Yullys Helsa., Yetty Ariani., Melva Zainil., & Sherlyane Hendri. 2019. "Mathematical Connection of Elementary School Students to Solve Mathematical Problem". *Journal on Mathematics Education*, 10(1):69-80.
- Listyani, E. 2012. "Implmentasi Model Pembelajaran Matematika dalam Pembentukan Karakter Peserta Didik". Makalah. Dipresentasikan dalam Seminar Nasional Matematika dan Pendidikan Matematika di Universitas Negeri Yogyakarta. Yogyakarta, 10 November 2012.
- Nurhasanah, A. 2017. "Pengembangan Bahan Ajar Pendidikan Matematika untuk Meningkatkan

- Kualitas Pembelajaran Mahasiswa PGSD Universitas Kuningan. *Jurnal Pendidikan Dasar*, 9(2): 68-69.
- Ranti, M.G. 2017. "Pengaruh Kemandirian Belajar (Self Regulated Learning) Terhadap Hasil Belajar Mahasiswa Pada Mata Kuliah Struktur Aljabar". *Math Didactic: Jurnal Pendidikan Matematika*, 3(1): 75-83.
- Resti, Y., Ena Suhena, P., & Muchamad, S. N. 2018. "Pengaruh Model Pembelajaran Missouri Mathematic Project Terhadap Kemampuan Koneksi Matematis dan Kemandirian Belajar Siswa SMP". *Jurnal Elemen*, 4(2): 131-144.
- Retno, M. 2017. Koneksi Matematis dan Berpikir Kreatif dalam Pembelajaran Matematika dengan Teori APOS. *Al-Khwarizmi: Jurnal Pendidikan Matematika dan Ilmu Pengetahuan Alam*, 5(1): 87-100.
- Rini, T. C., Trapsilasiwi, D., & Kurniati. D. 2015. "Implementasi Pembelajaran Berbasis Lesson Study Untuk Mengembangkan Karakter Kemandirian Belajar Siswa Kelas VII C SMP Negeri 9 Jember Tahun Ajaran 2013/2014 Pada Sub Pokok Bahasan Garis dan Sudut". *Kadikma*, 6(2): 109-118.
- Rusmini dan Surya, E. 2017. The Effect of Contextual Learning Approach to Mathematical Connection Ability and Student Self-Confidence Grade VIII SMP Negeri 8 Medan. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, 35 (2), 249-262.
- Siregar, N.D & Surya, E. 2017. Analysis of Students' Junior High School Mathematics Connection Ability. *International Journal of Sciences: Basic and Applied Research (IJSBAR)*, 3(2), 309-320
- Saminanto & Kartono. 2015. Analysis of Mathematical Connection Ability in Linear Equation with One Based on Connectivity Theory. *International Journal of Education and Research*, 3(4), 259-270.
- Yerizon. 2013. "Peningkatan Kemandirian Belajar Mahasiswa Melalui Penggunaan Pendekatan M-APOS". Makalah.Prosiding Semirata FMIPA Universitas Lampung. Lampung: Universitas Lampung.